

Screening Masses of Hot $SU(2)$ Gauge Theory from the 3d Adjoint Higgs Model

F.Karsch^{1*}, M.Oevers^{1,2†}, P. Petreczky^{1,3‡},

¹Fakultät für Physik, Universität Bielefeld,
P.O. Box 100131, D-33501 Bielefeld, Germany

²Department of Physics and Astronomy,
University of Glasgow, Glasgow, G12 8QQ, U.K.

³Dept. of Atomic Physics, Eötvös University,
H-1088 Puskin 5-7, Budapest Hungary

Abstract

We study the Landau gauge propagators of the lattice $SU(2)$ 3d adjoint Higgs model, considered as an effective theory of high temperature 4d $SU(2)$ gauge theory. From the long distance behaviour of the propagators we extract the screening masses. The propagators are studied both in the symmetric and the broken phases of the 3d Higgs model. It is shown that the pole masses extracted from the propagators in the symmetric phase agree well with the screening masses obtained recently in finite temperature $SU(2)$ theory, while propagators measured in the broken phase show quite a different behaviour. This suggests that the symmetric phase of the 3d model corresponds to the deconfined phase of the 4d $SU(2)$ gauge theory. The relation of the propagator masses to the masses extracted from gauge invariant correlators and the mass gap of pure 3d $SU(2)$ gauge theory is also discussed.

PACS: 11.10.Wx, 12.38.Mh, 11.15.Ha

Keywords: QCD, screening masses, lattice Monte-Carlo simulation, 3d effective theories

*karsch@Physik.Uni-Bielefeld.DE

†oevers@Physik.Uni-Bielefeld.DE

‡petreczk@Physik.Uni-Bielefeld.DE

petr@hercules.elte.hu

1 Introduction

The screening of static chromo-electric fields is one of the most outstanding properties of QCD and its investigation is important both from a theoretical and phenomenological point of view (for phenomenological applications see e.g. [1]). In leading order of perturbation theory the associated inverse screening length (Debye mass) is defined as the IR limit of the longitudinal part of the gluon self energy $\Pi(k_0 = 0, \mathbf{k} \rightarrow 0)$. However, as the screening phenomenon is related to the long distance behaviour of QCD the naive perturbative definition of the Debye mass is obstructed by severe IR divergences of thermal field theory and beyond leading order the above definition is no longer applicable. It was suggested by Rebhan [2] to define the Debye mass as a pole of the longitudinal part of the gluon propagator. Such a definition implies a self-consistent resummation of the perturbative series and ensures the gauge-independence and IR finiteness of the Debye mass. However, it requires the introduction of a so-called magnetic screening mass, a concept introduced long ago [3] to cure the IR singularities of finite temperature non-Abelian theories. Analogously to the electric (Debye) mass the magnetic mass can be defined as a pole of the transverse part of the finite temperature gluon propagator. Though the magnetic mass is not calculable in perturbation theory different self-consistent resummation schemes of the perturbative series give a non-vanishing magnetic mass [4, 5, 6, 7, 8]. The non-zero magnetic mass is also clearly seen in lattice studies of propagators in $SU(2)$ gauge theory [9, 10].

As the screening masses are static quantities it is expected that they can be determined in a 3d effective theory of QCD , the 3d $SU(3)$ adjoint Higgs model, provided the temperature is high enough. However, in the case of QCD one may worry whether the standard arguments of dimensional reduction apply. First of all the coupling constant is large $g \sim 1$ for any physically interesting temperature and thus the requirement $gT \ll \pi T$ is not really satisfied. Moreover, it is at present not clear which phase of the effective theory corresponds to the high temperature phase of QCD . The 3d adjoint Higgs model is known to have two phases, the symmetric (confinement) phase and the Higgs (Coulomb) phase [11, 12, 13]. The perturbative calculation of the effective potential [13, 14] in the effective theory suggests that the Higgs phase corresponds to the deconfined phase of 4d $SU(3)$ theory. This conclusion seems to be supported by the 2-loop level dimensional reduction performed in [13]. On the other hand in the dimensional reduction approach applied in [15, 16, 17] the symmetric phase turns out to be the physical one and a good description of spatial Wilson loops and Polyakov loop correlators has been obtained.

The aim of the present paper is to clarify whether the screening masses, defined as poles of the corresponding lattice propagators in Landau gauge, can be determined in the effective theory for the simplest case of the $SU(2)$ gauge group, where precise 4d data on screening masses exist for a huge temperature range [10]. The generalization to the $SU(3)$ case and the inclusion of fermions¹ is then straightforward.

There are several reasons which make the study of the screening masses, defined as poles of the corresponding propagators, in the effective theory interesting. First of all it is possible to compare masses defined in this way directly with 4d measurements and thus clarify the status of the 3d adjoint Higgs model as an effective theory. Secondly the masses defined

¹Fermionic fields do not appear in the effective theory, their role is only to modify the parameters of the effective theory.

in this way are closest to the spirit of resummation techniques, which implicitly rely on dimensional reduction (see e.g. discussion in [18]). A third reason is that gauge invariant definitions of the Debye mass rely on the 3d effective theory [13, 19, 20] and although these definitions are the best in the sense that they are explicitly gauge invariant, the corresponding correlators have not been measured yet in the 4d theory. Gauge invariant definitions yield larger masses than the pole masses [13, 20] and further studies are required to establish the connection between them. We will try to relate these masses in the last section of our paper in the spirit of the so-called constituent model [21].

2 The 3d SU(2) Adjoint Higgs Model on Lattice

The lattice action for the 3d adjoint Higgs model used in the present paper has the form

$$S = \beta \sum_P \frac{1}{2} \text{Tr} U_P + \beta \sum_{\mathbf{x}, \hat{i}} \frac{1}{2} \text{Tr} A_0(\mathbf{x}) U_i(\mathbf{x}) A_0(\mathbf{x} + \hat{i}) U_i^\dagger(\mathbf{x}) + \sum_{\mathbf{x}} \left[-\beta \left(3 + \frac{1}{2} h \right) \frac{1}{2} \text{Tr} A_0^2(\mathbf{x}) + \beta x \left(\frac{1}{2} \text{Tr} A_0^2(\mathbf{x}) \right)^2 \right], \quad (1)$$

where U_P is the plaquette, U_i are the usual link variables and the adjoint Higgs field is parameterized, as in [13, 15, 17] by anti-hermitian matrices $A_0 = i \sum_a \sigma^a A_0^a$ (σ^a are the usual Pauli matrices). Furthermore β is the lattice gauge coupling, x parameterizes the quartic self coupling of the Higgs field and h denotes the bare Higgs mass squared. In principle the parameters appearing in eq. (1) can be related to the parameters of the original 4d theory via the procedure of dimensional reduction [13, 15, 16, 17], which is essentially perturbative. Another possible procedure is to find the above parameters by matching some quantities which are equally well calculable both in the full 4d lattice theory and in the effective 3d lattice theory, i.e. to perform a non-perturbative matching. Since the validity of the perturbative dimensional reduction, as was discussed before, is not obvious in the case of *QCD* (or more precisely for $SU(N_c)$ gauge theories) we will try to explore the latter approach in the present paper. In general this would require a matching analysis in a 3d parameter space (β, x, h) , which is clearly difficult in general. We thus followed a more moderate approach and fix two of three parameters namely β and x , to the values obtained from the perturbative procedure of dimensional reduction. The values of these parameters at 2-loop level are [13]

$$\beta = \frac{4}{g_3^2 a},$$

$$g_3^2 = g^2(\mu) T \left[1 + \frac{g^2(\mu)}{16\pi^2} \left(L + \frac{2}{3} \right) \right], \quad (2)$$

$$x = \frac{g^2(\mu)}{3\pi^2} \left[1 + \frac{g^2(\mu)}{16\pi^2} (L + 4) \right], \quad (3)$$

$$L = \frac{44}{3} \ln \frac{\mu}{7.0555T}, \quad (4)$$

with a and T denoting the lattice spacing and temperature, respectively. The coupling constant of the 4d theory $g^2(\mu)$ is defined through the 2-loop formula

$$g^{-2}(\mu) = \frac{11}{12\pi^2} \ln \frac{\mu}{\Lambda_{\overline{MS}}} + \frac{17}{44\pi^2} \ln \left[2 \ln \frac{\mu}{\Lambda_{\overline{MS}}} \right]. \quad (5)$$

In order to be able to compare the results of the 3d simulation with the corresponding ones in the 4d theory it is necessary to fix the renormalization and the temperature scale. We choose the renormalization scale to be $\mu = 2\pi T$, which ensures that corrections to the leading order results for the parameters g_3^2 and x of the effective theory are small. Furthermore we use the relation $T_c = 1.06\Lambda_{\overline{MS}}$ from [10]. Now the temperature scale is fixed completely and the physical temperature may be varied by varying the parameter x . The lattice spacing was chosen according to the criterium $a \ll m^{-1} \ll Na$, where N is the extension of the lattice and m is the mass we want to measure.

The main goal of the present investigation is to study the propagators of scalar and vector (gauge) fields. For this purpose one has to fix a specific gauge, which is chosen to be the Landau gauge $\partial_\mu A_\mu = 0$ ². On the lattice this is realized by maximizing the quantity:

$$Tr \left[\sum_{\mathbf{x}, i} \left(U_i(\mathbf{x}) + U_i^\dagger(\mathbf{x}) \right) \right] \quad (6)$$

The gauge fixing is performed using the overrelaxation algorithm, which in our case is as efficient as combined overrelaxation and *FFT* algorithm used in [9, 10]. The vector field is defined in terms of link variables as

$$A_i(\mathbf{x}) = \frac{1}{2i} (U_i(\mathbf{x}) - U_i^\dagger(\mathbf{x})) \quad (7)$$

We are interested in extracting the electric (Debye) m_D and the magnetic m_T screening masses from the long distance behaviour of the scalar and vector propagators defined as

$$G_D(z) = \langle Tr A_0(z) A_0^\dagger(0) \rangle \sim \exp(-m_D z), \quad (8)$$

$$G_T(z) = \frac{1}{2} (G_1(z) + G_2(z)) \sim \exp(-m_T z), \quad (9)$$

with

$$\begin{aligned} G_i(z) &= \langle Tr A_i(z) A_i(0) \rangle, \\ A_\mu(z) &= \sum_{x,y} A_\mu(x, y, z), \quad \mu = 0, 1, 2 \end{aligned} \quad (10)$$

Note that due to the Landau gauge condition $G_3(z)$ should be constant. This fact can be used to test the precision and validity of the gauge fixing procedure. In our case this condition is satisfied with an accuracy of 0.1%.

Besides the scalar and vector propagators of the adjoint Higgs model we also calculate the gauge invariant scalar correlators and analyze the propagators in the limit of a 3d pure gauge theory.

²the choice of this gauge is motivated by the fact that 4d Landau gauge condition used in [9, 10] for static field configuration is equivalent to the 3d Landau gauge condition

To extract the masses from the correlation functions we have used the general fitting ansatz

$$A \left[\frac{\exp(-mz)}{z^b} + \frac{\exp(-m(N_z - z))}{(N_z - z)^b} \right] \quad (11)$$

motivated in [26] through the analysis of the gauge dependence of the electron propagator in QED. Previous investigations of gauge boson and quark propagators in Landau gauge [9, 26, 27] have shown that effective masses extracted from the correlation functions rise with increasing Euclidean time separation and eventually reach a plateau. This is contrary to local masses extracted from gauge invariant correlation functions which approach a plateau from above and is a direct consequence of a non-positive transfer matrix in Landau gauge. A fit with $b \neq 0$ can account for the observed drift in local masses and allows to extract stable masses already at shorter distances.

Most of our numerical studies have been performed on lattices of size³ $32^2 \times 64$. From previous studies of gluon propagators in the 4-dimensional $SU(2)$ gauge theory we know that such large lattices are needed to observe a plateau in local masses [10]. This is in particular the case for the rather small magnetic screening mass which leads to a rather slow decay of the correlation functions. We thus use the same spatial lattice size as in those studies.

We have used correlated (Michael-McKerrell) fits with eigenvalue smearing [22]. Our fits have been constrained to the region where local masses show a plateau (typically $z \sim 15$ for the magnetic mass and $z \sim 5$ for the electric mass). In this region fits with $b = 0$ and $b \neq 0$ yield consistent results within statistical errors. From fits with $b \neq 0$ we find best fits with $b < 0$, in accordance with the behaviour of local masses discussed above. However, although these fits with $b < 0$ can start at shorter distances, z , and still yield a good $\chi^2/d.o.f$, the magnitude of b is not well determined within our present statistical accuracy. In the following we thus will quote results from fits with $b = 0$.

3 Numerical Results for the Propagators of the 3d $SU(2)$ Higgs Model

The deconfined high temperature phase of the 4d $SU(2)$ field theory corresponds to some surface $h = h(x, \beta)$ in the parameter space (β, x, h) of the adjoint Higgs model. This is the surface of 4d physics and may lie in the symmetric or the broken phase. For fixed value of β (lattice spacing) the 4d physics is described by a line on this surface. Since most of our simulation were done for $\beta = 16$ we will refer later to this line as the line of 4d physics. In the present paper three choices for the line of 4d physics $h(x)$ are explored. A comparison with 4d simulations should allow to determine the physical line $h(x)$ which reproduces the result of the 4d analysis. The first choice for $h(x)$ is the perturbative line of 4d physics, calculated in [13] and lying in the broken phase, the other two choices are in the symmetric phase. The three choices for the line of 4d physics are illustrated on Figure 1 for $\beta = 16$. The actual procedure we used to choose the parameter h in the symmetric phase is the following.

³We also have performed additional calculations on a $16^2 \times 32$ lattice and checked explicitly that results for the electric mass show no volume dependence. Furthermore some calculations have been performed on a $32^2 \times 96$ lattice to check that the lattice size used for our calculations was sufficient for the determination of the magnetic mass.

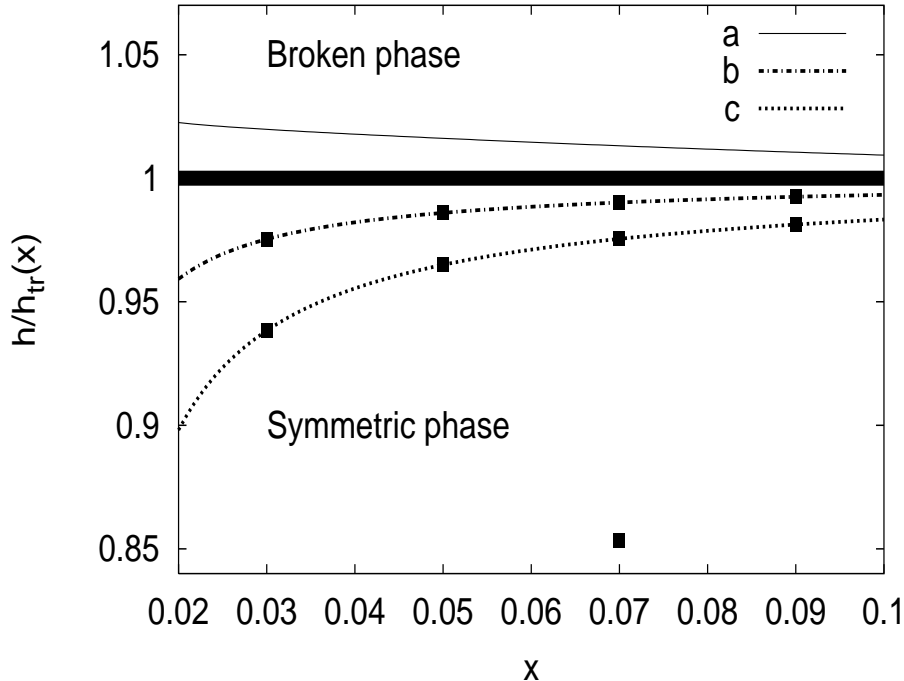


Figure 1: The bare masses normalized by the critical mass $h_{tr}(x)$ used in our matching analysis (squares): perturbative line (a) and two sets I (line b) and II (line c). For a discussion of their choice see text. For $x = 0.07$ also a point deeper in the symmetric phase has been selected. The thick solid line is the transition line and the thickness of the line indicate the uncertainty in its value.

First we have determined the transition line $h_{tr}(x)$. The transition line as function of x in the infinite volume limit was found in [13] in terms of the renormalized mass parameter $y = m^2/g_3^4$ (m is the continuum renormalized mass). The transition line in terms of y turns out to be independent of β . Then using eq. (5.7) from [13] one can calculate $h_{tr}(x)$. The usage of the infinite volume result for the transition line seems to be justified because most of our simulation were done on a $32^2 \times 64$ lattice. The two sets of $h(x)$ values, which appear on Figure 1, were chosen so that the renormalized mass parameter y (calculated using eq. (5.7) of [13]) always stays 10% and 25% away from the transition line. These values of h are of course *ad hoc* and one should use them only as trial values. The values of the parameters in the symmetric phase are shown in Table 1 where the two sets of h values are denoted as (I) and (II) and also the values of h_{tr} corresponding to the transition line are given.

Temperature scale		h		
x	T/T_c	I	II	transition
0.09	4.433	- 0.2652	- 0.2622	- 0.2672(4)
0.07	12.57	- 0.2528	- 0.2490	- 0.2553(5)
0.05	86.36	- 0.2365	- 0.2314	- 0.2399(6)
0.03	8761	- 0.2085	- 0.2006	- 0.2138(9)

Table 1: *The two sets of the bare mass squared used in the simulation and those which correspond to the transition line for $\beta = 16$*

Let us first discuss our calculations in the broken phase. In the broken phase the simulations were done for two sets of parameters: $\beta = 16$, $x = 0.03$, $h = -0.2181$ and

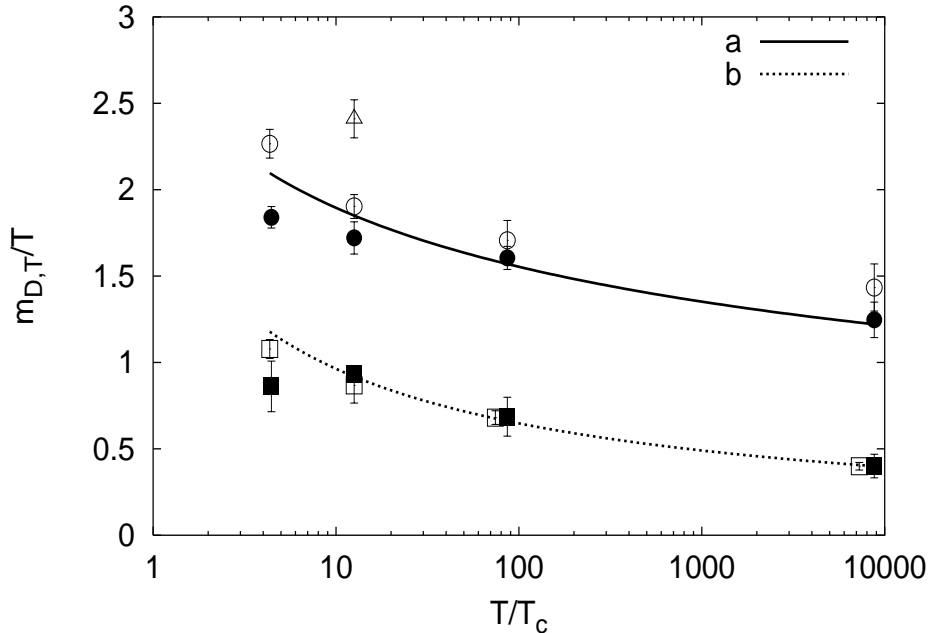


Figure 2: The screening masses in units of the temperature. Shown are the Debye mass m_D for the first (filled circles) and the second (open circles) set of h , and the magnetic mass m_T for the first (filled squares) and the second (open squares) set of h . The line (a) and line (b) represent the fit for the temperature dependence of the Debye and the magnetic mass from 4d simulations from [10]. The magnetic mass for the set II at the temperature $T \sim 90T_c$ and $\sim 9000T_c$ was shifted in the temperature scale for better visualization. The open triangle is the value of the Debye mass for $x = 0.07$ and $h = -0.2179$

$\beta = 8$, $x = 0.09$, $h = -0.5159$, here h was chosen along the perturbative line of 4d physics, which has been calculated in [13] to 2-loop order. The propagators obtained by us in the broken phase show a behaviour which is very different from that in the symmetric phase and that in the 4d case studied in Refs. [9, 10]. The magnetic mass extracted from the gauge field propagators is $0.104(20)g_3^2$ for the first set of parameters and $0.094(8)g_3^2$ for the second set of parameters. It thus is a factor 4 to 5 smaller than the corresponding 4d result. Moreover, the propagator of the A_0 field does not seem to show a simple exponential behaviour, this fact actually is in qualitative agreement with findings of Ref. [2]. Taken together these facts suggest that the broken phase does not correspond to the physical phase.

Let us now turn to a discussion of our results in the symmetric phase. In order to find the parameter range of interest for h we first have analyzed three different values of h at $\beta = 16$ and $x = 0.07$. In addition to the two values given in Table 1 we also chose an even larger value, $h = -0.2179$. The location of these values relative to the transition line is shown in Figure 1. For the electric screening mass we find, for increasing values of h , $m_D/T = 1.72(10)$, $1.90(7)$ and $2.41(11)$. We note that m_D/T increases with increasing distance from the transition line. These results should be compared to the 4d data. From the fit given in Ref.[10], we find at $T/T_c = 12.57$ for the electric screening mass $m_D/T = 1.85$. This shows that our third value for h clearly is inconsistent with the 4d result. One has to choose values for h close to the transition line in order to get agreement between the 3d and 4d results. From a linear interpolation between the results at the three different values for h we find the best matching value, i.e. a point on the line of 4d physics, $h(x = 0.07) = -0.2496$.

The analysis described above motivated our choice of trial values for h at other values of x as given in Table 1. The temperature dependence of the screening masses obtained in the symmetric phase for these two sets of parameters, which stay close to the transition line, is shown in Figure 2. Also given there is the result of the 4d simulations [10], $m_D^2/T^2 = Ag^2(T)$, with $A = 1.70(2)$ for the electric mass and $m_T/T = Cg^2(T)$, with $C = 0.456(6)$ for the magnetic mass. As can be seen both masses can be described consistently with a single choice of the coupling h for temperatures larger than $10T_c$. Although even at $T \simeq 4T_c$ we find reasonable agreement with the 4d fits, we note that the dependence of the results on the correct choice of h becomes stronger and a simultaneous matching of the electric and magnetic masses seems to be difficult. For larger temperatures we find that the magnetic mass shows little dependence on h (in the narrow range we have analyzed) and the determination of the correct choice of h thus is mainly controlled by the variation of the electric mass with h .

Let us summarize our findings for the screening masses in the symmetric phase. For $T \geq 10T_c$ the screening masses can be described very well in the effective theory, provided that values of h are close to the transition line. The suitable values of h can be found using the interpolation procedure outlined above for $x = 0.07$. This procedure can be also done for $x = 0.05$ and 0.03 , however there the 4d data are well described by values corresponding to the set I (see Figure 2), therefore the following h values can be considered as ones corresponding to 4d physics, $h(x = 0.07) = -0.2496$, $h(x = 0.05) = -0.2365$ and $h(x = 0.03) = -0.2085$. An interpolation between these values gives the line of 4d physics $h_{4d}(x)$.

Before closing this section we briefly want to address the question of a possible gauge dependence of the screening masses extracted by us. The pole of the propagator was proven to be gauge invariant in perturbation theory [23, 24, 25]. As concerns the results on the propagator pole masses extracted from lattice simulation the situation is less clear. Here one faces the numerical problem to isolate the asymptotic large distance behaviour of the correlation function from possible short distance (powerlike) corrections which are gauge dependent [26]. In [27] quark-propagators in axial, Coulomb and Landau gauges were studied and the effective masses extracted at quite short physical distances from the propagators were found to depend on gauge. In [26] the quark and gluon propagators were investigated in so-called λ -gauges. Masses extracted from the propagators using exponential fits like Eq. (11) also show a dependence (although mild) on the gauge parameter. To study the gauge dependence of our results following [26] we have introduced λ -gauges which in our case are defined by the condition

$$\lambda\partial_3 A_3 + \partial_2 A_2 + \partial_1 A_1 = 0. \quad (12)$$

and measured the propagators on $32^2 \times 96$ lattice. The masses were extracted from the propagators using the functional form given in Eq. (11). It turns out that screening masses measured at present for one parameter set and $\lambda = 0.5, 1.0$ and 2.0 are the same within our statistical errors of about (5-10)%. Such an accuracy is sufficient for the matching analysis and the resulting conclusions given above. Nonetheless, a more detailed analysis of the gauge dependence of the propagator pole masses is certainly needed. Work on this is in progress and will be presented elsewhere.

4 Magnetic Mass in 3d $SU(2)$ Pure Gauge Theory

The magnetic mass found in the previous section seems to scale with the 3d gauge coupling g_3^2 . This behaviour can be understood in the following way: If the temperature is high enough, the separation of different length scales holds, i.e. $g^2 T \ll gT \ll 2\pi T$ and besides non-static modes with mass $\sim 2\pi T$ the heavy A_0 field with mass $\sim gT$ can also be integrated out. In this limit the *IR* behaviour of high temperature $SU(N_c)$ gauge theory is described by 3d pure gauge theory in which the only mass scale is g_3^2 . Unfortunately for *QCD* (or $SU(2)$ gauge theory) the above arguments fail to hold because the coupling remains large for any realistic temperature. A non-perturbative study is therefore needed to establish the relation between the magnetic mass found in finite temperature $SU(2)$ theory and the mass gap of 3d pure gauge theory.

We have measured the Landau gauge propagators for the 3d $SU(2)$ gauge theory and from its large distance behaviour extracted the magnetic mass. The results for different values of β are listed in Table 2.

β	m_T/g_3^2	$\chi^2/d.o.f$
12.00	0.48 ± 0.036	0.580
16.00	0.42 ± 0.070	0.497
20.00	0.44 ± 0.068	1.439

Table 2: *The results of the fit for the magnetic mass in 3d pure gauge theory*

Using the data from Table 2 one finds $m_T = 0.46(3)g_3^2$. This value is in good agreement with the 3d adjoint Higgs model result. The magnetic mass thus is rather insensitive to the dynamics of the A_0 field. This finding is in accordance with the gap equation study of the adjoint Higgs model [18]. It is instructive to consider also the ratio of the magnetic mass and the string tension. For $SU(2)$ pure gauge theory the latter was found in Ref. [28] $\sqrt{\sigma_3} = 0.334(3)g_3^2$, which yields $m_T/\sqrt{\sigma_3} = 1.39(9)$. This should be compared with the ratio of the magnetic mass and the spatial string tension of the high temperature $SU(2)$ theory $m_T/\sqrt{\sigma_s} = 1.27(1)(1 + 0.11(2)g^2(T))$ [9].

5 Gauge Invariant Correlators and the Constituent Model

Let us finally discuss the relation between gauge invariant and gauge dependent correlators. Gauge invariant correlators for the $SU(2)$ Higgs model were studied in detail in [13]. We have studied here the Polyakov loop correlator, which in the effective theory following [15, 16] is defined as

$$\frac{\langle L_{eff}(\mathbf{x})L_{eff}(0) \rangle}{\langle \bar{L}_{eff} \rangle^2}, \quad L_{eff}(\mathbf{x}) = \frac{1}{2}Tr \exp(A_0(\mathbf{x})), \quad \bar{L}_{eff} = \sum_{\mathbf{x}} L_{eff}(\mathbf{x}) \quad . \quad (13)$$

Furthermore we have analyzed the correlation function of the scalar operator $Tr A_0^2$ whose large distance behaviour gives the mass of the $A_0 - A_0$ bound state $m(A_0)$. The mass of this bound state is also expected to determine the exponential fall off of the Polyakov loop

correlator [13]. We have measured these correlators on a $16^2 \times 32$ lattice in the symmetric phase. As expected both yield the same mass. The results for the masses extracted from the correlation function of $Tr A_0^2$ are shown in Table 3. The values of these masses are consistent with those obtained in [13]. For $x = 0.09$, which corresponds to the temperature $T \sim 4T_c$, we find $m(A_0)/T = 3.89(11)$ which should be compared with the Polyakov loop correlator in the high temperature 4d $SU(2)$ theory $m_P/T \sim 4$ [29]. Let us discuss the relation of $m(A_0)$ and the Debye mass defined through the propagator. If the coupling constant is small enough then $m(A_0) \sim 2\sqrt{\frac{N}{3}}gT \equiv 2m_{D0}$. In our case this perturbative relation is not satisfied. However, in terms of the constituent model [21] the mass of the $A_0 - A_0$ bound state can be represented by the sum of two constituent scalar masses, which are m_D .

The mass of the lowest lying glueball state was measured in [13] and was found to be rather independent of the couplings h and x of the A_0 field, the value of this mass is $m_G \sim 2.0g_3^2$. In terms of the constituent model this state can be viewed as a bound state of four constituent gluons [21] with a constituent mass equal to m_T , thus $m_G \sim 4m_T$. The masses predicted by the constituent model compared with the results of direct measurements are shown in Table 3.

<i>parameters</i>			$m(A_0)/g_3^2$		m_G/g_3^2	
β	x	h	<i>measured</i>	<i>constituent model</i>	<i>measured</i>	<i>constituent model</i>
16	0.09	-0.2622	1.54(15)	1.78(6)		1.68(9)
16	0.05	-0.2314	2.28(20)	2.38(16)	2.0	1.88(17)
24	0.03	-0.1475	3.03(65)	3.28(30)		1.84(10)

Table 3: *The masses of the scalar bound state measured on $16^2 \times 32$ lattice and the glueball mass taken from [13] in units of g_3^2 compared with the predictions of the constituent model*

Another gauge invariant operator, whose correlation function was measured in [13], is $h_i = \epsilon_{ijk} Tr A_0 F_{jk}$. The mass extracted from this correlator is the mass of the bound state of the scalar field and the light glue, thus its mass in terms of the constituent model is expected to be $m_h = m_D + m_T$. Our numerical simulations show that $m_D > m_T$ in the entire temperature range, therefore one would expect that $m(A_0) > m_h$. On the other hand the numerical simulation in Ref. [13] shows that this condition is satisfied only in the parameter range which does not corresponds to the physical situation. In this respect the constituent model seems to fail to explain the spectrum of the theory.

6 Conclusions

In this paper we have investigated the propagators of the 3d $SU(2)$ adjoint Higgs model in Landau gauge. Masses extracted from them are compared with the corresponding ones from recent simulations of the high temperature $SU(2)$ theory [10]. The gauge coupling and the Higgs quartic coupling of the effective theory were fixed via the procedure of dimensional reduction, while the bare Higgs mass h was left free to allow for a non-perturbative matching. We have considered the values of h which correspond to the broken phase and two sets of h in the symmetric phase. The screening masses in the symmetric phase are

in good agreement with the results of the 4d theory with both set of h , which means that the propagator masses in the symmetric phase are not too sensitive to the value of the bare mass. In the broken phase the propagators are quite different from those in the symmetric phase and 4d simulation. In particular the scalar propagator does not seem to show an exponential behaviour, while the vector propagators yield several times smaller masses than what is obtained in 4d simulation. The magnetic mass in the adjoint Higgs model scales with g_3^2 and its value is very close to the value of the mass gap obtained in the pure 3d gauge theory. We have also measured gauge invariant correlators of the scalar field. The resulting mass as well the mass of the lightest glueball can be related to the propagator masses via the constituent model proposed in [21]. The mass of the vector bound state, measured in [13], however, cannot be understood in this way.

Acknowledgements: This work was partially supported by the TMR Network *Finite Temperature Phase Transition in Particle Physics*, EU contract no. ERBFMRX-CT97-0122 and the ZiF project *Multi-scale Phenomena on Massively Parallel Computers*. We thank K. Rummukainen for providing us with data allowing to check our program. P.P. thanks W. Buchmüller, Z. Fodor, A. Jakovác, D. Miller and A. Patkós for usefull discussions.

References

- [1] X.-N. Wang, Phys. Rep. **280** (1997) 287
- [2] A. Rebhan, Nucl. Phys. **B430** (1994) 319
- [3] A. Linde, Phys. Lett. **B96** (1980) 289
- [4] F. Eberlein, *Two-Loop Gap Equations for the Magnetic Mass*, hep-ph/9804460
- [5] R. Jackiw and S.-Y. Pi, Phys. Lett. **B403** (1997) 297
- [6] G. Alexanian and V.P. Nair, Phys. Lett. **B352** (1995) 435
- [7] W. Buchmüller, Z. Fodor, T. Helbig and D. Walliser, Ann. Phys. (N.Y.) **234** (1994) 260
- [8] W. Buchmüller and O. Philipsen, Nucl. Phys. **B443** (1995) 47
- [9] U.M. Heller, F. Karsch and J. Rank, Phys. Lett. **B355** (1995) 511
- [10] U.M. Heller, F. Karsch and J. Rank, Phys. Rev. **D57** (1998) 1438
- [11] S. Nadkarni, Nucl. Phys. **B334** (1990) 559
- [12] A. Hart, O. Philipsen, J.D. Stack and M. Tepper, Phys. Lett. **B396** (1997) 217
- [13] K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. **B503** (1997) 357
- [14] J. Polónyi and S. Vazquez, Phys. Lett. **B240** (1990) 183
- [15] P. Lacock, D.E. Miller and T. Reisz, Nucl. Phys. **B369** (1992) 501

- [16] L. Kärkkäinen, P. Lacock, B. Peterson and T.Reisz, Nucl.Phys. **B395** (1993) 733
- [17] L. Kärkkäinen P. Lacock, D.E. Miller, B. Peterson and T.Reisz, Nucl. Phys. **B418** (1994) 3
- [18] A. Patkós, P. Petreczky and Zs. Szép, Eur. Phys. J. **C5** (1998) 337.
- [19] P. Arnold and L.G. Yaffe, Phys. Rev. **D52** (1995) 7208
- [20] K. Kajantie, M. Laine, J. Peisa, A. Rajantie, K. Rummukainen and M. Shaposhnikov, Phys. Rev. Lett. **79** (1997) 3130
M. Laine and O. Philipsen, Nucl.Phys. B523 (1998) 267
- [21] W. Buchmüller and O. Philipsen, Phys. Lett. **B397** (1997) 112
- [22] C. Michael and A. McKerrell, Phys. Rev. **D51** (1995) 3745
- [23] R. Kobes, G. Kunstatter, A. Rebhan, Phys. Rev. Lett. **64** (1990) 2992
- [24] R. Kobes, G. Kunstatter, A. Rebhan, Nucl. Phys. **B355** (1991) 1
- [25] A.S. Kronfeld, Phys. Rev. D58 (1998) 051501.
- [26] C. Bernard, A. Soni and K. Yee, Nucl. Phys. B (Proc. Suppl.) **20** (1991) 410
- [27] W. Dimm, P. Lepage and P.B. Mackenzie, Nucl. Phys. B (Proc. Suppl.) **42** (1995) 403
- [28] M. Teper, Phys. Lett. **B289** (1992) 115
- [29] J. Engels, V.K. Mitrjushkin and T. Neuhaus, Nucl. Phys. **B440** (1995) 555