

Lepton Asymmetry of the Universe and Charged Quark-Gluon Plasma

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Abstract

The lepton asymmetry of order one is not excluded experimentally and, if present, can lead to interesting phenomena in the early Universe. It is shown that, when the temperature is above the quark-hadron transition, the lepton asymmetry induces chemical potentials for electric charge and for baryon number, and makes the quark-gluon plasma electrically charged.

1 Introduction

The observational data do not severely restrict lepton asymmetry of the Universe which, if present, resides almost entirely in the neutrino sector. In particular, the chemical potentials and the temperature of relic neutrinos can be comparable in magnitude. The nucleosynthesis bounds on the neutrino degeneracy parameters, $\xi_{\nu_f} = \mu_{\nu_f}/T_\nu$, are $-0.06 < \xi_{\nu_e} < 1.1$, $|\xi_{\nu_{\mu,\tau}}| < 6.9$ [1]. Current measurements of the cosmic microwave background anisotropy put an upper limit of 3 – 5 on degeneracy parameters for all neutrino species [2, 3].

The large lepton asymmetry could have profound consequences in the early history of the Universe, when the temperature was of order of the electroweak scale. The lepton charge alters the pattern of electroweak symmetry breaking, because the lepton number density forces the Higgs field to Bose condense and therefore can impede restoration of electroweak symmetry at high temperature [4, 5, 6]. If the lepton number density is sufficiently large, the Universe has always been in the symmetry-broken phase and there was no electroweak phase transition [7]. This observation provides a possible resolution of the monopole and the domain wall problems [8].

Although it seems natural to assume that the lepton and the baryon asymmetries of the Universe are of the same order of magnitude, mechanisms that could generate primordial lepton asymmetry of order one without generating large baryon asymmetry are known [9, 10, 11, 12], and degeneracy of relic neutrinos remains at least a logical possibility. The assumption of large neutrino degeneracy was invoked, in particular, to explain an origin of the ultra-high energy cosmic rays [13].

In this paper I will study the consequences of the large lepton asymmetry at temperatures above the quark-hadron transition but below the electroweak scale. It appears that, when quarks are deconfined, the lepton asymmetry indirectly affects hadronic (strongly interacting) sector in such a way that all light quark species acquire large chemical potentials. A deviation of the lepton number density from zero and an overall electric neutrality cause an asymmetric distribution of the electric charge between the lepton and the hadron sectors. As a consequence, the quark-gluon plasma is electrically charged when lepton asymmetry is of order one.

2 Chemical potentials

There are five charges that are conserved or almost conserved when the temperature drops below the electroweak scale: electric charge, baryon number and lepton numbers for each of the three generations of fermions. Accordingly, one can introduce five chemical potentials to describe a state with non-zero densities of these charges. On average, two of the conserved charge densities are zero, because the Universe must be electrically neutral and the baryon asymmetry of the Universe is negligibly small. But densities of lepton numbers can potentially be large enough to require chemical potentials comparable to the temperature.

The chemical potential for each of the particle species is a linear combination of the chemical potentials for conserved charges:

$$\mu_i = \sum_{\alpha} q_i^{\alpha} \mu_{\alpha}. \tag{1}$$

Here $q_i^\alpha = (Q_i, B_i, L_i^e, L_i^\mu, L_i^\tau)$ are charges of i th species. In assumption of large lepton asymmetry, chemical potentials for lepton numbers are non-zero at present and cause asymmetries in equilibrium distributions of the relic neutrinos. When the temperature was higher than the mass of the electron, the lepton number chemical potential alone would lead to an asymmetric distribution of electrons and would induce a non-zero density of electric charge. The electric neutrality thus requires to set $\mu_Q = \mu_{Le}$, which makes the chemical potential for electrons equal to zero.

The situation drastically changes when quarks are deconfined. Light quarks can be considered essentially massless at temperatures above the quark-hadron transition and, since they carry both the electric charge and the baryon number, non-zero μ_Q would induce large baryon asymmetry unless μ_B is also non-zero. The conditions of electric neutrality and baryon symmetry then become more complicated and all chemical potentials generically become of the same order of magnitude.

I will consider temperatures of order of few GeV, when the temperature suppresses strong interactions due to the asymptotic freedom, and the ideal gas approximation for the quark-gluon plasma is more or less accurate. The active species at such temperatures are neutrinos, electrons, muons, photons, gluons and u, d and s quarks. Weak processes at the QCD epoch are rapid enough to maintain thermal equilibrium, and I will also assume that strong interactions do not considerably distort equilibrium distributions and will neglect masses of all active particle species.

The excess of particles over anti-particles per unit volume for two-component non-interacting fermions with chemical potential μ at temperature T is

$$n_+ - n_- = \frac{T^3}{6} \left(\kappa + \frac{\kappa^3}{\pi^2} \right). \quad (2)$$

The degeneracy parameters at the QCD epoch will be denoted by κ : $\kappa = \mu/T$, to distinguish them from the degeneracy parameters now, which are denoted by ξ . With the help of this equality, the electric neutrality, absence of considerable baryon number and requirement that there is a given lepton asymmetry yield a set of five equations for five chemical potentials:

$$\begin{aligned} \frac{T^3}{6} \sum_i Q_i \left(\kappa_i + \frac{\kappa_i^3}{\pi^2} \right) &= 0 \\ \frac{T^3}{6} \sum_i B_i \left(\kappa_i + \frac{\kappa_i^3}{\pi^2} \right) &= 0 \\ \frac{T^3}{6} \sum_i L_i^f \left(\kappa_i + \frac{\kappa_i^3}{\pi^2} \right) &= n_{Lf} - n_{\bar{L}f}, \end{aligned} \quad (3)$$

where n_{Lf} ($n_{\bar{L}f}$) are number densities of leptons (anti-leptons) of flavor f .

Though the lepton numbers are not conserved exactly, and some lepton charge could be produced by neutrino oscillations during nucleosynthesis [14, 15, 16], the asymmetry generated in this way is generically much smaller than one [17, 18]. So, if large density of lepton number was present at the QCD epoch, later it was just diluted by the expansion of the Universe. Since the entropy density,

$$s = \frac{P + \rho - \mu(n_+ - n_-)}{T}, \quad (4)$$

scales in the same way*, the ratio of the lepton number density to the entropy is time-independent:

$$n_{Lf} - n_{\bar{L}f} = s \left(\frac{n_{Lf} - n_{\bar{L}f}}{s} \right)_{\text{now}}. \quad (5)$$

Taking into account that the entropy of a gas of non-interacting two-component fermions is

$$s_{\text{ferm}} = T^3 \left(\frac{7\pi^2}{90} + \frac{\kappa^2}{6} \right), \quad (6)$$

and that each bosonic degree of freedom contributes $2\pi^2 T^3/45$, we find:

$$s = T^3 \left(\frac{247\pi^2}{90} + \sum_i \frac{\kappa_i^2}{6} \right) \quad (7)$$

at the QCD epoch. The entropy at present is carried by photons and by neutrinos:

$$s_{\text{now}} = \frac{4\pi^2}{45} T_\gamma^3 + T_\nu^3 \sum_f \left(\frac{7\pi^2}{90} + \frac{\xi_{\nu f}^2}{6} \right) = T_\nu^3 \left(\frac{43\pi^2}{90} + \sum_f \frac{\xi_{\nu f}^2}{6} \right), \quad (8)$$

where the relation $T_\gamma^3 = 11T_\nu^3/4$ was used in the last equality. The present value of the lepton asymmetry depends on the neutrino chemical potentials as follows:

$$(n_{Lf} - n_{\bar{L}f})_{\text{now}} = \frac{T_\nu^3}{6} \left(\xi_{\nu f} + \frac{\xi_{\nu f}^3}{\pi^2} \right). \quad (9)$$

Finally, we get:

$$n_{Lf} - n_{\bar{L}f} = \frac{T^3}{6} \left(\frac{247\pi^2}{15} + \sum_i \kappa_i^2 \right) \frac{\xi_{\nu f} + \frac{\xi_{\nu f}^3}{\pi^2}}{\frac{43\pi^2}{15} + \xi_{\nu e}^2 + \xi_{\nu \mu}^2 + \xi_{\nu \tau}^2}. \quad (10)$$

Plugging quantum numbers[†] of active species in eq. (1), using eq. (10) and the identities for electric charges of light quarks:

$$\sum_{u,d,s} Q_i = 0, \quad \sum_{u,d,s} Q_i^2 = \frac{2}{3}, \quad \sum_{u,d,s} Q_i^3 = \frac{2}{9}, \quad \sum_{u,d,s} Q_i^4 = \frac{2}{9}, \quad (11)$$

we get a system of five equations for dimensionless degeneracy parameters $\kappa_\alpha = \mu_\alpha/T$:

$$\begin{aligned} \frac{8}{3} \kappa_Q - \kappa_{L^e} - \kappa_{L^\mu} + \frac{2}{9\pi^2} (\kappa_Q^3 + \kappa_Q^2 \kappa_B + \kappa_Q \kappa_B^2) - \frac{1}{\pi^2} (\kappa_{L^e} - \kappa_Q)^3 - \frac{1}{\pi^2} (\kappa_{L^\mu} - \kappa_Q)^3 &= 0 \\ \kappa_B^3 + (6\kappa_Q^2 + 9\pi^2) \kappa_B + 2\kappa_Q^3 &= 0 \\ 2(\kappa_{L^e} - \kappa_Q) + \frac{2}{\pi^2} (\kappa_{L^e} - \kappa_Q)^3 + \kappa_{L^e} + \frac{1}{\pi^2} \kappa_{L^e}^3 &= \eta_e \\ 2(\kappa_{L^\mu} - \kappa_Q) + \frac{2}{\pi^2} (\kappa_{L^\mu} - \kappa_Q)^3 + \kappa_{L^\mu} + \frac{1}{\pi^2} \kappa_{L^\mu}^3 &= \eta_\mu \\ \kappa_{L^\tau} + \frac{1}{\pi^2} \kappa_{L^\tau}^3 &= \eta_\tau, \end{aligned} \quad (12)$$

*This assumption implies an entropy conservation and, in particular, an absence of considerable entropy production at the QCD phase transition.

[†]I normalize the baryon charge so that quarks have baryon number 1/3.

where

$$\eta_f = \left[\frac{247\pi^2}{15} + \frac{4}{3}\kappa_Q^2 + \frac{2}{3}\kappa_B^2 + 2(\kappa_{L^e} - \kappa_Q)^2 + 2(\kappa_{L^\mu} - \kappa_Q)^2 + \kappa_{L^e}^2 + \kappa_{L^\mu}^2 + \kappa_{L^\tau}^2 \right] \times \frac{\xi_{\nu_f} + \frac{\xi_{\nu_f}^3}{\pi^2}}{\frac{43\pi^2}{15} + \xi_{\nu_e}^2 + \xi_{\nu_\mu}^2 + \xi_{\nu_\tau}^2}. \quad (13)$$

The system of equations (12) allows to express the degeneracy parameters at the time in the history of the Universe when the temperature was of order of few GeV in terms of neutrino degeneracy parameters now. This system, upon closer look, does not contain any large numerical coefficients. So, if ξ_{ν_f} are of order one, all κ_α , including κ_B and κ_Q , will also be of order one. For instance, in the linear approximation, $\kappa_B = 0$ and $\kappa_Q = 1.44(\xi_{\nu_e} + \xi_{\nu_f})$.

I have solved the system of equations for the degeneracy parameters numerically in two representative cases. In the first case, neutrino degeneracies were chosen 'democratically' – equal for all three neutrino species: $\xi_e = \xi_\mu = \xi_\tau \equiv \xi$. The dependence of the baryon chemical potential and the chemical potential for electric charge at the QCD epoch on ξ are shown in Fig. 1. Fig. 2 shows the same quantities in the second case, when I took $\xi_e = 0$ and $\xi_\tau = 0$.

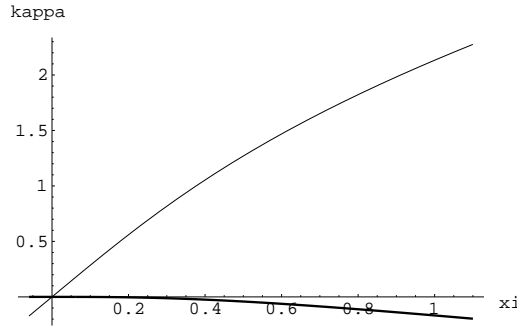


Figure 1: The electric charge chemical potential and the baryon number chemical potential (bold line) at the QCD epoch, $\kappa = \mu/T$, as the functions of neutrino degeneracy parameters at present: $\xi_e = \xi_\mu = \xi_\tau \equiv \xi$.

3 Discussion

The main result of this paper is that an overall charge neutrality, equilibrium with respect to the weak interactions and a non-zero density of the lepton number imply that the quark-gluon sector of the primordial plasma in the early Universe carries large (comparable to the entropy) electric charge, which compensates the opposite charge in the lepton sector. It is necessary to mention that, in explicit calculations of the chemical potentials carried out in the previous section, the quark-gluon plasma was treated as an ideal gas, which is an accurate approximation only at sufficiently high temperature. In practice, the ideal gas approximation should work well only for extensive quantities (entropy, charge density, etc.)

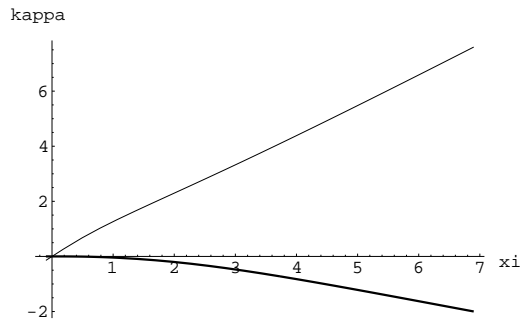


Figure 2: The same as in Fig. 1, but for $\xi_e = 0$, $\xi_\tau = 0$, $\xi_\mu \equiv \xi$.

and at temperature of order of few GeV or higher. The most interesting phenomena happen at lower temperatures when the ideal gas approximation definitely breaks down. Still, the qualitative conclusion that the quark-gluon plasma is charged in the presence of lepton asymmetry should remain unchanged, because it relies solely on the fact that quarks carry electric charges and can be thermally excited above the QCD phase transition.

The physics of charged quark-gluon plasma can differ considerably from that of the better studied neutral plasma. The charge density potentially can affect the fate of various topological defects [19, 20] and can change the nature of the quark-hadron transition. At zero chemical potentials, the quark-gluon and the hadron phases are likely to be smoothly connected, so that there is a smooth crossover instead of the phase transition [21]. It is difficult to say without detailed study whether large chemical potentials can make the transition stronger, but if this happens, it can have important cosmological consequences.

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