GLUON EXCITATIONS OF THE STATIC-QUARK POTENTIAL

K.J. Juge, J. Kuti, and C. Morningstar^{*} University of California at San Diego, La Jolla, California 92093-0319

ABSTRACT

The spectrum of gluon excitations in the presence of a static quark-antiquark pair is presented. Our results are obtained from computer simulations of gluons on anisotropic space-time lattices using an improved gauge-field action. Measurements for quark-antiquark separations r ranging from 0.1 fm to 4 fm and for various orientations on the lattice are made. Discretization errors and finite volume effects are taken into account. Surprisingly, the spectrum does not exhibit the expected onset of the universal π/r Goldstone excitations of the effective QCD string, even for r as large as 4 fm.

1. Introduction

Accurate knowledge of the properties of the stationary states of glue in the presence of the simplest of color sources, that of a static quark and antiquark separated by some distance r, is an important stepping stone on the way to understanding confinement. It is generally believed that at large r, the linearly-growing ground-state energy of the glue is the manifestation of the confining flux whose fluctuations can be described in terms of an effective string theory. The lowest-lying excitations are then the Goldstone modes associated with spontaneously-broken transverse translational symmetry. Expectations are less clear for small r. The determination of the energies of glue in the presence of a static quark-antiquark pair is also the first step in the Born-Oppenheimer treatment of conventional and hybrid heavy-quark mesons.¹ A better understanding of hybrid quarkonium should provide valuable insight into the nature of light hybrid mesons, which are currently of great experimental and theoretical interest.

Even the simplest property, the energy spectrum, of the stationary states of glue interacting with a static quark-antiquark pair is not accurately known. The main goal of this work is to remedy this. Here, we present, for the first time, a comprehensive determination of the low-lying spectrum of gluonic excitations in the presence of a static quark-antiquark pair. In this initial study, the effects of light quark-antiquark pair creation are ignored. A few of the energy levels for r less than 1 fm have been studied before.² Our results for these quantities have significantly improved precision, and we have extended the range in r to 4 fm. Most of the energy levels presented here have never been studied before. Some of our results were previously reported.³

^{*}Talk presented by C. Morningstar

2. Computation of the energies

We adopt the standard notation from the physics of diatomic molecules and use Λ to denote the magnitude of the eigenvalue of the projection $\vec{J}_g \cdot \hat{\mathbf{r}}$ of the total angular momentum \vec{J}_g of the gluons onto the molecular axis $\hat{\mathbf{r}}$. The capital Greek letters $\Sigma, \Pi, \Delta, \Phi, \ldots$ are used to indicate states with $\Lambda = 0, 1, 2, 3, \ldots$, respectively. The combined operations of charge conjugation and spatial inversion about the midpoint between the quark and the antiquark is also a symmetry and its eigenvalue is denoted by η_{CP} . States with $\eta_{CP} = 1(-1)$ are denoted by the subscripts g(u). There is an additional label for the Σ states; Σ states which are even (odd) under a reflection in a plane containing the molecular axis are denoted by a superscript + (-). Hence, the lowlying levels are labelled $\Sigma_g^+, \Sigma_g^-, \Sigma_u^+, \Sigma_u^-, \Pi_g, \Pi_u, \Delta_g, \Delta_u$, and so on. For convenience, we use Γ to denote these labels in general.

The glue energies $E_{\Gamma}(\vec{r})$ were extracted from Monte Carlo estimates of generalized Wilson loops. The expectation value of the path-ordered exponential of the gauge field along a closed loop is known as a Wilson loop. The familiar static-quark potential $E_{\Sigma_g^+}(r)$ can be obtained from the large-*t* behaviour $\exp[-tE_{\Sigma_g^+}(r)]$ of the Wilson loop for a rectangle of spatial length *r* and temporal extent *t*. The two spatial segments of such a Wilson loop can be replaced by a complicated *sum* of spatial paths, all sharing the same starting and terminating sites, in order to extract the $E_{\Gamma}(\vec{r})$ energies, as long as the sum of paths transforms as Γ under all symmetry operations. Using several different sums of paths (we typically use 3 to 22 such sums) then produces a matrix of Wilson loop correlators $W_{\Gamma}^{ij}(r, t)$.

Monte Carlo estimates of the $W_{\Gamma}^{ij}(r,t)$ matrices were obtained in eight simulations using an improved gauge-field action.⁴ Our use of anisotropic lattices in which the temporal lattice spacing a_t was much smaller than the spatial spacing a_s was crucial for resolving the glue spectrum, particularly for large r. To hasten the onset of asymptotic behaviour, iteratively-smeared spatial links⁴ were used in the generalized Wilson loops. The temporal segments in the Wilson loops were constructed from thermally-averaged links, whenever possible, to reduce statistical noise. To facilitate the removal of finitespacing errors by extrapolating to the continuum limit, results for several values of the lattice spacing were obtained; simulations were done using $a_s \approx 0.29, 0.27, 0.22, 0.19$, and 0.12 fm. Two runs corresponding to the same a_s but different a_t were done to provide a measure of the a_t^2 errors in our results (our action has $O(a_t^2, a_s^4)$ errors). Such information is important for carrying out the $a_s \rightarrow 0$ extrapolations. Agreement of energies obtained using different quark-antiquark orientations on the lattice was used to check the smallness of finite-spacing errors and to help identify the continuum Λ value corresponding to each level (there are only discrete symmetries on the lattice). For this reason, results were obtained not only for cases in which the molecular axis coincided with an axis of the lattice, but also for separation vectors such as $(r, r, r)/\sqrt{3}$ and $(r, r, 0)/\sqrt{2}$.

The matrices $W_{\Gamma}^{ij}(r,t)$ were reduced in the data fitting phase to single correlators and 2×2 correlation matrices using the variational method. The lowest-lying glue energies were then extracted from these reduced correlators by fitting a single exponential and a sum of two exponentials, the expected asymptotic forms, in various ranges t_{\min} to t_{max} of the source-sink separation. The two-exponential fits were used to check for consistency with the single-exponential fits, and in cases of favourable statistics, to extract the first-excited state energy in a given channel.

Three additional runs on small lattices were done to verify that finite-volume errors in our results were negligible. We confirmed the smallness of the a_s/a_t renormalization for two values of the QCD coupling by extracting the ground state potential from Wilson loops in different orientations. The hadronic scale parameter $r_0 \approx 0.5$ fm was used to determine the lattice spacing.⁴ The additive ultraviolet-divergent selfenergies of the static sources were removed by expressing all of our results with respect to $\Sigma_g^+(r_0)$. Finite-lattice spacing errors were removed by extrapolating our simulation results for $r_0[E_{\Gamma}(r) - E_{\Sigma_g^+}(r_0)]$ to the $a_s \to 0$ continuum limit. These extrapolations were carried out by fitting all of our simulation results to an ansatz $F_{\rm cont}(r) + a_s^4 F_{\rm latt}(r)$: a ratio of a polynomial of degree p + 1 over a polynomial of degree p, where p = 1 or 2, was found to work well for the continuum limit form $F_{\rm cont}(r)$, and $F_{\rm latt}(r)$ was chosen empirically to be a sum of three terms $1/\sqrt{r}$, 1/r, and $1/r^2$. All fits yielded $\chi^2/\text{dof near}$ unity. Continuum Λ values were easily identified in all cases but one: we were unable to distinguish between a Π'_u and Φ_u interpretation for the on-axis E'_u level.

3. Results

Our continuum-limit extrapolations are shown in Fig. 1. The ground-state Σ_g^+ is the familiar static-quark potential. A linearly-rising behaviour dominates the Σ_g^+ potential once r exceeds about 0.5 fm and we find no deviations from the linear form up to 4 fm. The lowest-lying excitation is the Π_u . There is definite evidence of a band structure at large r: the Σ_g' , Π_g , and Δ_g form the first band above the Π_u ; the Σ_u^+ , Σ_u^- , Π_u'/Φ_u , and Δ_u form another band. The Σ_g^- is the highest level at large r. This band structure breaks down as r decreases below 2 fm. In particular, two levels, the Σ_g^- and Σ_u^- , drop far below their large-r partners as r becomes small. Note that for r above 0.5 fm, all of the excitations shown are stable with respect to glueball decay. As r decreases below 0.5 fm, the excited levels eventually become unstable as their gaps above the ground state Σ_q^+ exceed the mass of the lightest glueball.

A universal feature of any low-energy description of a thin fluctuating flux tube is the presence of Goldstone excitations associated with the spontaneously-broken transverse translational symmetry. These transverse modes have energy separations above the ground state given by multiples of π/r . The level orderings and approximate degeneracies of the gluon energies at large r match, without exception, those expected of the Goldstone modes. However, the precise $m\pi/r$ gap behaviour is not observed (see Fig. 1). For separations less than 2 fm, the gluon energies lie well below the Goldstone energies and the Goldstone degeneracies are no longer observed. The two Σ^- states are in violent disagreement with expectations from a fluctuating string. The gluon energies also cannot be explained in terms of a Nambu-Goto string (whose quantization in four dimensions is problematical, besides).

These results are rather surprising. Our results cast serious doubts on the validity of treating glue in terms of a fluctuating string for quark-antiquark separations less than 2 fm. Note that such a conclusion does not contradict the fact that the $\Sigma_g^+(r)$ energy rises linearly for r as small as 0.5 fm. A linearly-rising term is not necessarily indicative



Fig. 1. Plot of the continuum-limit extrapolations (with uncertainties as indicated) for $r_0[E_{\Gamma}(r) - E_{\Sigma_g^+}(r_0)]$ against $r/(2r_0)$ for various Γ . The dashed lines indicate the locations of the $m\pi/r$ gaps above the Σ_g^+ curve for m = 1, 2, 3, and 4. The dotted curves are the naive Nambu-Goto energies in four-dimensions. Note that we cannot rule out a Φ_u interpretation for the curve labelled Π'_u .

of a string: for example, the adiabatic bag model predicts a linearly-rising ground state much before the onset of string-like behaviour, even in the spherical approximation.¹ For r greater than 2 fm, there are some tantalizing signatures of Goldstone mode formation, yet significant disagreements still remain. To what degree these discrepancies can be explained in terms of a distortion of the Goldstone mode spectrum arising from the spatial fixation of the quark and antiquark sources is currently under investigation. This work was supported by the U.S. DOE, Grant No. DE-FG03-97ER40546.

References

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