

Towards $SU(2)$ invariant formulation of the monopole confinement mechanism

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The type of the vacuum is studied numerically in the maximally Abelian (MA) gauge and in the Landau (LA) gauge of $SU(2)$ gluodynamics. The type of the vacuum is determined by a ratio between the dual coherence and the dual penetration lengths. The dual penetration length is determined from correlations between Wilson loops and electric fields in both gauges. The dual coherence length is found from correlations between Wilson loops and dimension-2 operators both in the MA and the LA gauges. This determination of the coherence length is supported by theoretical and numerical observation that the dimension-2 gluon operators in the studied gauges have a strong correlation with the monopole current determined in the MA gauge. We find numerically that the dual penetration lengths and the dual coherence lengths in the LA and the MA gauges are almost the same. Therefore we conclude, that in both gauges the type of the vacuum in the confinement phase is near to the border between the type 1 and the type 2 dual superconductors.

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1. Introduction

It is conjectured that the dual Meissner effect caused by the monopole condensation is the color confinement mechanism [1]. The conjecture seems to be realized if we perform Abelian projection in the maximally Abelian (MA) gauge where Abelian component of the gluon field and Abelian monopoles are found to be dominant [2]. Numerical calculations show that the vacuum of quenched $SU(2)$ QCD ($SU(2)$ gluodynamics) is near the border between the type 1 and the type 2 dual superconductor [3], although there are some claims that it is a superconductor of weakly type 1, Refs. [4, 5]. Since the explicit definition of a dual Higgs field is unknown the coherence length ξ is usually calculated using classical Ginzburg-Landau equations, while the penetration length λ can be calculated directly measuring the correlations between Wilson loops and (non-)Abelian electric fields. Below we show that the coherence length ξ can be derived in the MA gauge also from the measurement of the monopole density around a chromomagnetic flux.

The MA gauge is just one gauge among infinite possible gauges. Since the physics should be gauge-independent, it is important to know the confinement mechanism as well as the type of the vacuum in another gauge. This problem has been discussed recently in Ref. [6] where the Landau (LA) gauge is considered and for Abelian components the dual Meissner effect is observed. A magnetic displacement current plays the role of the solenoidal supercurrent which squeezes the Abelian electric fields. The observation of the dual Meissner effect in the LA gauge suggests that there exists a gauge-independent definition of the monopole condensation.

In order to fix the type of the vacuum in the MA and in the LA gauges we use the following scheme. First, we demonstrate numerically a strong correlation between the dimension 2 gluon operators (namely, the operator $A^+A^-(s) \equiv \sum_\mu [(A_\mu^1(s))^2 + (A_\mu^2(s))^2]$ in the MA gauge and $A^2(s) = A^+A^-(s) + A^3A^3(s)$ in the LA gauge [7]) and the monopole currents $|k_\mu(s)|$ (defined in the MA gauge). Then we show that the monopole density is strongly correlated with the position of the QCD string. Indeed, in the dual Ginzburg-Landau model [8] of the $SU(2)$ vacuum, the expectation value of the (squared) monopole density around the string worldsheet Σ is given by the sum of the solenoidal current and the quantum correction, respectively [9]:

$$\begin{aligned} \langle k_\mu^2 \rangle_\Sigma &\equiv (k_\mu^{\text{string}})^2 + (k_\mu^{\text{quant}})^2 = [\eta^2 m_B K_1(m_B \rho)]^2 + \frac{g^2 |\Phi(\rho)|^4 \Lambda^2}{16\pi^2} + \dots \\ &\rightarrow \frac{g^2 \Lambda^2 \eta^4}{16\pi^2} \left[1 - 4 \sqrt{\frac{\pi \xi}{2\rho}} e^{-\rho/\xi} \right] + \dots \quad [\text{in the limit } \rho \gg \xi]. \end{aligned} \quad (1.1)$$

Here $m_B \equiv 1/\lambda$ is the mass of the dual gauge boson, η is the expectation value of the Higgs field, Λ is an UV-cutoff, and ρ is the distance to the string worldsheet (the string is taken to be infinitely long, straight and static for the sake of simplicity). Moreover, the limit (1.1) demonstrates that the leading behavior of the monopole density at large distances is controlled by the coherence length ξ and not by the penetration length λ .

Thus, coherence lengths in the MA and LA gauges can be found from the correlation between the corresponding dimension 2 operators and the Wilson loops. The comparison of the penetration length and the coherence lengths reveals that they are almost the same. Consequently, we conclude that the vacuum is near the border between the type 1 and the type 2 dual superconductors in the MA gauge. Below we show the results of the numerical simulations which support our conclusion.

2. Numerical results

2.1 Method

We use an improved gluonic action found by Iwasaki [10] which was already implemented in Ref. [6]: $S = \beta \{C_0 \sum \text{Tr}(\text{plaquette}) + C_1 \sum \text{Tr}(\text{rectangular})\}$. The mixing parameters are fixed as $C_0 + 8C_1 = 1$ and $C_1 = -0.331$. We adopt the coupling constant $\beta = 1.2$ which corresponds to the lattice spacing $a(\beta = 1.2) = 0.0792(2)\text{fm}$. The lattice size is 32^4 and we use around 5000 thermalized configurations for measurements. To get a good signal-to-noise ratio, the APE smearing technique [11] is used when evaluating Wilson loops $W(R, T) = W^0 + iW^a \sigma^a$. The thermalized vacuum configurations are gauge-transformed in the MA(+ U1LA) gauge and in the LA gauge.

2.2 MA gauge

The MA gauge is defined by the maximization of the functional $R[U] = \sum_l R_l[U]$, where $R_l[U] = \frac{1}{2} \text{Tr}[U_l \sigma_3 U_l^\dagger \sigma_3]$, with respect to the $SU(2)$ gauge transformations, $U_{x,\mu}^\Omega = \Omega_x^\dagger U_{x,\mu} \Omega_{x+\hat{\mu}}$. In a naive continuum limit one can make an identification of the dimension-2 operator $A_\mu^+ A_\mu^-$ and a lattice quantity: $A_\mu^+(x) A_\mu^-(x) = \frac{1}{2}(1 - R_{x,\mu}[U])$ where no summation over μ is assumed.

The numerical measurements [12] of the local correlation between monopoles and the quantity R_l revealed that the $A^+ A^-$ condensate is enhanced on monopoles. Moreover, according to Fig. 1 the correlation between the $A^+ A^-$ condensate and the monopole is short ranged with the correlation length $\zeta_{\text{cond}} \approx 0.06 \text{ fm}$. Note that $\zeta_{\text{cond}} \approx \zeta_{\text{Action}} \approx 0.05 \text{ fm}$ where the ζ_{Action} is the scale of correlations between the monopole density and the $SU(2)$ action [13].

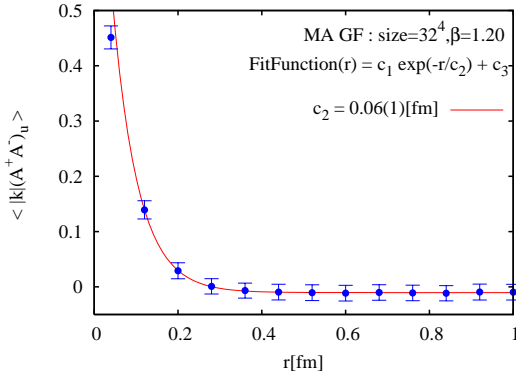


Figure 1: The correlation between the monopole density $|k_\mu|$ and the operator $A^+ A^-$ in the MA gauge. The solid line denotes the best exponential fit.

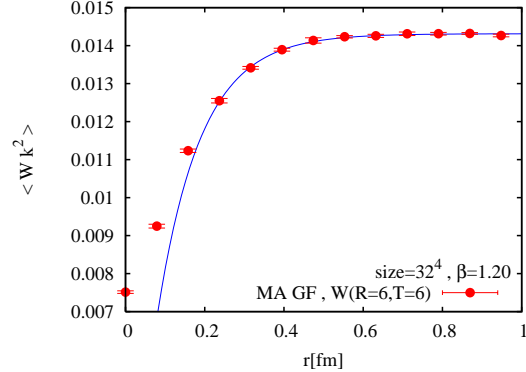


Figure 2: The correlation between the $R \times T = 6 \times 6$ Wilson loop and the square monopole density in the MA gauge. The solid line denotes the best exponential fit.

2.2.1 Correlations of monopoles and condensates with chromoelectric strings

Let us then derive the coherence length in the MA gauge. The correlations between the Wilson loop and the monopole density is plotted in Fig. 2. The static quarks are represented by the Wilson loop $W(R, T)$. The measurements of correlations are mainly done on the perpendicular plane at

the midpoint between the quark pair. An exponential fit of the correlation function provides the correlation length of the vacuum ξ according to our discussion above.

Since the monopoles and the dimension-2 condensate are strongly correlated, the coherence length can also be calculated from the correlations between the Wilson loop and the dimension-2 quantity $[A^+A^-]_\theta = \sum_\mu \{[\theta_\mu^1(s)]^2 + [\theta_\mu^2(s)]^2\}$ which uses the angles $\theta_\mu(s)$ given by the relation $U_\mu(s) = \exp(i\theta_\mu^a(s)\sigma^a)$. The quantity $[A^+A^-]_\theta$ (measured in the MA+U1LA gauge) is identical in naive continuum limit to the quantity $A^+A^-(s)$ defined in the MA gauge. The corresponding correlation function is shown in Fig. 3. We find the coherence lengths determined by the use of the quantities $[A^+A^-]_\theta$ and k^2 coincide within the error bars.

To derive the penetration length we study the correlation of the non-Abelian electric fields which are defined from 1×1 plaquette $U_{\mu\nu}(s) = U_{\mu\nu}^0(s) + iU_{\mu\nu}^a(s)\sigma^a$. A typical example is shown in Fig. 4. Note that electric fields perpendicular to the $Q\bar{Q}$ axis are found to be negligible. An exponential fit of this correlation function provides the penetration length of the vacuum, λ .

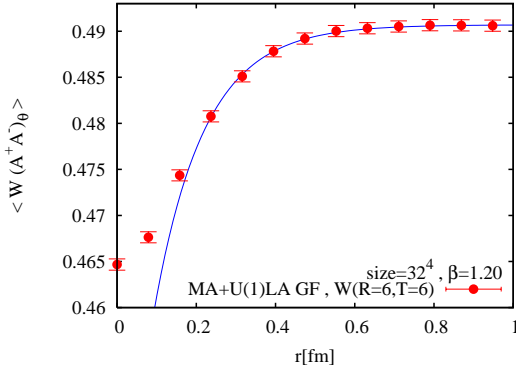


Figure 3: The correlation between the $R \times T = 6 \times 6$ Wilson loop and the $A^+A^-_\theta$ in the MA + U1LA gauge. The solid line denotes the best exponential fit.

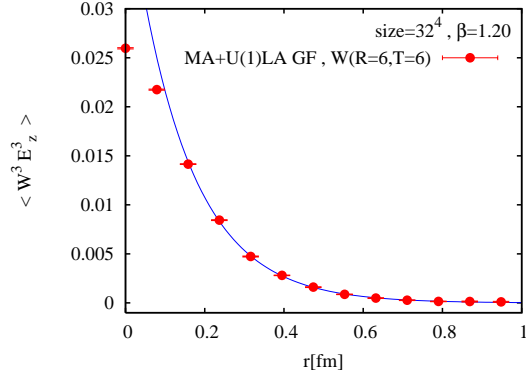


Figure 4: The non-Abelian \vec{E} electric field profile in the MA + U1LA gauge obtained with the use of the $R \times T = 6 \times 6$ Wilson loop. The solid line denotes the best exponential fit.

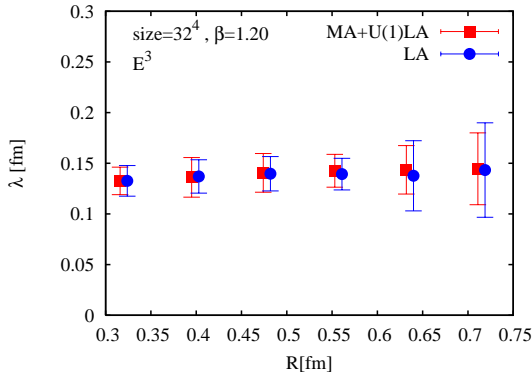


Figure 5: The penetration lengths of the non-Abelian electric field in the Landau gauge and in the MA + U1LA gauge for various R .

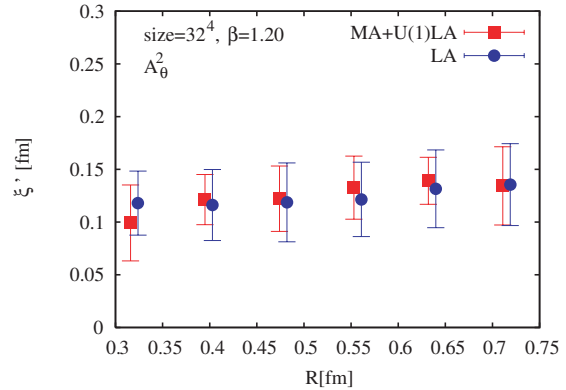


Figure 6: The coherence lengths of the dimension 2 gluon operator in the Landau gauge and in the MA + U1LA gauge for various R .

2.3 LA gauge

In the LA gauge the functional $\sum_{s,\mu} \text{Tr}[U_\mu(s) + U_\mu^\dagger(s)]$ is maximized with respect to all gauge transformations. Similarly to the case of the MA gauge, we find the coherence length from the measurement of the correlation between the chromoelectric string and the dimension-2 operator

$$A^2(s) \equiv \sum_{\mu=1}^4 \sum_{a=1}^3 (\theta_\mu^a(s))^2. \quad (2.1)$$

Indeed, in the LA gauge the operator $A^2(s)$ (or its square-root) is physically relevant and may have information about properties of a dual Higgs field characterizing the QCD vacuum.

The $A^2(s)$ profile around the string in the LA gauge is very similar to that shown in Fig. 3 for the MA gauge. This is very exciting, since the behavior of the profile is just what we expect from a profile of a Higgs field. The coherence length is obtained from the exponential fitting of the correlation function similarly to the MA case.

To derive the penetration length we study the correlation of the Wilson loops with electric fields E_{Ai}^a defined in the LA gauge. The correlations in the LA gauge are very similar to the case of the MA gauge shown in Fig. 4.

2.4 The vacuum type: comparison between MA gauge and LA gauge

In order to study the gauge-(in)dependence of the dual superconductor picture, we show in Fig. 5 the penetration lengths determined in the MA + U1LA gauge and in the LA gauge. We also plot the coherence lengths in Fig. 6. From these figures, we observe that the coherence and correlation lengths calculated in different gauges coincide with each other. Note that we measure the correlation between the gauge-invariant Wilson loop and the gauge-invariant (“gauge-singlet”) pieces of the a gauge-variant operators A^2 . Since the A^2 operators are defined with respect to different gauges their gauge-singlet parts are non-local and different. Therefore the observed equivalence of the correlations lengths is a non-trivial fact.

The Ginzburg-Landau parameter (*i.e.*, the ratio of penetration length and the coherence length) determines the type of the $SU(2)$ vacuum. According to our measurements

$$\begin{aligned} \kappa_{MA} &= 1.04(\pm 0.07 \text{ statistic})(\pm 0.1 \text{ systematic}) \quad [\text{MA gauge}], \\ \kappa_{LA} &= 1.04(\pm 0.05 \text{ statistic})(\pm 0.1 \text{ systematic}) \quad [\text{LA gauge}]. \end{aligned}$$

3. Conclusions

1. The coherence lengths of the vacuum of the $SU(2)$ gluodynamics in the MA gauge can *equivalently* be fixed either (i) from the correlations between the Wilson loops and the monopole density, or (ii) from the correlations between the Wilson loops and the dimension 2 operators.
2. The coherence lengths measured in the MA gauge and in the LA gauge are the same.
3. The penetration lengths measured in the MA gauge and in the LA gauge are the same.
4. The type of the vacuum in both gauges is determined to be near the border between type 1 and type 2. The Ginzburg-Landau parameters in both gauges coincide with each other.

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