

PRIMORDIAL GALACTIC MAGNETIC FIELDS FROM THE QCD PHASE TRANSITION

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In this letter, we propose a mechanism to generate large-scale magnetic fields with correlation lengths of 100 kpc. Domain walls with QCD scale internal structure form and coalesce obtaining Hubble scale correlations and align nucleon spins. Due to strong CP violation, nucleons in these walls have anomalous electric and magnetic dipole moments and thus the walls are ferromagnetic. This induces electromagnetic fields with Hubble size correlations. The same CP violation also induces a maximal helicity (Chern-Simons) correlated through the Hubble volume which supports an “inverse cascade” allowing the initial correlations to grow to 100 kpc today. We estimate the generated electromagnetic fields in terms of the QCD parameters and discuss the effects of the resulting fields.

1 Introduction

The source of cosmic magnetic fields with large scale correlations has remained somewhat of a mystery¹. There are two possible origins for these fields: primordial sources and galactic sources. Primordial fields are produced in the earlier universe, then evolve, and are thought to provide seeds which gravitational dynamos later amplify. Galactic sources would produce the fields as well as amplify them. Many mechanisms have been proposed^{2,3,4,5}, however, most fail to convincingly generate fields with large enough correlation lengths to match the observed microgauss fields with ~ 100 kpc correlations. We present here a mechanism which, although probably requiring a dynamo to produce microgauss fields, generates fields with hundred kiloparsec correlations. We present this mechanism as an application of our recent understanding of QCD domain walls, which will be described in detail elsewhere⁶.

1. Sometime near the QCD phase transition, $T_{\text{QCD}} \approx 1$ GeV, QCD domain walls form.
2. These domain walls rapidly coalesce until there remains, on average, one domain wall per Hubble volume with Hubble scale correlations.
3. Baryons interact with the domain walls and align their spins along the domain walls.
4. The magnetic and electric dipole moments of the baryons induce helical magnetic fields correlated with the domain wall.
5. The domain walls decay, leaving a magnetic field.

6. As the universe expands, an “inverse cascade” mechanism transfers energy from small to large scale modes, effectively increasing the resulting correlation length of the observed large scale fields.

We shall start by discussing the “inverse cascade” mechanism because it seems to be the most efficient mechanism for increasing the correlation length of magnetic turbulence. After presenting some estimates to show that this mechanism can indeed generate fields of the observed scales, we shall discuss the domain wall mechanism for generating the initial fields and some relevant astrophysical phenomena associated with this mechanism.

2 Evolution of Magnetic Fields

As suggested by Cornwall³, discussed by Son⁴ and confirmed by Field and Carroll⁵, energy in magnetic fields can undergo an apparent “inverse cascade” and be transferred from high frequency modes to low frequency modes, thus increasing the overall correlation length of the field faster than the naïve scaling by the universe’s scale parameter $R(T)$. There are two important conditions: turbulence must be supported as indicated by a large Reynolds number Re , and magnetic helicity (Abelian Chern-Simons number) $H = \int \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} d^3x$ is approximately conserved. The importance of helicity was originally demonstrated by Pouquet and collaborators⁷. The mechanism is thus: the small scale modes dissipate, but the conservation of helicity requires that the helicity be transferred to larger scale modes. Some energy is transferred along with the helicity and hence energy is transported from the small to large scale modes. This is the “inverse cascade”^{3,4,5}

In the early universe, Re is very large and supports turbulence. This drops to $Re \approx 1$ at the e^+e^- annihilation epoch,⁴ $T_0 \approx 100$ eV. After this point (and throughout the matter dominated phase) we assume that the fields are “frozen in” and that the correlation length expands as R while the field strength decays as R^{-2} . Note that the “inverse cascade” is only supported during the radiation dominated phase of the universe.

Under the assumption that the field is maximally helical, these conditions imply the following relationships^{4,5} between the initial field $B_{\text{rms}}(T_i)$ with initial correlation $l(T_i)$ and present fields today ($T_{\text{now}} \approx 2 \times 10^{-4}$ eV) $B_{\text{rms}}(T_{\text{now}})$ with correlation $l(T_{\text{now}})$:

$$B_{\text{rms}}(T_{\text{now}}) = \left(\frac{T_0}{T_{\text{now}}} \right)^{-2} \left(\frac{T_i}{T_0} \right)^{-7/3} B_{\text{rms}}(T_i) \quad (1)$$

$$l(T_{\text{now}}) = \left(\frac{T_0}{T_{\text{now}}} \right) \left(\frac{T_i}{T_0} \right)^{5/3} l(T_i). \quad (2)$$

As pointed out by Son⁴, the only way to generate turbulence is either by a phase transition T_i or by gravitational instabilities. We consider the former source. As we shall show, our mechanism generates Hubble size correlations l_i at a phase transition T_i . In the radiation dominated epoch, the Hubble size scales as T_i^{-2} . Combining this with (2), we see that $l_{\text{now}} \propto T_i^{-1/3}$; thus, the earlier the phase transition, the smaller the possible correlations.

The last phase transition is the QCD transition, $T_i = T_{\text{QCD}} \approx 0.2$ GeV with Hubble size $l(T_{\text{QCD}}) \approx 30$ km. We calculate (24) the initial magnetic field strength to be $B_{\text{rms}}(T_i) \approx e\Lambda_{\text{QCD}}^2/(\xi\Lambda_{\text{QCD}}) \approx (10^{17}\text{G})/(\xi\Lambda_{\text{QCD}})$ where ξ is a correlation length that depends on the dynamics of the system as discussed below and $\Lambda_{\text{QCD}} \approx 0.2$ GeV. With these estimates, we see that we can achieve

$$B_{\text{rms}} \sim \frac{10^{-9}\text{G}}{\xi\Lambda_{\text{QCD}}}, \quad l \sim 100 \text{ kpc} \quad (3)$$

today. One might consider the electroweak transition which might produce 100 pc correlations today, but this presupposes a mechanism for generating fields with Hubble scale correlations. Such a mechanism does not appear to be possible in the Standard Model. Instead, the fields produced are correlated at the scale T_i^{-1} which can produce only ~ 1 km correlations today.

These are crude estimates, and galactic dynamos likely amplify these fields. The important point is that we can generate easily the 100 kpc correlations observed today *provided* that the fields were initially of Hubble size correlation. Unless another mechanism for amplifying the correlations of magnetic fields is discovered, we suggest that, in order to obtain microgauss fields with 100 kpc correlation lengths, helical fields must be generated with Hubble scale correlations near or slightly after the QCD phase transition T_{QCD} . The same conclusion regarding the relevance of the QCD scale for this problem was also reached by Son, Field and Carroll^{4,5}. The rest of this work presents a mechanism that can provide the desired Hubble size fields, justifying the estimate (3). We shall explain the mechanism and give simple estimates here, but present details of the calculations elsewhere⁶.

3 Domain Walls

The key players in our mechanism are domain walls formed at the QCD phase transition that possess an internal structure with QCD scale. We shall present a full exposition about these types of walls in another paper⁶ but, to be specific, here we shall discuss the so-called axion- η' ($a_{\eta'}$) domain wall⁶.

We start with a similar effective Lagrangian to that used by Huang and Sikivie except that we included the effects of the η' singlet field which they

neglected. The Lagrangian density is

$$\mathcal{L}_{\text{eff}} = \frac{f_a^2}{2} |\partial_\mu e^{ia}|^2 + \frac{f_\pi^2}{4} \text{Tr} |\partial_\mu \mathbf{U}|^2 - V(\mathbf{U}, a) \quad (4)$$

where a is the dimensionless axion field and the matrix $\mathbf{U} = \exp(i\eta' + i\pi^f \lambda^f)$ contains the pion and η' fields (to simplify the calculations, we consider only the $SU(2)$ flavor group). The η' field is not light, but as we shall see, is the dominant player in aligning the magnetic fields so we include it. The potential V is given by

$$V = \frac{1}{2} \text{Tr} (\mathbf{M} \mathbf{U} e^{ia} + \text{h.c.}) - E \cos \left(\frac{i \ln[\det(\mathbf{U})]}{N_c} \right) \quad (5)$$

which was first introduced by Halpern and Zhitnitsky.⁹ It should be realized that $i \ln[\det(\mathbf{U})] \equiv i \ln(\det(\mathbf{U})) + 2\pi n$ is a multivalued function and we must choose the minimum valued branch. Details about this potential are discussed in the original paper⁹ but several points will be made here. All dimensionful parameters are expressed in terms of the QCD chiral and gluon vacuum condensates and are well known numerically: $\mathbf{M} = -\text{diag}(m_q^i |\langle \bar{q}^i q^i \rangle|)$ and $E = \langle b\alpha_s / (32\pi) G^2 \rangle$.

The result is that two different types of axion domain walls form.⁶ One is almost identical to the one discussed by Huang and Sikivie⁸ with small corrections due to the η' field. We shall call this the axion/pion (a_π) domain wall. The second type, which we shall call the axion/eta' ($a_{\eta'}$) domain wall is a new solution characterized by a transition in both the axion and η' fields (see our other paper⁶ for a complete description of this wall). The boundary conditions (vacuum states) for this wall are $a(-\infty) = \eta'(-\infty) = 0$ and $a(\infty) = \eta'(\infty) = \pm\pi$ with $\pi^0 = 0$ at both boundaries.

The main difference between the structures of the two walls is that, whereas the a_π domain wall has structure only on the huge scale of m_a^{-1} , the η' transition in the $a_{\eta'}$ has both scales, the axion scale, m_a^{-1} , as well as $\Lambda_{\text{QCD}}^{-1}$ scale. Therefore, the $a_{\eta'}$ domain wall has a “sandwich” structure. The reason is that, in the presence of the non-zero axion (θ) field, the pion becomes effectively massless due to its Goldstone nature. The η' is not so sensitive to θ parameter and so its mass never becomes zero. It is crucial that the walls have a structure of scale $\Lambda_{\text{QCD}}^{-1}$: thus there is no way for the a_π wall to trap nucleons because of the huge difference in scales but the $a_{\eta'}$ wall has exactly this structure and can therefore efficiently align the nucleons. The QCD domain walls (which were also discussed in⁶) have the same property as $a_{\eta'}$ walls. Namely, they have the structure of scale $\Lambda_{\text{QCD}}^{-1}$ and they can play the same role as $a_{\eta'}$ walls. In what follows, for more concreteness, we use $a_{\eta'}$ walls.

The model we propose is this: Immediately after the phase transition, the universe is filled with domain walls on the scale of T_{QCD}^{-1} . As the temperature drops, these domain walls coalesce, resulting in an average of one large domain wall per Hubble volume with Hubble scale correlations^{10,11}. It is these Hubble scale $a_{\eta'}$ domain walls which align the dipole moments of the nucleons producing the seed fields.

The following steps are crucial for this phenomenon:

1. The coalescing of QCD domain wall gives the fields π, \dots, η' Hubble scale correlations.
2. These fields interact with the nucleons producing Hubble scale correlations of nucleon spins residing in the vicinity of the domain wall. (The spins align perpendicular to the wall surface.)
3. Finally, the nucleons, which carry electric and magnetic moments (due to strong CP violation), induce Hubble scale correlated magnetic and electric fields.
4. These magnetic and electric fields eventually induce a nonzero helicity which has the same correlation. This helicity enables the inverse cascade.

4 Domain Wall Properties

We present here a method for simplifying the calculations of the bulk properties of domain walls. This method makes the approximation that the domain wall is flat and that translational and rotational symmetries are preserved in the plane of the wall which we take to be the x - y plane. These approximations are valid in the case of domain walls whose curvature is large in comparison to the length scale of the pertinent physics.

Once this approximation is made, we can reformulate the problem in $1+1$ dimensions (z and t) and calculate the density of the desired bulk properties along the domain wall. To regain the full four-dimensional bulk properties, we must estimate the density of the particles in the x - y plane to obtain the appropriate density and degeneracy factors for the bulk density. Thus, the final results are not independent of physics in the x - y plane, but rather, these effects are accounted for only through the degeneracy factors.

4.1 Alignment of Spins in the Domain Wall Background

We proceed to demonstrate this technique by calculating the alignment of fermionic spins along the wall. To estimate the strengths of the fields involved, we consider only the η' transition because it has a similar structure in both

the $a_{\eta'}$ domain wall and the QCD domain walls. We take the following simple interaction between the η' field and the nucleons:

$$\bar{\Psi} \left[i\not{\partial} - m_N e^{i\eta'(z)\gamma_5} \right] \Psi. \quad (6)$$

For our approximations, we assume that fluctuations in the nucleon fields do not affect the domain walls and, thus, treat the domain walls as a background field^a. The strategy is to break (6) into two 1 + 1 dimensional components by setting $\partial_x = \partial_y = 0$ and then by manipulating the system of equations that result to obtain an equivalent two-dimensional system.

First, we introduce the following chiral components of the Dirac spinors

$$\Psi_+ \equiv \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}, \quad \Psi_- \equiv \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}, \quad \Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} \Psi_+ + \Psi_- \\ \Psi_+ - \Psi_- \end{pmatrix} \quad (7)$$

Secondly, we assume that our system is effectively two-dimensional (an infinite domain wall lying in the x - y plane) and hence neglect the conserved x and y momenta. We find that the Dirac equations which follow from (7) are equivalent to the coupled system

$$\begin{aligned} [i\partial_0 + i\sigma_3\partial_3] \Psi_+ &= m_N e^{-i\eta'} \Psi_- \\ [i\partial_0 - i\sigma_3\partial_3] \Psi_- &= m_N e^{+i\eta'} \Psi_+ \end{aligned} \quad (8)$$

in Ψ_+ and Ψ_- . At this stage it proves convenient to rearrange these equations by introducing new “two-dimensional” Dirac spinors $\Psi^{(1)}$ and $\Psi^{(2)}$

$$\Psi^{(1)} = \begin{pmatrix} \chi_1 \\ \eta_1 \end{pmatrix}, \quad \Psi^{(2)} = \begin{pmatrix} \eta_2 \\ \chi_2 \end{pmatrix} \quad (9)$$

Using the definitions (9), we put system (8) into the form of two two-dimensional (2D) Dirac equations

$$\begin{aligned} [i\hat{\gamma}_\mu \partial_\mu - m_N e^{-i\eta' \hat{\gamma}_5}] \Psi^{(1)} &= 0 \\ [i\hat{\gamma}_\mu \partial_\mu - m_N e^{+i\eta' \hat{\gamma}_5}] \Psi^{(2)} &= 0 \end{aligned} \quad (10)$$

where we have introduced the 2D Dirac matrices

$$\hat{\gamma}_0 = \sigma_1, \quad \hat{\gamma}_1 = -i\sigma_2, \quad \hat{\gamma}_5 = \hat{\gamma}_0 \hat{\gamma}_1 = \sigma_3, \quad \hat{\gamma}_\mu \hat{\gamma}_\nu = g_{\mu\nu} + \epsilon_{\mu\nu} \hat{\gamma}_5. \quad (11)$$

^aA full account would take into account the effects of this back-reaction. We expect that they would affect the potential (5) by altering the form of the last term $E \cos(\cdot)$ and possibly adding higher order corrections, but that they would not alter the nature of the domain walls. Quantitatively this would alter the numerical results, but would not change the qualitative picture presented here.

Equations (10) are reproduced from the following effective 2D Lagrangian

$$\mathcal{L}_{2D} = \bar{\Psi}^{(1)} \left[i\hat{\gamma}_\mu \partial_\mu - m_N e^{i\eta' \hat{\gamma}_5} \right] \Psi^{(1)} + \bar{\Psi}^{(2)} \left[i\hat{\gamma}_\mu \partial_\mu - m_N e^{-i\eta' \hat{\gamma}_5} \right] \Psi^{(2)}, \quad (12)$$

where $\eta' = \eta'(z)$ is the background classical field with boundary conditions $\eta'(-\infty) = 0$, $\eta'(\infty) = \pi$. This 2D Lagrangian describes two species of 2D Dirac fermions of opposite chiral charge interacting with the external η' field.

Now we are ready to demonstrate that the domain walls align the spins of the fermions. The relevant operator (which becomes the spin operator for the nonrelativistic system) is

$$\Psi^\dagger \vec{\Sigma} \Psi = \bar{\Psi} \vec{\gamma} \gamma_5 \Psi = \Psi_+^\dagger \vec{\sigma} \Psi_+ + \Psi_-^\dagger \vec{\sigma} \Psi_-, \quad \vec{\Sigma} \equiv \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}, \quad (13)$$

and our goal is to demonstrate that the mean value $\langle \Sigma_z \rangle$ of this operator is generally non-zero in the domain wall. Thus, the nucleon spins are aligned in the z direction and have a correlation length similar to the domain wall.

The easiest way to demonstrate this phenomenon in our model (which is effectively 2D) is to use the Goldstone-Wilczek adiabatic approximation^{12,13} together with a bosonization trick

$$\begin{aligned} \bar{\Psi}^{(i)} i\hat{\gamma}_\mu \partial_\mu \Psi^{(i)} &\rightarrow \frac{1}{2} \partial_\mu \phi_i \partial_\mu \phi_i & \bar{\Psi}^{(i)} i\hat{\gamma}_5 \Psi^{(i)} &\rightarrow -\mu \sin(2\sqrt{\pi} \phi_i) \\ \bar{\Psi}^{(i)} \hat{\gamma}_\mu \Psi^{(i)} &\rightarrow \frac{1}{\sqrt{\pi}} \varepsilon_{\mu\nu} \partial_\nu \phi_i & \bar{\Psi}^{(i)} \Psi^{(i)} &\rightarrow -\mu \cos(2\sqrt{\pi} \phi_i) \end{aligned} \quad (14)$$

($\mu \sim m_N$ is a scale parameter). The Lagrangian (12) after bosonization is

$$\begin{aligned} \mathcal{L}_{2D} &= \frac{1}{2} [(\partial_\mu \phi_1)^2 + (\partial_\mu \phi_2)^2] - U(\phi_1, \phi_2) \\ U(\phi_1, \phi_2) &= -\mu m_N [\cos(2\sqrt{\pi} \phi_1 - \eta') + \cos(2\sqrt{\pi} \phi_2 + \eta')] \end{aligned} \quad (15)$$

The adiabatic approximation¹³ is to neglect the kinetic terms in the analysis of the dynamics of ϕ_1 and ϕ_2 fields in Equation (15). In this case, the mean values $\langle \phi_1(z) \rangle$ and $\langle \phi_2(z) \rangle$ will follow the background field $\eta'(z)$, and can be found by minimizing the potential U in Equation (15).

$$\langle \phi_1(z) \rangle = \frac{\eta'(z)}{2\sqrt{\pi}}, \quad \langle \phi_2(z) \rangle = -\frac{\eta'(z)}{2\sqrt{\pi}} \quad (16)$$

To calculate the induced spin $\Psi^\dagger \vec{\Sigma} \Psi$ in our theory we should present the spin operator in terms of 2D fields ϕ_1 and ϕ_2 , and replace these fields by their mean values (16) in the domain wall background:

$$\Psi^\dagger \Sigma_z \Psi = \bar{\Psi}^{(1)} \hat{\gamma}_0 \Psi^{(1)} - \bar{\Psi}^{(2)} \hat{\gamma}_0 \Psi^{(2)} = \frac{1}{\sqrt{\pi}} (\partial_z \phi_1(z) - \partial_z \phi_2(z)), \quad (17)$$

where, in the last step, we used the bosonic representation for the 2D $\Psi^{(i)}$ fields^b. The last step is to replace these fields by their mean-values (16)

$$\langle \Psi^\dagger \Sigma_z \Psi \rangle = N \times \frac{1}{\pi} \frac{\partial \eta'(z)}{\partial z}, \quad (18)$$

where N is the appropriate normalization and degeneracy factor for the Ψ field which has canonical dimension 3/2 in four dimensions while the 2D $\Psi^{(i)}$ fields have canonical dimension 1/2.

4.2 Fermion Degeneracy in the Domain wall Background

We have assumed that locally the domain walls have only a spatial z dependence. This implies that there is still a 2-dimensional translational and rotational symmetry in the x - y plane. These translational degrees of freedom imply that momentum in the plane is conserved and hence we can treat the neglected degrees of freedom for the fermions as free degrees. The degeneracy in a region of area S will simply be a sum over these degrees with a discrete factor $g = 4 = 2 \times 2$ for spin and isospin degeneracy

$$N = g \int \frac{dx dy dp_x dp_y}{(2\pi)^2} = \frac{g p_F^2}{4\pi} S \simeq \frac{\Lambda_{QCD}^2}{\pi} S \quad (19)$$

where we estimated the Fermi energy $p_F \simeq \Lambda_{QCD} \simeq 150$ MeV. As expected, the degeneracy is proportional to the area of the domain wall S . Now it is clear that an appropriate normalization for the two dimensional $\Psi^{(i)}$ fields can be achieved by adding a factor $1/\sqrt{S}$ in the definition (9). In this case these 2D fields have correct canonical dimension 1/2. Now we are ready to estimate the original four-dimensional expectation value (18):

$$\langle \Psi^\dagger \Sigma_z \Psi \rangle_{4D} = \frac{1}{S} \times N \times \frac{1}{\pi} \frac{\partial \eta'(z)}{\partial z} \simeq \frac{\Lambda_{QCD}^2}{\pi^2} \frac{\partial \eta'(z)}{\partial z} \sim \frac{\Lambda_{QCD}^2 m_{\eta'}}{\pi^2}, \quad (20)$$

which has correct dimension 3. Using the same technique one can estimate other matrix elements in the domain wall background which have non-zero magnitude and thus demonstrate that they have a large correlation L on the of the size of the domain wall in the x - y direction. In particular, the result for the mean value $\langle \bar{\Psi} \gamma_5 \sigma_{xy} \Psi \rangle$ is:

$$\langle \bar{\Psi} \sigma_{xy} \gamma_5 \Psi \rangle_{4D} \sim \mu \frac{\Lambda_{QCD}^2}{\pi}, \quad (21)$$

^bThe 2D problem under discussion is quite familiar to physics community: namely, the calculation of the induced fermion charge in a solitonic background.

where the factor $\sim \Lambda_{\text{QCD}}^2/\pi$ has the same origin as in Equation (20) and is related to the degeneracy of the system (19), while the factor $\mu \sim m_N$ is a dimensional parameter originating from the bosonic representation (15) of the effective two-dimensional theory.

5 Magnetic Field Generation Mechanism

Here we estimate the strengths of the induced fields in terms of the QCD parameters. We consider two types of interactions. First, the nucleon spins align with the domain wall. We assume that the fluctuations in the nucleon field Ψ are rapid and that these effects cancel, leaving the classical domain wall background unaltered. Thus, we are able to estimate many mean values correlated on a large scale on the domain walls such as $\langle \bar{\Psi} \gamma_5 \sigma_{xy} \Psi \rangle$ (21) and $\langle \bar{\Psi} \gamma_z \gamma_5 \Psi \rangle$ (20) through these interactions as described above. These mean values are only nonzero within a distance $\Lambda_{\text{QCD}}^{-1}$ of the domain wall and are correlated on the same Hubble scale as the domain wall.

From now on we treat the expectation value (21) as a background classical field correlated on the Hubble scale. Once these sources are known, one could calculate the generated electromagnetic field by solving Maxwell's equations with the interaction

$$\mathcal{L}_{\text{int}} = \frac{1}{2} (d_\Psi \bar{\Psi} \sigma_{\mu\nu} \gamma_5 \Psi + \mu_\Psi \bar{\Psi} i \sigma_{\mu\nu} \Psi) F_{\mu\nu} + \bar{\Psi} (iD)^2 \Psi \quad (22)$$

where d_Ψ (μ_Ψ) is effective electric (magnetic) dipole moments of the field Ψ . Due to the CP violation (nonzero θ) along the axion domain wall, the anomalous nucleon dipole moment in (22) $d_\Psi \sim \mu_\Psi \sim \frac{e}{m_N}$ is also nonzero¹⁴. This is an important point: if no anomalous moments were induced, then only charged particles could generate the magnetic field: the walls would be diamagnetic not ferromagnetic as argued by Voloshin¹⁵ and Landau levels would exactly cancel the field generated by the dipoles.

Solving the complete set of Maxwell's equations, however, is extremely difficult. Instead, we use simple dimensional arguments. For a small planar region of area ξ^2 filled with aligned dipoles with constant density, we know that the net magnetic field is proportional to ξ^{-1} since the dipole fields tend to cancel, thus for a flat section of our domain wall, the field would be suppressed by a factor of $(\xi \Lambda_{\text{QCD}})^{-1}$. For a perfectly flat, infinite domain wall ($\xi \rightarrow \infty$), there would be no net field as pointed out¹⁵. However, our domain walls are far from flat. Indeed, they have many wiggles and high frequency modes, thus, the size of the flat regions where the fields are suppressed is governed by a correlation ξ which describes the curvature of the wall. Thus, the average

electric and magnetic fields produced by the domain wall are of the order

$$\langle F_{\mu\nu} \rangle \simeq \frac{1}{\xi^* \Lambda_{\text{QCD}}} [d_\Psi \langle \bar{\Psi} \sigma_{\mu\nu} \gamma_5 \Psi \rangle + \mu_\Psi \langle \bar{\Psi} i \sigma_{\mu\nu} \Psi \rangle] \quad (23)$$

where ξ^* is an effective correlation length related to the size of the dominant high frequency modes.

To estimate what effective scale ξ^* has, however, requires an understanding of the dynamics of the domain walls. Initially, the domain walls are correlated with a scale of $\Lambda_{\text{QCD}}^{-1}$. As the temperature cools, the walls smooth out and the lower bound $\xi(t)$ for the scale of the walls correlations increases from $\xi(0) \simeq \Lambda_{\text{QCD}}^{-1}$. This increase is a dynamical feature, however, and is thus slow. In addition, the walls coalesce and become correlated on the Hubble scale generating large scale correlations. Thus the wall has correlations from $\xi(t)$ up to the upper limit set by the Hubble scale. Thus, the effective $\xi^* \ll$ Hubble size at the time that the fields are aligned and so the suppression is not nearly as great as implied by Voloshin¹⁵. Note that, even though the effects are confined to the region close to the wall, the domain walls are moving and twisted so that the effects occur throughout the entire Hubble volume.

The picture is thus that fields of strength

$$\langle E_z \rangle \simeq \langle B_z \rangle \sim \frac{1}{\xi^* \Lambda_{\text{QCD}}} \frac{e}{m_N} \frac{m_N \Lambda_{\text{QCD}}^2}{\pi} \sim \frac{e \Lambda_{\text{QCD}}}{\xi^* \pi} \quad (24)$$

are generated with short correlations ξ^* , but then domains are correlated on a large scale by the Hubble scale modes of the coalescing domain walls. Thus, strong turbulence is generated with correlations that run from Λ_{QCD} up to the Hubble scale.

Finally, we note that this turbulence should be highly helical. This helicity arises from the fact that both electric and magnetic fields are correlated together along the entire domain wall, $\langle \vec{\mathbf{E}} \rangle \sim \langle \vec{\mathbf{A}} \rangle / \tau$ where $\langle \vec{\mathbf{A}} \rangle$ is the vector potential and τ is a relevant timescale for the electrical field to be screened (we expect $\tau \sim \Lambda_{\text{QCD}}^{-1}$ as we discuss below). The magnetic helicity density is thus:

$$h \sim \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \sim \tau \langle E_z \rangle \langle B_z \rangle \sim \tau \frac{e^2}{\pi^2} \frac{\Lambda_{\text{QCD}}^2}{\xi^{*2}}. \quad (25)$$

Note carefully what happens here: The total helicity was zero in the quark-gluon-plasma phase and remains zero in the whole universe, but the helicity is separated so that in one Hubble volume, the helicity has the same sign. The reason for this is that, as the domain walls coalesce, initial perturbations cause either a soliton or an antisoliton to dominate and fill the Hubble volume. In the neighboring space, there will be other solitons and antisolitons so

that there is an equal number of both, but they are separated and this spatial separation prevents them from annihilating. This is similar to how a particle and anti-particle may be created and then separated so they do not annihilate. In any case, the helicity is a pseudoscalar and thus maintains a constant sign everywhere along the domain wall: thus, the entire Hubble volume is filled with helicity of the same sign. This is the origin of the Hubble scale correlations in the helicity and in B^2 . The correlation parameter ξ which affects the magnitude of the fields plays no role in disturbing this correlation.

As we mentioned, eventually, the electric field will be screened. The timescale for this is set by the plasma frequency for the electrons (protons will screen much more slowly) ω_p which turns out to be numerically close to Λ_{QCD} near the QCD phase transition. The nucleons, however, also align on a similar timescale $\Lambda_{\text{QCD}}^{-1}$, and the helicity is generated on this scale too, so the electric screening will not qualitatively affect the mechanism. Finally, we note that the turbulence requires a seed which remains in a local region for a timescale set by the conductivity¹⁶ $\sigma \sim cT/e^2 \sim \Lambda_{\text{QCD}}$ where for $T = 100$ MeV, $c \approx 0.07$ and is smaller for higher T . Thus, even if the domain walls move at close to the speed of light (due to vibrations), there is still enough time to generate turbulence.

For this mechanism to work and not violate current observations, it seems that the domain walls must eventually decay. Several mechanisms have been discussed for the decay of axion domain walls^{10,17} and the timescales for these decays are much larger than $\Lambda_{\text{QCD}}^{-1}$, ie. long enough to generate these fields but short enough to avoid cosmological problems. In addition, we have found some additional structures which may help solve this problem. We shall present these elsewhere⁶. In any case, we assume that some mechanism exists to resolve the domain wall problem in an appropriate timescale. Thus, all the relevant timescales are of the order $\Lambda_{\text{QCD}}^{-1}$ except for the lifetime of the walls which is substantially longer and thus, although the discussed interactions will affect the qualitative results, they will not affect the mechanism or substantially change the order of the effects.

6 Conclusion.

We have shown that this mechanism can generate the magnetic fields (3) with large correlations. It seems that galactic dynamos should still play an important amplification role. It seems that the crucial conditions for the dynamo to take place are fields $B > 10^{-20}$ G with large (100 kpc) correlations. From (3) we see that we have a huge interval $10^{-10} \ll \xi^* \Lambda_{\text{QCD}} \leq 1$ of ξ^* to seed these dynamos. Also, if ξ^* is small, then this mechanism may generate measurable

extra-galactic fields.

We mention two new points that distinguish this mechanism from previous proposals¹⁸. First, the key nucleon is the neutron which generates the fields due to an anomalous dipole moment induced by the CP violating domain walls. The nucleons thus make the wall ferromagnetic, not diamagnetic as discussed in¹⁵. Second, the interaction between the domain walls and nucleons are substantial because of the similar scale ($\Lambda_{\text{QCD}}^{-1}$) of the η' transition in the $a_{\eta'}$ domain wall. There is no way that axion domain walls with scales $\sim m_a^{-1}$ can efficiently align nucleons at a temperature T_{QCD} .

The presence of the magnetic fields generated by our mechanism may have several observable effects. First, large magnetic fields may alter nucleosynthesis production ratios¹⁹. Secondly, large scale magnetic fields may distort the CMB spectrum in a measurable manner²⁰. These place upper bounds on the strength of the fields. Even the maximal fields (24) with $\xi^* \Lambda_{\text{QCD}} = 1$ generated by domain walls lie within these bounds. Also, if ξ^* turns out to be quite small, then, unless the distribution of galaxies is correlated with the domain walls, this mechanism might generate measurable extra-galactic fields.

Two other effects may be closely related to magnetic fields generated from domain walls. One is the observation of ultra-high energy cosmic rays past the GZK cutoff²¹. Magnetic fields on the scale of those discussed here may hold a key to explaining this mystery. The other is an apparent anisotropy of radiation propagation over large distances resulting in a constant offset in Faraday measurements²². One possible explanation involves the introduction of a Chern-Simons term by hand²³. This type of term might arise naturally from CP violating domain walls.

Domain walls at the QCD phase transition provide a nice method of generating magnetic fields on 100 kpc correlations today (3). In addition, the fields and domain walls key to this mechanism may play a role in a number of unexplained astrophysical phenomena. We conclude on this optimistic note.

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