

RELATIVISTIC INSTANT-FORM APPROACH TO THE STRUCTURE OF TWO-BODY COMPOSITE SYSTEMS

A.F.Krutov*

Samara State University, 443011, Samara, Russia

V.E.Troitsky†

Nuclear Physics Institute, Moscow State University, 119899, Moscow, Russia
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A new approach to the electroweak properties of two-particle composite systems is developed. The approach is based on the use of the instant form of relativistic Hamiltonian dynamics. The main novel feature of this approach is the new method of construction of the matrix element of the electroweak current operator. The electroweak current matrix element satisfies the relativistic covariance conditions and in the case of the electromagnetic current also the conservation law automatically. The properties of the system as well as the approximations are formulated in terms of form factors. The approach makes it possible to formulate relativistic impulse approximation in such a way that the Lorentz-covariance of the current is ensured. In the electromagnetic case the current conservation law is ensured, too. The results of the calculations are unambiguous: they do not depend on the choice of the coordinate frame and on the choice of "good" components of the current as it takes place in the standard form of light-front dynamics. Our approach gives good results for the pion electromagnetic form factor in the whole range of momentum transfers available for experiments at present time, as well as for lepton decay constant of pion.

I. INTRODUCTION.

All atoms, nuclei, and the main part of the so called elementary particles are composite systems. That is why the constructing of correct quantitative methods of calculation for composite-particle structure is an important line of investigations in particle physics. In nonrelativistic dynamics there exist different correct methods which use model or phenomenological interaction potentials. However, in the case of high energy one needs to develop relativistic methods. It is worth to note that now the experiments on accelerators, in particular at JLab are performed with such an accuracy that the treatment of traditionally "nonrelativistic" systems (e.g. the deuteron)

needs to take into account relativistic effects. Relativistic effects are important also in the treatment of composite systems of light quarks. In this case the relativistic effects are significant even at low energy. However, the relativistic treatment of hadron composite systems is a rather complicated problem. To solve it one needs, in fact, to solve a many-particle relativistic problem with, in addition, not precisely known interaction. Let us note that the use of the methods of the field theory in this case encounters serious difficulties. For example, it is well known that the perturbative QCD can not be used in the case of quark bound states (see, e.g., [1,2]).

In the present paper we will use the relativistic constituent model which describes the hadron properties at quark level in terms of degrees of freedom of constituent quarks. The constituent quarks are considered as extended objects, the internal characteristics of which (MSR, anomalous magnetic moments, form factors) are parameters of model. As relativistic variant of constituent model we choose the method of relativistic Hamiltonian dynamics (RHD) (see, e.g., [3-6] and the references therein).

Our aim is to construct a relativistic invariant approach to electroweak structure of two-particle composite systems. The main problem here is the construction of the current operators [7-11].

It seems to us that RHD is the method the most adequate for our purpose. The use of RHD enables one to separate the main degrees of freedom and so to construct convenient models.

We use one of the forms of RHD, namely the version of the instant form (IF).

Our approach has a number of features that distinguish it from other forms of dynamics and other approaches in the frames of IF.

- The electroweak current matrix element satisfies the relativistic covariance conditions and in the case of the electromagnetic current also the conservation law automatically.
- We propose a modified impulse approximation (MIA). It is constructed in relativistic invariant

*Electronic address: krutov@ssu.samara.ru

†Electronic address: troitsky@theory.npi.msu.su

way. This means that our MIA does not depend on the choice of the coordinate frame, and this contrasts principally with the "frame-dependent" impulse approximation usually used in instant form (IF) of dynamics.¹

- Our approach provides with correct and natural nonrelativistic limit ("the correspondence principle" is fulfilled).
- For composite systems (including the spin 1 case) the approach guarantees the uniqueness of the solution for form factors and it does not use such concepts as "good" and "bad" current components.
- The approach describes correctly the spin Wigner rotation and this fact makes it possible to obtain the correct (QCD) asymptotic.

The RHD method as the relativistic theory of composite systems is based on the direct realization of the Poincaré algebra on the set of dynamical observables on the Hilbert space (see, e.g., [3–6] and the references therein). RHD theory of particles lie between local field theoretic models and nonrelativistic quantum mechanical models.

Let us describe briefly the main statements of RHD.

As it is known (see, e.g., [12]), the relativistic invariance of a theory means that there exists (on the Hilbert space of states) the unitary representation of the inhomogeneous group $SL(2, C)$, which is the universal covering of the Poincaré group. The relativistic invariance means the validity of the Poincaré algebra commutation relations for the generators of space-time translations \hat{P}^μ and rotations $\hat{M}^{\mu\nu}$. To construct the representation $SL(2, C)$ means to obtain these generators in terms of dynamical variables of the system. In the case of free particle system this program is not difficult to be realized and the generators \hat{P}^μ , $\hat{M}^{\mu\nu}$ have clear physical meaning: $\hat{P}^0 \equiv \hat{H}$ — is the total energy operator, $\vec{\hat{P}} = (\hat{P}^1, \hat{P}^2, \hat{P}^3)$ — is the operator of the total 3-momentum, $\vec{\hat{J}} = (\hat{M}^{23}, \hat{M}^{31}, \hat{M}^{12})$ — the operator of the total angular momentum, $\vec{\hat{N}} = (\hat{M}^{01}, \hat{M}^{02}, \hat{M}^{03})$ — the generators of Lorentz boosts.

In the case of interacting systems the situation is different and the construction of the generators in terms of dynamical variables encounters some difficulties. To make clear the sense of these difficulties let us compare RHD with the nonrelativistic quantum mechanics. In nonrelativistic case it is the Galilean group that is the group of invariance. In the case of nonrelativistic interacting system the interaction operator enters (in additive way) only the generator of time translations (energy operator). The interaction operator satisfies the usual conditions of invariance under translations and rotations and of independence on the choice of inertial coordinate frame. Under these conditions the algebraic relations for the generators of the Galilei group written in terms of dynamical variables remain to be valid after the interaction including. This is the meaning of the Galilean invariance of the theory. So, in nonrelativistic theory of interacting particles it is only one generator of Galilei group — the Hamiltonian — that contains the interaction. Other generators have the same form as in the case of free particles. This way of interaction including in the observables algebra is unique in nonrelativistic case and gives the unique nonrelativistic dynamics — the dynamics governed by the Schrödinger equation.

The situation in the relativistic case is quite different. The structure of the Poincaré algebra is of such kind that the interaction including in the total energy operator alone results in the breaking of algebraic structure, i.e. in the relativistic invariance breaking. This is the consequence of the fact that the Lorentz transformations mix space and time. To preserve the algebra it is necessary to include the interaction in the other generators, too. Now the generators of Poincaré algebra can be divided into two classes: containing interaction (interacting generators) – hamiltonians, and without interaction (non-interacting generators). The latter present the so called kinematical subgroup. This division, that is the interaction inclusion in the Poincaré group, is not unique. The different ways of such a division preserving the Poincaré algebra result in different types of relativistic dynamics (see, e.g., [4]).

The idea of this approach — RHD — is originated by Dirac. In [13] he considered different ways of description of the evolution of classical relativistic systems — different forms of dynamics. Dirac separated the concept of time as a coordinate and that of time as a parameter defining the system evolution. Consequently, he defined three main forms of dynamics with different evolution parameters: point (PF), instant (IF) and light-front (FF) dynamics. Each of these forms has its relative advantages and disadvantages. The total number of possible dynamics is actually five [3] so that the unique non-relativistic Hamilton description is changed in relativistic

¹ It is known that correct impulse approximation (IA) realization in the frame of traditional version of IF dynamics encounters difficulties: the standard IA depends on the choice of the coordinate frame. We show below that IA can be formulated in an invariant way, the composite system form factors being defined by the one-particle currents alone.

tic case for, generally, five possibilities. Each of these possible dynamics is connected with a three-dimensional hypersurface in the four dimensional space. The initial conditions are given on these hypersurfaces and the evolution of the hypersurfaces is described. The kinematical subgroups are the invariance groups of the hypersurfaces. In particular, these hypersurfaces are: hyperboloid $x^\mu x_\mu = a^2$, $t > 0$ (PF), hyperplane $t = 0$ (IF), light-cone surface $x^0 + x^3 = 0$ (FF).

It is worth to notice that RHD and the field theory are quite different approaches. The establishment of the connection between RHD and field theory is a difficult and as yet unresolved problem. Contrary to field theory, RHD is dealing with finite number of degrees of freedom from the very beginning. This is certainly a kind of a model approach. The preserving of the Poincaré algebra ensures the relativistic invariance (see for details the Sec.II). So, the covariance of the description in the frame of RHD is due to the existence of the unique unitary representation of the inhomogeneous group $SL(2, C)$ on the Hilbert space of composite system states with finite number of degrees of freedom. The success of composite models shows the validity of approximate relativistic invariant description with fixed number of particles and a finite number of degrees of freedom. The similar situation (as was pointed out in [3]) takes place in the solid state theory: many of solid state properties are connected directly with its symmetry group and the actual form of the particle interaction plays less important role. RHD is based on the simultaneous action of two fundamental principles: relativistic invariance and Hamiltonian principle — and presents the tool the most adequate to treat the systems with finite number of degrees of freedom. It is worth to notice that the mathematics of RHD is similar to that of nonrelativistic quantum mechanics and permits to assimilate the sophisticated methods of phenomenological potentials and can be generalized to describe three or more particles.

Now the problem of the choice of the actual form of RHD arises.

Some time ago it was proved that S -matrices are equivalent in the different dynamics forms [14]. This fact is interesting but it does not mean the absolute equivalence of the forms. First, there are problems which can not be reduced to S -matrix, e.g., the calculation of form factors. Second, one has to keep in mind that any concrete calculation uses some approximations; the approximations usually used in different forms of dynamics are nonequivalent.

Our point of view is the following. One must choose the form of dynamics adequate to the problem in question and to the approximations to be done. It seems us that this is in the spirit of RHD — the choosing of the adequate

degrees of freedom.

RHD is widely used in the theory of electromagnetic properties of composite quark and nucleon systems [6], [8], [11], [15–28]. It was shown that RHD not only presents an interesting relativistic model but is a fruitful tool and can compete with other approaches in describing the existing experimental data (especially at small and moderate momentum transfers). At present time the FF dynamics is the most developed and used for composite systems [8,11,15,16,18,19]. In fact, the light front dynamics has obvious advantages: a) the minimal number of operators containing the interaction (three); b) the simple relativistic invariant separation of the variables into classes of "internal" and "external" variables (while considering the approach as Hamiltonian theory with fixed particle number); c) the simple vacuum structure for the light-front perturbative field theory. However, there are some difficulties in the FF RHD approach. In particular, it was shown [15,29] that the calculated electromagnetic form factors for the systems with the total angular momentum $J = 1$ (the deuteron, the ρ - meson) vary significantly with the rotation of the coordinate frame. This ambiguity is caused by the breaking of the so called angle condition [15,29], that is really by the breaking of the rotation invariance of the theory. Some of the difficulties of FF dynamics are discussed in [30]. A possible way to solve the problem by adding some new (nonphysical) form factors to the electromagnetic current was proposed (see [31] and references therein).

A different approach to the problem was proposed recently in Ref. [11], where a new method of construction of electromagnetic current operators in the frame of FF dynamics was given. The method of [11] gives unambiguous deuteron form factors. However, as the authors of [11] note themselves, their current operator and the one used in Ref. [8] are different, since both of them are obtained from the free one, but in different reference frames, related by an interaction dependent rotation.

All these facts naturally cause authors to consider other forms of relativistic Dirac dynamics.

Recently the PF dynamics was considered in the papers [6,23,26,27]. The authors used PF dynamics to calculate the processes under investigations in JLab experiments. It is worth to notice that these experiments enlarge the interest to the RHD approach — the relativistic theory which can be used in the region of soft processes.

Now we present a relativistic treatment of the problem of soft electroweak structure in the framework of another form — IF of RHD. IF of relativistic dynamics, although not widely used, has some advantages. The calculations can be performed in a natural straightforward way without special coordinates. IF is particularly con-

venient to discuss the nonrelativistic limit of relativistic results. This approach is obviously rotational invariant, so IF is the most suitable for spin problems.

We describe the dynamics of composite systems (the constituent interaction) in the frame of general RHD axiomatics. However, our approach differs from the traditional RHD by the way of constructing of matrix elements of local operators. In particular, our method of description of the electromagnetic structure of composite systems permits the construction of current matrix elements satisfying the Lorentz-covariance condition and the current conservation law.

To construct the current operator in the frame of IF RHD we use the general method of relativistic invariant parameterization of matrix elements of local operators proposed as long ago as in 1963 by Cheshkov and Shirokov [32].

The method of [32] gives matrix elements of the operators of arbitrary tensor dimension (Lorentz-scalar, Lorentz-vector, Lorentz-tensor) in terms of a finite number of relativistic invariant functions – form factors. The form factors contain all the dynamical information on the transitions defined by the operator. That is why a system can be described in terms of form factors.

The method of parameterization is similar in spirit to the method of presentation of matrix elements of irreducible tensor operators on the rotation group in terms of reduced matrix elements. This method extracts from the matrix element of a tensor operator a part defining symmetry properties and selection rules following the well known Wigner-Eckart theorem.

In the review [4] two possible variants of such kind of representation of matrix elements in terms of form factors are presented – the elementary-particle parameterization and the multipole parameterization. The variant of parameterization given in [32] is an alternative one. In [32] the authors propose the construction of matrix elements in canonical basis so it can be called canonical parameterization. This method was developed for the case of composite systems in [33,34]. The composite-system form factors in this approach are in general case the distributions (generalized functions), they are defined by continuous linear functionals on a space of test functions. Thus, for example, the current matrix elements for composite systems are functionals, generated by some Lorentz-covariant distributions, and the form factors are functionals generated by regular Lorentz-invariant generalized functions. We demonstrate these facts below, in Sec.III, using a simple model as an example.

It is worth to notice that the statement that the form factors of a composite system are generalized functions is not something exotic. This fact takes place in the standard nonrelativistic potential theory, too (see Sec.III(F)).

The use of canonical parameterization permits to describe the electroweak properties of composite systems satisfying the Lorentz-covariance condition on each stage of calculation and to satisfy the electromagnetic current conservation law when describing the electromagnetic properties. In our formalism it is necessary to formulate the composite model features in slightly unusual way in terms of matrix elements which are generalized functions.

In particular, the relativistic impulse approximation (IA) has to be reformulated.

Let us remind the physics of IA. In IA a test particle interacts mainly with each component separately, that is the electromagnetic current of the composite system can be described in terms of one-particle currents. In fact, the composite-system current is approximated by the corresponding free-system current. This means that exchange currents are neglected, or, in other words, that there is no three-particle forces in the interaction of a test particle with constituents. It is well known that the traditional IA breaks the Lorentz-covariance of the composite-system current and the conservation law for the electromagnetic current (see, e.g., [4] for details). As we show in the Sec.III(C) one can overcome these difficulties if one formulates IA in terms of form factors.

It is worth to notice that all known approaches (including the perturbative quantum field theory (QFT)) encounter difficulties while constructing a composite-system current operator satisfying Lorentz-covariance and conservation conditions. This problem is now discussed widely in the literature [7–11]. To satisfy the conservation law in the frame of Bethe-Salpeter equation and quasipotential equations, for example, it is necessary to go beyond IA: one has to add the so called two-particle currents to the current operator. In the case of nucleon composite systems these currents are interpreted as meson exchange currents [9]. In the case of deuteron this means the simultaneous interaction of virtual γ - quanta with proton and neutron. However, in Ref. [35] it is shown that the current conservation law can be satisfied without such processes, although they contribute to the deuteron form factor. It seems that at the present time there is an intention to formulate IA with transformed conservation properties without dynamical contribution of exchange currents [11,26,31].

Our formalism also gives, in fact, the description of the covariance properties of the operators in terms of many-particle as well as one-particle currents. However, the important feature of our formalism is the fact that form factors or reduced matrix elements describing the dynamics of transitions contain only the contributions of one-particle currents.

So, our approach to the construction of the current

operator includes the following main points:

1. We extract from the current matrix element of composite system the reduced matrix elements (form factors) containing the dynamical information on the process. In general these form factors are generalized functions.
2. Along with form factors we extract from the matrix element a part which defines the symmetry properties of the current: the transformation properties under Lorentz transformation, discrete symmetries, conservation laws etc.
3. The physical approximations which are used to calculate the current are formulated not in terms of operators but in terms of form factors.

In this paper we present the main points of our approach. To make it transparent we consider here only simple systems with zero total angular momenta, so that technical details do not mask the essence of the method. We demonstrate the effectiveness of the approach by calculating the pion electroweak properties. In this case the canonical parameterization is very simple and can be realized without difficulties. The case of more complicated systems needs using a rather sophisticated mathematics for canonical parameterization of local operator matrix elements and will be considered elsewhere.

The paper is organized as follows. In Sect. II we remind briefly the basic statements of RHD, especially of IF RHD. Two bases in the state space of composite system are considered: the basis with individual spins and momenta and the basis with separated center-of-mass motion. The Clebsh–Gordan decomposition for the Poincaré group which connects these bases is given. The IF wave functions of composite systems are defined. In Sect. III our approach to relativistic theory of two-particle composite systems and their electroweak properties is presented. A simple model is considered in details: two spinless particles in the S -state of relative motion, one of the particles being uncharged. The electromagnetic form factor of the system is derived. The standard conditions for the current operator are discussed. The modified impulse approximation (MIA) is proposed. The results of IA and MIA are compared. The nonrelativistic limit is considered. The connection between the presented version of IF RHD and the dispersion relations in mass is established. In Sect. IV the developed formalism is used in the case of the system of two particles with spins 1/2. The pion electromagnetic form factor and the lepton decay constant are derived. The model parameters are discussed and the comparison of the results with the experimental data is given. The results of calculations in IA and MIA are compared and are shown to differ significantly. In Sect. V the conclusion is given.

II. RELATIVISTIC HAMILTONIAN DYNAMICS.

In this Section some basic equations of RHD and some relations from relativistic spin theory are briefly reviewed.

The relativistic invariance of a theory means that the unitary representation of the Poincaré group is realized on the Hilbert space of system states (see, e.g., [12]). In this case the structure of the Poincaré algebra can be defined on the set of observables ($\hat{M}^{\mu\nu}, \hat{P}^\sigma$)

$$[\hat{M}^{\mu\nu}, \hat{P}^\sigma] = -i(g^{\mu\sigma}\hat{P}^\nu - g^{\nu\sigma}\hat{P}^\mu),$$

$$[\hat{M}^{\mu\nu}, \hat{M}^{\sigma\rho}] = -i(g^{\mu\sigma}\hat{M}^{\nu\rho} - g^{\nu\sigma}\hat{M}^{\mu\rho}) - (\sigma \leftrightarrow \rho),$$

$$[\hat{P}^\mu, \hat{P}^\nu] = 0. \quad (1)$$

In (1) $g^{\mu\nu}$ is the metric tensor in Minkowski space. As we have mentioned, in the case of the particles with interaction the realization of the Poincaré algebra on the set of observables is more complicated than in the case of the free particles. Let us consider one key commutator from the set of commutators (1) (see, e.g., [2]):

$$[\hat{P}^j \hat{N}^k] = i \delta^{jk} \hat{H} \quad (2)$$

(we use notations given in the Introduction; $j, k = 1, 2, 3$).

Since \hat{H} is interaction dependent for non-trivial systems, either \hat{P} , \hat{N} , or some combination of \hat{P} and \hat{N} also must be interacting. To preserve the commutation relations (1) one has to make other generators depending on the interaction, too. So, the generators occur to fall into two groups: the generators which are independent of the interaction and form the so called kinematical subgroup, and the generators depending on the interaction – Hamiltonians. This division is not unique. Different ways to obtain kinematical subgroups result in different forms of dynamics.

In this paper we use the so called instant form dynamics (IF). In this form the kinematical subgroup contains the generators of the group of rotations and translations in the three-dimensional Euclidean space:

$$\vec{\hat{J}}, \vec{\hat{P}}. \quad (3)$$

The remaining generators are Hamiltonians (interaction depending):

$$\hat{P}^0, \hat{N}. \quad (4)$$

The additive including of interaction into the mass square operator (Bakamjian–Thomas procedure [36], see,

e.g., [4] for details) presents one of the possible technical ways to include interaction in the algebra (1):

$$\hat{M}_0^2 \rightarrow \hat{M}_I^2 = \hat{M}_0^2 + \hat{U}. \quad (5)$$

Here \hat{M}_0 is the operator of invariant mass for the free system and \hat{M}_I – for the system with interaction. The interaction operator \hat{U} has to satisfy the following commutation relations:

$$[\hat{\vec{P}}, \hat{U}] = [\hat{\vec{J}}, \hat{U}] = [\hat{\vec{\nabla}}_P, \hat{U}] = 0. \quad (6)$$

These constraints (6) ensure that the algebraic relations (1) are fulfilled for interacting system. The constraints (6) are not too strong. For instance, a large class of non-relativistic potential satisfies (6). The relations (6) mean that the interaction potential does not depend on the total momentum of the system. This fact is well established for a class of potential, for example, for separable potentials [37]. Nevertheless, the conditions (5) and (6) can be considered as the model ones. There exists another approach [38] where a potential depends on the total momentum but that approach is out of scope of this paper.

In RHD the wave function of the system of interacting particles is the eigenfunction of a complete set of commuting operators. In IF this set is:

$$\hat{M}_I^2, \quad \hat{J}^2, \quad \hat{J}_3, \quad \hat{\vec{P}}. \quad (7)$$

\hat{J}^2 is the operator of the square of the total angular momentum. In IF the operators \hat{J}^2 , \hat{J}_3 , $\hat{\vec{P}}$ coincide with those for the free system. So, in (7) only the operator \hat{M}_I^2 depends on the interaction.

To find the eigenfunctions for the system (7) one has first to construct the adequate basis in the state space of composite system. In the case of two-particle system (for example, quark-antiquark system $q\bar{q}$) the Hilbert space in RHD is the direct product of two one-particle Hilbert spaces: $\mathcal{H}_{q\bar{q}} \equiv \mathcal{H}_q \otimes \mathcal{H}_{\bar{q}}$.

As a basis in $\mathcal{H}_{q\bar{q}}$ one can choose the following set of two-particle state vectors:

$$\begin{aligned} |\vec{p}_1, m_1; \vec{p}_2, m_2\rangle &= |\vec{p}_1, m_1\rangle \otimes |\vec{p}_2, m_2\rangle, \\ \langle \vec{p}, m | \vec{p}', m'\rangle &= 2p_0 \delta(\vec{p} - \vec{p}') \delta_{mm'}. \end{aligned} \quad (8)$$

Here \vec{p}_1 , \vec{p}_2 are 3-momenta of particles, m_1 , m_2 — spin projections on the axis z , $p_0 = \sqrt{\vec{p}^2 + M^2}$, M is the constituent mass.

One can choose another basis where the motion of the two-particle center of mass is separated and where three operators of the set (7) are diagonal:

$$|\vec{P}, \sqrt{s}, J, l, S, m_J\rangle,$$

$$\langle \vec{P}, \sqrt{s}, J, l, S, m_J | \vec{P}', \sqrt{s'}, J', l', S', m_{J'} \rangle$$

$$= N_{CG} \delta^{(3)}(\vec{P} - \vec{P}') \delta(\sqrt{s} - \sqrt{s'}) \delta_{JJ'} \delta_{ll'} \delta_{SS'} \delta_{m_J m_{J'}},$$

$$N_{CG} = \frac{(2P_0)^2}{8k\sqrt{s}}, \quad k = \frac{1}{2}\sqrt{s - 4M^2}. \quad (9)$$

Here $P_\mu = (p_1 + p_2)_\mu$, $P_\mu^2 = s$, \sqrt{s} is the invariant mass of the two-particle system, l — the orbital angular momentum in the center-of-mass frame (C.M.S.), $\vec{S}^2 = (\vec{S}_1 + \vec{S}_2)^2 = S(S+1)$, S — the total spin in C.M.S., J — the total angular momentum with the projection m_J .

The basis (9) is connected with the basis (8) through the Clebsch–Gordan (CG) decomposition for the Poincaré group. The decomposition of the direct product (8) of two irreducible representations of the Poincaré group into irreducible representations (9) has the following form [34]:

$$|\vec{p}_1, m_1; \vec{p}_2, m_2\rangle = \sum |\vec{P}, \sqrt{s}, J, l, S, m_J\rangle$$

$$\times \langle J m_J | S l m_s m_l \rangle Y_{lm_l}^*(\vartheta, \varphi) \langle S m_S | 1/2 1/2 \tilde{m}_1 \tilde{m}_2 \rangle$$

$$\times \langle \tilde{m}_1 | D^{1/2}(P, p_1) | m_1 \rangle \langle \tilde{m}_2 | D^{1/2}(P, p_2) | m_2 \rangle. \quad (10)$$

Here the sum is over the variables \tilde{m}_1 , \tilde{m}_2 , m_l , m_s , l , S , J , m_J . $\vec{p} = (\vec{p}_1 - \vec{p}_2)/2$, $p = |\vec{p}|$, ϑ, φ are the spherical angles of the vector \vec{p} in the C.M.S., Y_{lm_l} — a spherical harmonics (star means the complex conjugation), $\langle S m_S | 1/2 1/2 \tilde{m}_1 \tilde{m}_2 \rangle$ and $\langle J m_J | S l m_s m_l \rangle$ are the CG coefficients for the group $SU(2)$, $\langle \tilde{m} | D^{1/2}(P, p) | m \rangle$ — the three-dimensional spin rotation matrix to be used for correct relativistic invariant spin addition.

Let us discuss briefly the relativistic properties of spins. It is known that the Lorentz transformation for spins is momentum depending (see, e.g., [12]). So, to perform Lorentz invariant spin addition for particles with different momenta \vec{p} and \vec{p}' one has to "shift" the spins to the frame where the momenta are equal to one another. The spin transforms following the so called small group which is isomorphic to rotation group and thus, the operator of such a "shift" is a 3-dimensional rotation matrix $D(\alpha, \beta, \gamma)$. The Euler angles α, β, γ can be written in terms of the components of the vectors \vec{p} and \vec{p}' . In this way the "transplantation" of spins on one and the same momentum is realized. To understand what means this "transplantation" let us consider an example: one particle has the momentum \vec{p}_1 , mass M_1 , spin j and spin projection m , while another particle with momentum \vec{p}_2 and mass M_2 has no spin. In the case of free particles the vector state of this system is

$$|\vec{p}_1, M_1, j, m; \vec{p}_2, M_2\rangle = |\vec{p}_1, M_1, j, m\rangle \otimes |\vec{p}_2, M_2\rangle. \quad (11)$$

The two-particle vector state in the case when it is the first particle that has no spin is

$$|\vec{p}_1, M_1; \vec{p}_2, M_2, j, m\rangle = |\vec{p}_1, M_1\rangle \otimes |\vec{p}_2, M_2, j, m\rangle. \quad (12)$$

In nonrelativistic angular momentum theory the states (11) and (12) are identical. They describe the two-particle system with momenta \vec{p}_1 and \vec{p}_2 and total spin j with the projection m . In both cases the total spin can be obtained in simple way and is equal to the spin j of the first particle for (11) or of the second particle for (12).

In relativistic theory the states (11) and (12) differ essentially. The difference is caused by the fact that the states transform from one inertial coordinate system to another in different ways. As was mentioned before the Lorentz transformation for spin depends on the particle momentum and the spins in (11) and (12) correspond to particles with different momenta. In relativistic case the states (11) and (12) coincide only if the particles momenta are equal. In general case, to connect the state vectors (11) and (12) one has to "shift" the spin, for example, to shift the spin in (12) into the frame where the second particle has the momentum \vec{p}_1 . This shifting transformation is realized by the matrix $D^j(p_2, p_1)$ which belongs to the small group. Let us consider the state vector

$$|\vec{p}_1, M_1\rangle \otimes D^j(p_2, p_1) |\vec{p}_2, M_2, j, m\rangle. \quad (13)$$

Let us remind that a transformation belonging to the small group does not act on momenta. It is easy to see that the resulting vector describes the same state as (11) and transforms from one frame to another in the same way as (11). To show this let us use the equation $D(p', p'') D(p'', p) = D(p', p)$. Now the following covariant equality is valid:

$$\begin{aligned} & |\vec{p}_1, M_1, j, m; \vec{p}_2, M_2\rangle \\ &= \sum_{m'} |\vec{p}_1, M_1; \vec{p}_2, M_2, j, m'\rangle \langle m' | D^j(p_2, p_1) | m \rangle. \end{aligned} \quad (14)$$

As one can see from (14) the spin has "changed" the momentum. As this operation is very important for the understanding of the parameterization below let us formulate it in other words, namely, in terms of the generators of the Lorentz transformations.

The Lorentz-transformation generator for the state (11) is of the form:

$$\hat{N} = \hat{N}_1^j + \hat{N}_2,$$

$$\hat{N}^j = ip_0 \frac{\partial}{\partial \vec{p}} - \frac{[\vec{j}\vec{p}]}{p_0 + M}, \quad \hat{N} = ip_0 \frac{\partial}{\partial \vec{p}}. \quad (15)$$

If we perform the transformation (14), the D -matrix transforms the generators in the following way:

$$D^j(p_2, p_1) (\hat{N}_1^j + \hat{N}_2) [D^j(p_2, p_1)]^{-1} = \hat{N}_1 + \hat{N}_2^j. \quad (16)$$

It is just the equations (14) and (16) (and only these equations) that present the exact statement that D -function shifts spin from one momentum to another one. Similar equations (however, more complicated) are valid in the case of two non-zero spins. In the case of spin 1/2 the D -function [34] has the form

$$\begin{aligned} D^{1/2}(p_1, p_2) &= \cos\left(\frac{\omega}{2}\right) - 2i(\vec{k}\vec{j}) \sin\left(\frac{\omega}{2}\right), \quad \vec{k} = \frac{[\vec{p}_1 \vec{p}_2]}{[|\vec{p}_1 \vec{p}_2|]}, \\ \omega &= 2 \arctan \frac{[|\vec{p}_1 \vec{p}_2|]}{(p_{10} + M_1)(p_{20} + M_2) - (\vec{p}_1 \vec{p}_2)}. \end{aligned} \quad (17)$$

We will use it below.

Let us make a remark concerning the invariance of the decomposition (10). The total spin S and the total orbital angular momentum l in (10) play the role of invariant parameters of degeneracy. However, the square of the total spin $\vec{S} = (\vec{S}_1 + \vec{S}_2)$ is not invariant. But one can define the total spin square in invariant way as follows:

$$\begin{aligned} & \left\{ [D^{S_1}(p_1, P)]^{-1} \vec{S}_1 [D^{S_1}(p_1, P)] \right. \\ & \left. + [D^{S_2}(p_2, P)]^{-1} \vec{S}_2 [D^{S_2}(p_2, P)] \right\}^2 = S(S+1). \end{aligned} \quad (18)$$

Here P is the center-of-mass momentum. One can see that in C.M.S. the definition (18) coincides with the definition (9). Similarly, one can define the orbital moment l in invariant way.

The described spin rotation effect in (10) is a purely relativistic effect. If one takes it into account, one obtains interesting observable effects [24].

To obtain the basis vectors (9) in terms of vectors (8) one has to inverse (10). The final equation has the form:

$$\begin{aligned} & |\vec{P}, \sqrt{s}, J, l, S, m_J\rangle \\ &= \sum_{m_1 m_2} \int \frac{d\vec{p}_1}{2p_{10}} \frac{d\vec{p}_2}{2p_{20}} |\vec{p}_1, m_1; \vec{p}_2, m_2\rangle \\ & \times \langle \vec{p}_1, m_1; \vec{p}_2, m_2 | \vec{P}, \sqrt{s}, J, l, S, m_J \rangle. \end{aligned} \quad (19)$$

Here

$$\langle \vec{p}_1, m_1; \vec{p}_2, m_2 | \vec{P}, \sqrt{s}, J, l, S, m_J \rangle$$

$$\begin{aligned}
&= \sqrt{2s}[\lambda(s, M^2, M^2)]^{-1/2} 2P_0 \delta(P - p_1 - p_2) \\
&\times \sum_{\tilde{m}_1 \tilde{m}_2} \langle m_1 | D^{1/2}(p_1 P) | \tilde{m}_1 \rangle \langle m_2 | D^{1/2}(p_2 P) | \tilde{m}_2 \rangle \\
&\times \sum_{m_l m_S} \langle 1/2 1/2 \tilde{m}_1 \tilde{m}_2 | S m_S \rangle Y_{lm_l}(\vartheta, \varphi) \\
&\times \langle S l m_s m_l | J m_J \rangle .
\end{aligned} \tag{23}$$

Here $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ac)$. To obtain (19) the decomposition in terms of spherical harmonics and the summation of all of the momenta to give the total momentum J were performed in the C.M.S. and then the obtained result was shifted to arbitrary coordinate frame by use of D -functions.

It is on the vectors (9), (19) that the Poincaré-group representation is realized in the vector state space of two free particles. The vector in representation is determined by the eigenvalues of the complete commuting set of operators:

$$\hat{M}_0^2 = \hat{P}^2, \hat{J}^2, \hat{J}_3. \tag{20}$$

The parameters S and l (as was mentioned) play the role of invariant parameters of degeneracy.

As in the basis (9) the operators $\hat{J}^2, \hat{J}_3, \hat{P}$ in (7) are diagonal, one needs to diagonalize only the operator \hat{M}_l^2 in order to obtain the system wave function.

The eigenvalue problem for the operator \hat{M}_l^2 in the basis (9) has the form of nonrelativistic Schrödinger equation (see, e.g., [4]).

The corresponding composite-particle wave function has the form

$$\begin{aligned}
&\langle \vec{P}', \sqrt{s'}, J', l', S', m'_J | p_c \rangle = \\
&= N_C \delta(\vec{P}' - \vec{p}_c) \delta_{JJ'} \delta_{m_J m'_J} \varphi_{l'S'}^{J'}(k'), \tag{21}
\end{aligned}$$

$$N_C = \sqrt{2p_{c0}} \sqrt{\frac{N_{CG}}{4k'}} ,$$

$|p_c\rangle$ is an eigenvector of the set (7); $J(J+1)$ and m_J are the eigenvalues of \hat{J}^2, \hat{J}_3 , respectively (Eqs. (7), (20)).

The two-particle wave function of relative motion for equal masses and total angular momentum and total spin fixed is:

$$\varphi_{lS}^J(k(s)) = \sqrt[4]{s} u_l(k) k, \tag{22}$$

and the normalization condition has the form:

$$\sum_l \int u_l^2(k) k^2 dk = 1. \tag{23}$$

Let us note that for composite quark systems one uses sometimes instead of equation (23) the following one:

$$n_c \sum_l \int u_l^2(k) k^2 dk = 1. \tag{24}$$

Here n_c – is the number of colours. The wave function (22) coincides with that obtained by "minimal relativization" in [39]. The normalization factors in (22) in this case correspond to the relativization obtained by the transformation to relativistic density of states

$$k^2 dk \rightarrow \frac{k^2 dk}{2\sqrt{(k^2 + M^2)}}. \tag{25}$$

It is worth to notice that wave functions in RHD (for example, the wave function (21) defined as the eigenfunction of the operators set (7)) in general are not the same as relativistic covariant wave functions defined as solutions of wave equations or as the matrix elements of local Heisenberg field.

The formalism of this Section is used in the next one to present the method of calculation of electroweak properties of composite systems. Particularly, the method of construction of electroweak current operators is described.

III. THE NEW RELATIVISTIC INSTANT-FORM APPROACH TO THE ELECTROWEAK STRUCTURE OF TWO BODY COMPOSITE SYSTEMS.

In this Section we present our approach to electroweak properties of relativistic two-particle systems. To demonstrate how one describes the electromagnetic properties of composite systems in our version of the RHD instant form we first use the following simple model. We consider the system of two spinless particles in the S -state of relative motion, one particle having no charge. Let us note that a similar model was used in [4] where the authors gave the description of constituent interaction in IF of RHD and obtained the mass spectrum. The application of our method in general case follows the scheme of this Section. The case of π -meson is investigated in Sec.IV and the $S = 1$ case in [40].

Electromagnetic properties of the system are determined by the current operator matrix element. This matrix element is connected with the charge form factor $F_c(Q^2)$ as follows:

$$\langle p_c | j_\mu(0) | p'_c \rangle = (p_c + p'_c)_\mu F_c(Q^2), \quad (26)$$

where p'_c , p_c are 4-momenta of the composite system in initial and final states, $Q^2 = -t$, $q^2 = (p_c - p'_c)^2 = t$, q^2 is the momentum-transfer square. The form (26) is defined by the Lorentz covariance and by the conservation law only and does not depend on the model for the internal structure of the system.

The Eq. (26) presents the simplest example of the extraction of a reduced matrix element. The 4-vector $(p_c + p'_c)_\mu$ describes symmetry and transformation properties of the matrix element. The reduced matrix element (the form factor) contains all the dynamical information on the process described by the current. Usually, one does not fix the dependence of form factor on a scalar mass of the composite system $p_c^2 = p'_c{}^2 = M_c^2$, because it is diagonal with respect to this variable. The representation of a matrix element in terms of form factors often is referred to as the parameterization of matrix element. The scattering cross section for elastic scattering of electrons by a composite system can be expressed in terms of charge form factor $F_c(Q^2)$. So, form factor can be obtained from experiment and it is interesting to calculate it in a theoretical approach.

In this Section we calculate the form factor of our simple composite system using the version of RHD IF based on the approach of the Section II.

Now let us list the conditions for the operator of the conserved electromagnetic current to be fulfilled in relativistic case (see, e.g., [10]).

(i). *Lorentz-covariance*:

$$\hat{U}^{-1}(\Lambda) \hat{j}^\mu(x) \hat{U}(\Lambda) = \Lambda^\mu_\nu \hat{j}^\nu(\Lambda^{-1}x). \quad (27)$$

Here Λ is the Lorentz-transformation matrix, $\hat{U}(\Lambda)$ – the operator of the unitary representation of the Lorentz group.

(ii). *Invariance under translation*:

$$\hat{U}^{-1}(a) \hat{j}^\mu(x) \hat{U}(a) = \hat{j}^\mu(x - a). \quad (28)$$

Here $\hat{U}(a)$ is the operator of the unitary representation of the translation group.

(iii). *Current conservation law*:

$$[\hat{P}_\nu, \hat{j}^\nu(0)] = 0. \quad (29)$$

In terms of matrix elements $\langle \hat{j}^\mu(0) \rangle$ the conservation law can be written in the form:

$$q_\mu \langle \hat{j}^\mu(0) \rangle = 0. \quad (30)$$

Here q_μ is 4-vector of the momentum transfer.

(iv). *Current-operator transformations under space-time*

reflections are:

$$\begin{aligned} & \hat{U}_P \left(\hat{j}^0(x^0, \vec{x}), \hat{\vec{j}}(x^0, \vec{x}) \right) \hat{U}_P^{-1} \\ &= \left(\hat{j}^0(x^0, -\vec{x}), -\hat{\vec{j}}(x^0, -\vec{x}) \right), \\ & \hat{U}_R \hat{j}^\mu(x) \hat{U}_R^{-1} = \hat{j}^\mu(-x). \end{aligned} \quad (31)$$

In (31) \hat{U}_P is the unitary operator for the representation of space reflections and \hat{U}_R is the antiunitary operator of the representation of space-time reflections $R = PT$.

(v). *Cluster separability condition*: If the interaction is switched off then the current operator becomes equal to the sum of the operators of one-particle currents.

(vi). *The charge is not renormalized by the interaction including*: The electric charge of the system with interaction is equal to the sum of the constituent electric charges.

In this paper the explicit equations for the form factors are obtained taking into account all the listed conditions.

A. Electromagnetic properties of the system of free particles

Let us consider first the simple two-particle system described in the beginning of Section III. The elastic scattering of a test particle, e.g., of an electron, by the system are defined by the operator of electromagnetic current $\hat{j}_\mu^{(0)}(0)$ of the two-particle free system. This operator can be calculated in the representation given by the basis (8) or in the representation given by the basis (9). In the first case the operator has the form $\hat{j}_\mu^{(0)} = j_{1\mu} \otimes I_2$. Here $j_{1\mu}$ is the electromagnetic current of the charged particle and I_2 is the unity operator in the Hilbert space of states of the uncharged particle.

$$\begin{aligned} & \langle \vec{p}_1; \vec{p}_2 | \hat{j}_\mu^{(0)}(0) | \vec{p}'_1; \vec{p}'_2 \rangle \\ &= \langle \vec{p}_2 | \vec{p}'_2 \rangle \langle \vec{p}_1 | j_{1\mu}(0) | \vec{p}'_1 \rangle. \end{aligned} \quad (32)$$

The matrix element of the one spinless particle current in the free case contains only one form factor – the charge form factor of the charged particle $f_1(Q^2)$:

$$\langle \vec{p}_1 | j_{1\mu}(0) | \vec{p}'_1 \rangle = (p_1 + p'_1)_\mu f_1(Q^2). \quad (33)$$

So, the electromagnetic properties of the system of two free particles (26) are defined by the form factor $f_1(Q^2)$, containing all the dynamical information on elastic processes described by the matrix element (32) [4]. Particularly, the charge of the system is defined by the value of this form factor at $Q^2 \rightarrow 0$:

$$\lim_{Q^2 \rightarrow 0} f_1(Q^2) = f_1(0) = e_c. \quad (34)$$

e_c is the system charge.

Now let us write the electromagnetic-current matrix element for the two-particle free system in the basis where the center-of-mass motion is separated (9):

$$\langle \vec{P}, \sqrt{s}, | j_\mu^{(0)}(0) | \vec{P}', \sqrt{s'} \rangle. \quad (35)$$

Here the variables which take zero values are omitted: $J = S = l = 0$. One can consider the matrix element (35) as a matrix element of an irreducible tensor operator on the Lorentz group and one can apply the Wigner-Eckart theorem. Under the condition of the theorem the matrix element of an irreducible tensor operator is the product of two factors: the invariant part (reduced matrix element) and the CG coefficient which defines the transformation properties of the matrix element. Thus, one can write (35) in the form

$$\begin{aligned} & \langle \vec{P}, \sqrt{s}, | j_\mu^{(0)}(0) | \vec{P}', \sqrt{s'} \rangle = \\ & = A_\mu(s, Q^2, s') g_0(s, Q^2, s'). \end{aligned} \quad (36)$$

The motivation for the parameterization (36) is easy to be understood for our simple system. The 4-vector A_μ describes the transformation properties of the matrix element and the invariant function $g_0(s, Q^2, s')$ contains the dynamical information on the process. We will refer to $g_0(s, Q^2, s')$ as to free two-particle form factor. For more complicated systems the parameterization corresponding to the Wigner-Eckart theorem for the Lorentz group can be performed using a special mathematical techniques as described in the papers [32], [34].

The vector $A_\mu(s, Q^2, s')$ which describes the matrix-element transformation properties is defined by the 4-momenta of initial and final states only: we have no other vectors to our disposal. So $A_\mu(s, Q^2, s')$ is a linear combination of 4-momenta of initial and final states and is defined by the current transformation properties (the Lorentz-covariance and the conservation law):

$$A_\mu = \frac{1}{Q^2} [(s - s' + Q^2)P_\mu + (s' - s + Q^2)P'_\mu]. \quad (37)$$

Thus, in the basis (9) the electromagnetic properties of the free two-particle system are defined by the free two-particle form factor $g_0(s, Q^2, s')$.

So, in both representations (defined by the basis (8) as well as by the basis (9)) we pass from the description of the system in terms of matrix elements to that in terms of Lorentz-invariant form factors.

One can see that (32) and (36) describe electromagnetic properties in terms of only one form factor. Both

of these descriptions are, certainly, equivalent from the physical point of view. Let us consider the difference between these descriptions. As we will show below by direct calculation the free two-particle form factor $g_0(s, Q^2, s')$ is not an ordinary function but has to be considered in the sense of distributions in variables s, s' , generated by a locally integrable function. So, $g_0(s, Q^2, s')$ is a regular generalized function. Let us remind that regular generalized function is that defined through an integral in the space of test functions. So, all the properties of $g_0(s, Q^2, s')$ have to be considered as the properties of a functional given by the integral over the variables s, s' of the function $g_0(s, Q^2, s')$ multiplied by a test function. As test functions it is sufficient to take a large class of smooth functions that give the uniconvergence of the integral. In particular, the limit (34) giving the total charge of the system through two-particle form factor is now the weak limit:

$$\lim_{Q^2 \rightarrow 0} \langle g_0(s, Q^2, s'), \phi(s, s') \rangle. \quad (38)$$

Here $\phi(s, s')$ is a function from the space of test functions. The precise definition of the functional will be given below.

As the invariant variables s, s' contain the energies of the relative motion of particles in initial and final states, one can consider the integral in (38) as an integral over these energies.

At the first glance it seems that the description of the two-particle free system in terms of the form factor $g_0(s, Q^2, s')$ is too complicated. However, so is the reality, as we will see later in the Subsection III.F. In fact, this kind of description is used implicitly for a long time in nonrelativistic theory of composite systems, without calling things by their proper names. It is this kind of description that makes it possible to construct the electromagnetic current operator with correct transformation properties for interacting systems.

B. The form factor of the system of two free particle.

The locally integrable function $g_0(s, Q^2, s')$ can be easily obtained by use of CG decomposition (19) for the Poincaré group. Using (19) we obtain for (36):

$$\begin{aligned} & \langle \vec{P}, \sqrt{s}, | j_\mu^{(0)}(0) | \vec{P}', \sqrt{s'} \rangle \\ & = \int \frac{d\vec{p}_1}{2p_{10}} \frac{d\vec{p}_2}{2p_{20}} \frac{d\vec{p}'_1}{2p'_{10}} \frac{d\vec{p}'_2}{2p'_{20}} \langle \vec{P}, \sqrt{s}, | \vec{p}_1; \vec{p}_2 \rangle \\ & \times \langle \vec{p}_1; \vec{p}_2 | j_\mu^{(0)}(0) | \vec{p}'_1; \vec{p}'_2 \rangle \langle \vec{p}'_1; \vec{p}'_2 | \vec{P}', \sqrt{s'} \rangle. \end{aligned} \quad (39)$$

To calculate the free two-particle form factor one has to use (32), (33), (36) and the explicit form of CG coefficients (19) for quantum numbers of the system. As the particles of the system under consideration are spinless, now (19) does not contain D -functions.

It is convenient to integrate in (39) using the coordinate frame with $\vec{P}' = 0$, $\vec{P} = (0, 0, P)$. As the result we obtain the following relativistic invariant form for the function $g_0(s, Q^2, s')$:

$$g_0(s, Q^2, s') = \frac{(s + s' + Q^2)^2 Q^2}{2\sqrt{(s - 4M^2)(s' - 4M^2)}} \frac{\vartheta(s, Q^2, s')}{[\lambda(s, -Q^2, s')]^{3/2}} f_1(Q^2). \quad (40)$$

Here $\vartheta(s, Q^2, s') = \theta(s' - s_1) - \theta(s' - s_2)$, and θ is the step function. The result, naturally, does not depend on the choice of the coordinate frame.

$$s_{1,2} = 2M^2 + \frac{1}{2M^2}(2M^2 + Q^2)(s - 2M^2) \mp \frac{1}{2M^2}\sqrt{Q^2(Q^2 + 4M^2)s(s - 4M^2)}.$$

The functions $s_{1,2}(s, Q^2)$ give in the plane (s, s') the kinematically available region. The position of this region depends strongly on the momentum-transfer square $t = -Q^2$. The simplest way to obtain the functions $s_{1,2}$ is a geometrical one [33].

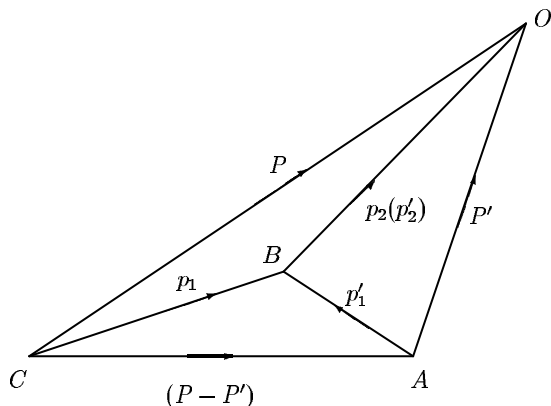


FIG. 1. Kinematical triangle

Let us construct the triangle schematically presented on the Fig.1. The side OB presents the vector $p_{2\mu}$, and $p'_{2\mu}$, as following (32) $p_{2\mu} = p'_{2\mu}$. The side CB presents the vector $p_{1\mu}$ and $AB - p'_{1\mu}$. Now the sides of the large triangle AOC present the vectors P'_μ, P_μ and $(P_\mu - P'_\mu)$, with the norms $\sqrt{s'}, \sqrt{s}, \sqrt{t}$, respectively. Vectors of initial state $p'_{1\mu}, p_{2\mu} = p'_{2\mu}, P'_\mu$ are fixed. Let us fix the norm of vector $(P_\mu - P'_\mu)$. So, because the norm of vector $p_{1\mu}, p_{1\mu}^2 = M^2$ is constant, the triangles ABO and ABC are determined unambiguously by three sides. But the triangle ABC can be rotated around the side AB ($p'_{1\mu}$). It is possible to find the minimal $\sqrt{s_1}$ and maximal $\sqrt{s_2}$ lengths of OC (norm of vector P_μ) under this rotation. The value of the s_1, s_2 give the kinematically available region in the plane (s, s') which is symmetric under interchange $s \leftrightarrow s'$.

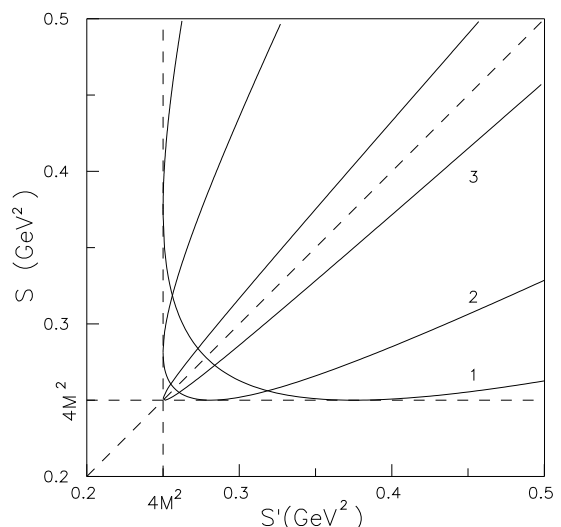


FIG. 2. The kinematically available region in the plane s, s' (inside the parabolae). The calculation is performed for: 1) $Q^2 = 2M^2$. 2) $Q^2 = M^2/2$. 3) $Q^2 = M^2/64$. The constituent mass is $M = 0.25$ GeV.

In Fig.2 the domain where the generalized function (40) is nonzero in the plane s, s' is given for different values of the momentum-transfer square. One can see that the free two-particle form factor $g_0(s, Q^2, s')$ (40) has in fact to be interpreted in terms of the distributions: The ordinary limit as $Q^2 \rightarrow 0$ is zero because of the cutting ϑ -functions and the static limit exists only as the weak limit (38).

Let us calculate this limit. Let us define the functional giving regular generalized function as a functional in \mathbf{R}^2 as follows:

² The ϑ -function is a purely kinematical factor for IA. This fact does not depend on relativism and takes place in nonrelativistic case, too. See Subsection III.F for details.

$$\langle \tilde{g}_0(s, Q^2, s'), \phi(s, s') \rangle = \int d\mu(s, s') \tilde{g}_0(s, Q^2, s') \phi(s, s'). \quad (41)$$

Here

$$\begin{aligned} & \tilde{g}_0(s, Q^2, s') \\ &= 16 \theta(s - 4M^2) \theta(s' - 4M^2) g_0(s, Q^2, s'). \end{aligned} \quad (42)$$

$$d\mu(s, s') = \sqrt[4]{ss'} d\mu(s) d\mu(s'), \quad d\mu(s) = \frac{1}{4} k d\sqrt{s}. \quad (43)$$

The θ – functions in (42) give the physical region of possible variations of the invariant mass squares in the initial and final states explicitly. The measure (43) is due to the relativistic density of states (22), (25). $\phi(s, s')$ is a function from the test function space. So, for example, the limit of $g_0(s, Q^2, s')$ as $Q^2 \rightarrow 0$ (the static limit) has the meaning only as the weak limit (compare with (34)):

$$\lim_{Q^2 \rightarrow 0} \langle \tilde{g}_0, \phi \rangle = \langle e\delta(\mu(s') - \mu(s))\vartheta(s - 4M^2), \phi \rangle. \quad (44)$$

It is this weak limit that gives the electric charge of the free two–particle system. If the test functions are normalized with the relativistic density of states, then the r.h.s. of the Eq. (44) is equal to the total charge of the system.

C. Electromagnetic structure of the system of two interacting particles.

Now let us consider the electromagnetic structure of our simple model (26) in the case of interacting particles.

As we have mentioned in Sec.II when constructing the bases (8) and (9) in the frame of RHD the state vector $|p_c\rangle$ belongs to the direct product of two one–particle spaces. We can write the decomposition of this vector with $J = l = S = m_J = 0$ in the basis (9). Now (26) has the form:

$$\begin{aligned} & \int \frac{d\vec{P} d\vec{P}'}{N_{CG} N'_{CG}} d\sqrt{s} d\sqrt{s'} \langle p_c | \vec{P}, \sqrt{s} \rangle \langle \vec{P}, \sqrt{s} | j_\mu(0) | \vec{P}', \sqrt{s'} \rangle \\ & \times \langle \vec{P}', \sqrt{s'} | p'_c \rangle = (p_c + p'_c)_\mu F_c(Q^2). \end{aligned} \quad (45)$$

Here $\langle \vec{P}', \sqrt{s'} | p'_c \rangle$ is the wave function in the sense of the instant form of RHD (21).

Using (21) we obtain for (45):

$$\int \frac{N_c N'_c}{N_{CG} N'_{CG}} d\sqrt{s} d\sqrt{s'} \varphi(s) \varphi(s')$$

$$\times \langle \vec{p}_c, \sqrt{s} | j_\mu(0) | \vec{p}'_c, \sqrt{s'} \rangle = (p_c + p'_c)_\mu F_c(Q^2). \quad (46)$$

We have omitted in the wave function (22) the variables with zero values: $J = S = l = 0$ (see (35) too).

Using (22), (43) we can rewrite (46) in the form of the functional in \mathbf{R}^2 :

$$\begin{aligned} & \int d\mu(s, s') u(k(s)) J_\mu(\vec{p}_c, \sqrt{s}; \vec{p}'_c, \sqrt{s'}) u(k(s')) \\ &= (p_c + p'_c)_\mu F_c(Q^2), \end{aligned} \quad (47)$$

$$J_\mu(\vec{p}_c, \sqrt{s}; \vec{p}'_c, \sqrt{s'}) = 16 \theta(s - 4M^2) \theta(s' - 4M^2)$$

$$\times \frac{N_c N'_c}{N_{CG} N'_{CG}} \langle \vec{p}_c, \sqrt{s} | j_\mu | \vec{p}'_c, \sqrt{s'} \rangle.$$

In the previous cases the state vectors and the operators entering matrix elements transformed following one and the same representation of the Poincaré group (namely, following the universal covering subgroup of the Poincaré group – nonuniform group $SL(2, C)$ [12]). Now in the matrix element in the integrand of (47) the state vectors and the operator transform following the different representations of this group. The current operator describes the transitions in the system of two interacting particles and transforms following the representation with the generators of Lorentz boosts depending on the interaction (7). The state vectors belong to the basis (9) and physically describe the system of two free particles and, so, transform following a representation with generators which do not depend on the interaction (20). That is why the current operator matrix element in (47) can not be represented in the form (36), (37): we can not construct the 4–vector defining the matrix–element transformation properties under Lorentz boosts from the variables which the state vectors depend on.

Nevertheless, as we show below, the problem of the parameterization of the current matrix element in (47) can be solved if one consider this equality as the equality of two functionals. The l.h.s. contains a functional in R^2 generated by the Lorentz–covariant function (current matrix element). Let us denote

$$\psi(s, s') = u(k(s)) u(k'(s')). \quad (48)$$

The functional in the l.h.s. of (47) is given on the set of test functions $\psi(s, s')$ through an integral in R^2 and defines a Lorentz–covariant (regular) generalized function with the values in the Minkowski space (see, e.g., [41]). Here Q^2 is a parameter. The test–function space can be (in general) larger than (48). However, the uniconvergence of (47) has to be guaranteed.

Let us write the matrix element in the form analogous to (36):

$$J_\mu(\vec{p}_c, \sqrt{s}; \vec{p}'_c, \sqrt{s'}) = B_\mu(s, Q^2, s') G(s, Q^2, s'). \quad (49)$$

The covariant part in (49) (as well as in (36)), the vector $B_\mu(s, Q^2, s')$, is supposed to be an ordinary smooth function and the invariant part $G(s, Q^2, s')$ is generalized function. In fact, $G(s, Q^2, s')$ is the reduced matrix element containing the information on the process. This kind of representation of a Lorentz covariant generalized function as a product of a Lorentz covariant ordinary smooth function and a Lorentz invariant generalized function was described in [41]. Let us remark that all normalization constants enter the vector B_μ . Contrary to (36), now it is impossible to construct the 4-vector B_μ , describing the transformation properties of the matrix element, in terms of the variables entering the state vectors (as was pointed out before).

Using (49) we can rewrite (47) in the following form:

$$\begin{aligned} \int d\mu(s, s') \psi(s, s') B_\mu(s, Q^2, s') G(s, Q^2, s') \\ = (p_c + p'_c)_\mu F_c[\psi](Q^2). \end{aligned} \quad (50)$$

To obtain the vector B_μ let us require the Eq.(50) to be covariant in the sense of distributions, that is to be valid for any test function $\psi(s, s')$ in any fixed frame. The variation of test function in the functional (50) means in fact, following (48), the variation of the wave function of the internal motion. Under such a variation the vector in the r.h.s of (50) is unchanged as it is constructed with 4-vectors describing the motion of the system as a whole, independent of the internal constituent motion. As to the form factor in the r.h.s it varies under the test function variation. So, under a variation of the test function the r.h.s. of (50) remains to be collinear to the vector $(p_c + p'_c)_\mu$. At the same time, under arbitrary variation of the test function the vector in the l.h.s. in general changes the direction. So, for the validity of the equality (50) with arbitrary test function it is sufficient to require that the following equation

$$B_\mu(s, Q^2, s') = (p_c + p'_c)_\mu \quad (51)$$

holds. This choice of the vector B_μ in (51) ensures that the l.h.s. of (47) satisfies the condition of Lorentz covariance for the current as well as the condition of current conservation.

Let us discuss the physical meaning of the representation (49), (51) for the matrix element. As this representation is explicitly Lorentz covariant and also satisfies the current conservation law, then it means that the current

operator for the composite system contains the contribution not only of one-particle currents but of two-particle currents, too (see, e.g., [4]):

$$j(x) = \sum_k j^{(k)}(x) + \sum_{k < m} j^{(km)}. \quad (52)$$

Here the first term is the sum of one-particle currents and the second – of two-particle currents. In the case of our simple model each sum in (52) contains only one term. It is well known that if one approximates $j(x) \approx \sum_k j^{(k)}(x)$ then the current operator in IF dynamics does not satisfy the condition of Lorentz covariance and the conservation law [4]. So, from the physical point of view, the covariant part of the current matrix element (51) which defines the transformation properties of the current in (47) is given by (52) and contains the contributions of one- and two-particle currents.

The invariant part of the decomposition (49) is the form factor or the reduced matrix element $G(s, Q^2, s')$ and contains the information on the dynamics of the scattering of test particle by each of the constituents (the first term in (52)), i.e. by the free two-particle system, as well as by two constituent simultaneously (the second term). So, the form factor contains the contribution of the free system form factor (40) and the contribution of some exchange currents analogous to meson currents in nucleon systems [7].

$$G(s, Q^2, s') = g_0(s, Q^2, s') + G_c(s, Q^2, s'). \quad (53)$$

Here G_c is the reduced matrix element containing the contribution of two-particle currents (52).

Using (22), (43), (48), (51) one can obtain from (50) the scalar equation of the following form:

$$\int d\sqrt{s} d\sqrt{s'} \varphi(s) G(s, Q^2, s') \varphi(s') = F_c(Q^2). \quad (54)$$

The form factor $G(s, Q^2, s')$ includes all possible mechanisms of the transition described by the matrix element (26). So, the representation (54) for the charge form factor of the system is quite general.

Now let us proceed with the approximate calculation of the form factor (54).

D. Modified impulse approximation (MIA)

The problem of the calculation of the form factor $G(s, Q^2, s')$ (54) including exchange currents is a very difficult problem. We propose an approximation which is a kind of analog of relativistic impulse approximation. We propose to omit the contribution of the two-particle currents to the form factor $G(s, Q^2, s')$.

However we will not change the covariant part B_μ of the current matrix element in (49), so that this covariant part will contain the contribution of the two-particle currents and so that the transformation properties of the matrix element will not be changed.

So, we approximately change the generalized function $G(s, Q^2, s')$ in (49), (53) for the generalized function $g_0(s, Q^2, s')$ (36), (40), which describes, as we have shown before, the electromagnetic properties of the free two-particle system. Nevertheless, the matrix element (45), (49) as a whole will contain the contributions of two-particle currents, although not the full contribution but such that ensures its correct transformation properties.

Let us note that our approximation does not contradict general statements (see [4]) that to obtain correct description of electromagnetic current of composite system which satisfy the Lorentz-covariance condition and the current conservation law one has to take into account many-particle currents.

Let us discuss now the meaning of our approximation from the point of view of the Wigner-Eckart theorem for the Lorentz group. The matrix element of a current including many-particle currents, following the Wigner-Eckart theorem for the Lorentz group, can be presented in the form (49), (51). The dynamical information on many-particle currents is contained in the reduced matrix element – the form factor, while the transformation properties of the contributions of many-particle currents are defined by the covariant part of the form (49). So, our approximation means that the dynamical part of the contribution of the many-particle currents to the total current is omitted while the covariant part of the contributions remains. The dynamics of many-particle currents remains out of the limits of the approximation, while the transformation properties of the total current remain intact.

Thus, in our approximation the scalar equality (54) transforms into approximate scalar equality which corresponds, from the physical point of view, to relativistic impulse approximation. In the developed mathematical formalism we have not broke the Lorentz covariance of the current nor the current conservation law. Let us point out that to calculate form factor we do not use a special current component as it is done in other mathematical formulations of RHD (see, e.g., [8]). Let us remark that, from the physical point of view, the form factor $g_0(s, Q^2, s')$ contains the contributions of one-particle currents only (see Equations (36), (39), (40)) and in this sense our approximation corresponds to the known impulse approximation. In order to emphasize that our approximation differs from the usual IA we will refer to it as to modified impulse approximation (MIA). The form factor of the composite system in MIA has the form:

$$F(Q^2) = \int d\sqrt{s} d\sqrt{s'} \varphi(s) g_0(s, Q^2, s') \varphi(s'). \quad (55)$$

It is worth to notice that the Eq.(51) and the form (55) can be formally obtained if we write in (45) the current of the free system (36) instead of that of the interaction system and change the covariant part of (37) for

$$A_\mu(s, Q^2, s') |_{P=p_c, P'=p'_c} = (p_c + p'_c)_\mu. \quad (56)$$

The Eq.(56) gives a simple prescription to write the current matrix elements for interacting system in the basis (9) in MIA using the current parameterization (36) for the free system. The prescription is as follows: in the vectors in the parameterization (36), (37) one has to use the momenta of composite system instead of the center-of-mass momenta of the free two-particle system. Note that this prescription works for more complicated systems, too.

We do not discuss in this paper the problem of going beyond the limits of MIA and of obtaining corrections to $g_0(s, Q^2, s')$ in (53), (55). This means that if considering, for example, nucleon systems we do not take into account meson current.

Let us consider now the fulfilling of the conditions (i)–(vi) for the electromagnetic current.

The conditions (i)–(iii) are satisfied by construction. For example the fulfilling of (i) and (iii) is ensured by the correct transformation properties of the 4-vectors in (36), (49), and (51).

The condition (iv) is satisfied immediately as the form factor $g_0(s, Q^2, s')$ in (36) and the form factor $G(s, Q^2, s')$ in (49) are scalars in our simple model³.

The condition of cluster separability (v) needs a more detailed consideration. At large distances (or if the interaction is switched off) the contribution of two-particle currents has to go to zero: $G_c(s, Q^2, s') \rightarrow 0$ in (53). This means that in the form (53) the form factor $G(s, Q^2, s')$ has to transform into $g_0(s, Q^2, s')$. Let us remark that the condition of cluster separability is fulfilled in MIA, too, as in this approximation the use of $g_0(s, Q^2, s')$ instead of $G(s, Q^2, s')$ is supposed from the very beginning. When the interaction is switched off the generalized function $g_0(s, Q^2, s')$ for the free two-particle system acts on a larger space of test functions than (48). As $g_0(s, Q^2, s')$ contains only the one-particle current contributions (39) the condition (v) is satisfied

³ The currents which do not conserve the parity also can be considered in our formalism. In that case one can separate not only the scalar part of current matrix element but the pseudoscalar part, too. This case is considered elsewhere.

and the composite-system current go to the sum of the one-particle currents.

The condition on the charge to be nonrenormalizable also is fulfilled directly in MIA because the weak limit (44) does exist on test functions (48).

So, our prescription for the construction of the current in MIA satisfies all the conditions for the current operator.

E. MIA versus IA

Let us compare the approximation MIA with the well known IA.

To do this let us first calculate the form factor in IF RHD not using the canonical parameterization. In particular, let us formulate IA in terms of operators as it is formulated usually (not in terms of form factors). Let us decompose the matrix element (26) through the complete set of states (8):

$$\begin{aligned} \langle p_c | j_\mu(0) | p'_c \rangle &= \int \frac{d\vec{p}_1 d\vec{p}_2}{2p_{10} 2p_{20}} \frac{d\vec{p}'_1 d\vec{p}'_2}{2p'_{10} 2p'_{20}} \\ &\times \langle p_c | \vec{p}_1; \vec{p}_2 \rangle \langle \vec{p}_1; \vec{p}_2 | j_\mu | \vec{p}'_1; \vec{p}'_2 \rangle \\ &\times \langle \vec{p}'_1; \vec{p}'_2 | p'_c \rangle. \end{aligned} \quad (57)$$

Here $\langle \vec{p}_1; \vec{p}_2 | p_c \rangle$ is wave function of constituents in composite system.

If the current matrix element in (57) is taken in the IA approximation (52) and contains one-particle currents only, then the Eq. (57) is selfcontradicting [4]. In fact, one can show this in the following way. In our simple model all the dynamical information about the current (i.e. the composite system form factor) can be obtained from only one matrix element in the Breit frame. However, to go to the Breit frame one has to perform the transformation which is interaction depending. This means that in Breit system two-particle currents appear along with one-particle ones. The form factors calculated in arbitrary coordinate frames using different matrix elements will be of different forms.

To write the form factor in terms of wave functions (21) one has to perform the CG decomposition of the basis (8) in terms of the basis (9) in the wave functions (57) and to use the explicit form for CG coefficients (10) for the quantum numbers of the system:

$$\langle \vec{p}_1; \vec{p}_2 | p_c \rangle = \sqrt{\frac{2}{\pi}} \langle \vec{P}, \sqrt{s}, J, l, S, m_J | p_c \rangle. \quad (58)$$

The current matrix element in (57) has the form (32). The one-particle currents are expressed through the form factors (33).

The Eq.(57) is an equality for two 4-vectors. Taking different components of this equality and exploiting δ -functions in integrals, one can calculate the form factor of the composite system. The result of calculation of the form factor in this way is not unambiguous. In particular, it depends on the actual choice of the component of the current (57) to be used in the calculation. Moreover, the result depends on the coordinate frame chosen to perform the integration in (57). This is the general feature of IA in the usual formulation of IF RHD (see, e.g., [4]).

Let us write the final result of the calculation of the form factor from the equation for the null-component of the current and performing the integration in the coordinate frame where $\vec{p}'_c = 0$, $\vec{p}_c = (0, 0, p)$. If now we write the integral in terms of the invariant variables s, s' the obtained form factor has the form:

$$\begin{aligned} F_c(Q^2) &= \frac{M_c}{4} \frac{\sqrt{2(2M_c^2 + Q^2)}}{4M_c^2 + Q^2} \\ &\times \int \sqrt{\frac{s}{s'}} \frac{d\sqrt{s} d\sqrt{s'}}{\sqrt{(s-4M^2)(s'-4M^2)}} \frac{(s+s'+Q^2)^4 Q^2}{[\lambda(s, -Q^2, s')]^{3/2}} \\ &\times \frac{1}{(s s')^{1/4}} \frac{\theta(s, Q^2, s')}{\sqrt{s'}(s+Q^2)} \varphi(s) \varphi(s') f_1(Q^2). \end{aligned} \quad (59)$$

The Eq.(59) differs from (55), obtained with the use of the two-particle free form factor. In the case of wave functions satisfying the conditions (22), (23), the form factor (59) satisfies the normalization: $F_c(0) = e_c$. Let us note that the form factor obtained in this way from the third current component in (57) does not satisfy this condition.

Let us compare IA and MIA results. Let us note once again that in MIA we separate (by use of the scheme of canonical parameterization) the covariant part of the current matrix element in (50) prior to perform any calculations. This covariant part ensures the correct transformation properties of the corresponding decompositions in terms of free-particle states. The difference between (55) and (59) is:

$$\begin{aligned} \Delta F_c(Q^2) &= \int d\sqrt{s} d\sqrt{s'} \varphi(s) \varphi(s') \\ &\times g_0(s, Q^2, s') [1 - R(s, Q^2, s')]. \end{aligned} \quad (60)$$

$$\begin{aligned} R(s, Q^2, s') &= \frac{M_c}{2} \frac{\sqrt{2(2M_c^2 + Q^2)}}{4M_c^2 + Q^2} \sqrt{\frac{s}{s'}} \\ &\times \frac{(s+s'+Q^2)^2}{(s s')^{1/4}} \frac{1}{\sqrt{s'}(s+Q^2)}. \end{aligned} \quad (61)$$

The value $R(s, Q^2, s')$ presents an additional factor to one-particle currents, that is in reality the two-particle current contributions. This term ensures the Lorentz covariance of the electromagnetic current matrix element and the current conservation law in (47). Let us note that this additional term contains no dynamical information on the interaction of test particle with two constituents simultaneously. It does not depend, for example, on the interaction constants for such a process.

So, to summarize, we can write the following schematic equations:

$$(IA)_{Breit} \neq (IA)_{Lab}$$

$$(MIA)_{Breit} = (MIA)_{Lab}$$

It is well known that the standard IA depends strongly on the coordinate frame used for the calculation. The MIA results do not depend on it at all. So, the differences between IA and MIA results for different IA coordinate frame can be rather significant.

Notice that IA and MIA coincide in the nonrelativistic limit. As this takes place, the nonrelativistic limits of form factors, which were obtained from the different current components, are identical. Hence the difference between IA and MIA is connected with the breaking of relativistic covariance conditions really.

We give the quantitative comparison of the form factors obtained in IA and in MIA in the Section IV where the realistic calculation of the pion electromagnetic structure is given.

F. The nonrelativistic limit

The description of composite-system form factors in terms of distributions is not a specific feature of our relativistic approach. The similar formalism is used in non-relativistic theory of composite systems [42] for a rather long time (although not referring to the mathematics of distributions). In the nonrelativistic limit our approach gives the formalism developed in [42].

In the nonrelativistic limit the relativistic charge form factor (55) has the following form:

$$F_{NR}(Q^2) = \int k^2 dk k'^2 dk' u(k) g_{0NR}(k, Q^2, k') u(k'), \quad (62)$$

$$g_{0NR}(k, Q^2, k') = \frac{f_1(Q^2)}{k k'} \theta(k, Q^2, k'), \quad (63)$$

$$\vartheta(k, Q^2, k') = \theta\left(k' - \left|k - \frac{Q}{2}\right|\right) - \theta\left(k' - k - \frac{Q}{2}\right).$$

Here $g_{0NR}(k, Q^2, k')$ is the free relativistic form factor obtained from (40) in the nonrelativistic limit. $f_1(Q^2)$ is the charged-particle form factor. The obtained result coincides with that derived in standard nonrelativistic calculations [42].

In [42] the same formulae are obtained from the equations for form factors in terms of coordinate representation wave functions:

$$F_{NR}(Q^2) = f_1(Q^2) \int_0^\infty dr r u^2(r) j_0\left(\frac{Qr}{2}\right). \quad (64)$$

The Eqs.(62), (63) can be obtained from (64) by use of the Bessel transformation:

$$u(k) = \sqrt{\frac{2}{\pi}} \int_0^\infty r dr u(r) j_0(kr) \quad (65)$$

and the normalization condition:

$$\int_0^\infty u^2(r) dr = \int_0^\infty k^2 u^2(k) dk = 1.$$

Rigorously speaking, the Eq.(62) has to be interpreted as a functional in the sense of distributions generated by the function $g_{0NR}(k, Q^2, k')$ and defined on test functions $u(k) u(k')$. The ordinary function (63) generates regular generalized function defined generally on the larger class of test functions $\psi(k, k')$ in R^2 , providing the uniform convergence of the integral. One needs the uniform convergence to take limits in the integrands.

Let us define the functional in R^2 by the following regular distribution (compare with (41)–(43)):

$$\begin{aligned} & \langle \tilde{g}_{0NR}(k, Q^2, k'), \psi(k, k') \rangle \\ &= \int d\mu(k, k') \tilde{g}_{0NR}(k, Q^2, k') \psi(k, k'), \end{aligned} \quad (66)$$

$$\tilde{g}_{0NR}(k, Q^2, k') = \vartheta(k) \vartheta(k') g_{0NR}(k, Q^2, k'),$$

$$d\mu(k, k') = d\mu(k) d\mu(k'), \quad d\mu(k) = k^2 dk.$$

The function $g_{0NR}(k, Q^2, k')$ which appears in [42] quite formally, here has a definite physical meaning and describes the electromagnetic properties of non-relativistic free system of two spinless particles in the S - state, one of particle having no charge (compare with $g_0(s, Q^2, s')$ in (36), (40), (41)). The static limit $\lim_{Q^2 \rightarrow 0} g_{0NR}(k, Q^2, k')$ giving the system charge exists only in the weak sense as the limit of the functional (66):

$$\lim_{Q^2 \rightarrow 0} \langle \tilde{g}_{0NR}(k, Q^2, k'), \psi(k, k') \rangle$$

$$= \langle e_c \delta(\mu(k') - \mu(k)) \vartheta(k), \psi(k, k') \rangle. \quad (67)$$

On the test functions $\psi(k, k') = u(k)u(k')$ (with $u(k)$ – being the normalized bound state wave function), the functional (66) defines the bound state form factor in the nonrelativistic IA (62). The weak limit (67) is equal to the system charge:

$$\begin{aligned} & \lim_{Q^2 \rightarrow 0} \langle \tilde{g}_{0NR}(k, Q^2, k'), \psi(k, k') \rangle \\ &= e_c \int_0^\infty k^2 dk u^2(k) = e_c. \end{aligned} \quad (68)$$

So, one can see that the description of the system in terms of form factors in IA by the Eq.(62) (as in [42]) in fact defines the form factor in the sense of distributions as a functional defined on the set of the wave functions of bound system.

To go beyond nonrelativistic IA one has to addend some terms to $g_{0NR}(k, Q^2, k')$. For example, such terms cause the meson exchange currents in two–nucleon systems. So, in the standard nonrelativistic theory the dynamical treatment of exchange currents is performed in the same way as in our relativistic approach (53).

So, to conclude, one can consider our approach to IA to be a relativistic generalization of nonrelativistic IA, and our equations for form factors in this approximation to be a relativistic generalization of the equations of [42]. Let us remark that in more complicated systems (e.g., in ρ – meson and deuteron) our relativistic form factors also have correct nonrelativistic limits which coincide with [42].

G. A bridge to dispersion relations

Let us discuss now one of the unsolved problems of RHD – the possible links between RHD and quantum field theory (QFT) [4]. The fact, that RHD, contrary to QFT itself, operates with the finite number of degrees of freedom, makes it to be in some way similar to the dispersion approach of QFT, which is dealing in principle with a finite number of degrees of freedom, too. However, the dispersion relations, based on the analytic properties of the scattering amplitudes, matrix elements, form factors in the complex energy plane, are rather correctly derived in the frame of QFT [43]. So, it seems to us, that one can look for links between RHD and QFT not only directly but through the dispersion approach, too.

Here, using the simple model of the previous Subsections, we compare our version of RHD with the so called modified dispersion approach. Dispersion-relation integrals over composite-particle mass are used in this approach. This approach enabled one to write the deuteron

electromagnetic form factors in terms of the physical hadron scattering phase shift and gave the results for the elastic ed -scattering in good agreement with experimental data. The details of the modified dispersion approach can be found in [33,44–46] (see also [47]). Let us note that an immediate application of the approach to quark systems is difficult to realize because of the fact of quark confinement. However, there are some investigations based on similar ideas where the form factors of hadrons as constituent–quark bound states are considered in the frame of the dispersion technique of the integral over composite particle mass [48].

For convenience of reader let us describe briefly, omitting the proofs, the essence of the modified dispersion approach and obtain the electromagnetic form factor of composite system for our simple model following the paper [33].

The Heisenberg current operator of the system of two particles interacting as in (52) can be written in the form

$$j_\mu = j_\mu^{(0)} + j_\mu^{(int)}. \quad (69)$$

Here $j_\mu^{(0)}$ is the current of the free two–particle system (see (32), (36)). The operator $j_\mu^{(int)}$ is interaction dependent. Let us suppose that our model constituent system has scattering states and let us calculate the matrix element of the operator (69) between *in*– and *out*– states. Let us suppose that the scattering states are the S – states of relative motion. The matrix element of the operator $j_\mu^{(0)}$ can be written in terms of the free two–particle form factor (36) calculated previously (40). The matrix element of the operator containing the interaction

$$\langle \vec{P}(\pm) | j_\mu^{(int)} | \vec{P}'(\pm) \rangle$$

can be written in terms of the form factor

$$G_i(s \mp i\varepsilon, Q^2, s' \pm i\varepsilon). \quad (70)$$

Here sign ”+” stands for *in*– state and sign ”–” for *out*–state. The form factor in (70) has kinematic cuts in the complex plane of the variables s , s' . The cuts go along the real axis from the point $4M^2$ to infinity. The notation $G_i(s + i\varepsilon, Q^2, s' - i\varepsilon)$ means that there exists the analytic continuation of this form factor from the physical region of the variable s into the upper complex half–plane, and to the lower half–plane in s' . One can check this fact considering simple models. The form factor entering the parameterization of the total current (69) can be written as the sum of the form factors:

$$G(s, Q^2, s') = g_0(s, Q^2, s') + G_i(s, Q^2, s'). \quad (71)$$

Let us consider the matrix element of the total current. Let us fix the variable s and let us connect by S –matrix the *in*– and *out*– vectors of the basis in the variable s' :

$$\langle \vec{P} | j_\mu | \vec{P}'(+) \rangle = \langle \vec{P} | j_\mu | \vec{P}'(-) \rangle S(s'). \quad (72)$$

$S(s) = \exp(2i\delta)$, δ is the scattering phase shift. S -matrix can be written in the form:

$$S(s) = \frac{B(s - i\varepsilon)}{B(s + i\varepsilon)}. \quad (73)$$

Here $B(s)$ is the relativistic analog of the Jost function. Taking into account Eqs.(70), (73) one can rewrite (72) as:

$$\begin{aligned} G_i(s, Q^2, s' - i\varepsilon) B(s' - i\varepsilon) - G_i(s, Q^2, s' + i\varepsilon) B(s' + i\varepsilon) \\ = -g_0(s, Q^2, s') (B(s' - i\varepsilon) - B(s' + i\varepsilon)). \end{aligned} \quad (74)$$

The equation (74) presents the so called Riemann–Hilbert problem for half-axis. The solution has the form [49]:

$$G_i(s, Q^2, s') B(s') = \tilde{G}(s, Q^2, s') + C_1(s, Q^2, s'), \quad (75)$$

$$\begin{aligned} \tilde{G}(s, Q^2, s') = -\frac{1}{2\pi i} \int_{4M^2}^{\infty} \frac{ds'' g_0(s, Q^2, s'') \Delta(s'')}{s' - s''}, \\ (76) \end{aligned}$$

$$\Delta(s) = (B(s + i\varepsilon) - B(s - i\varepsilon)),$$

Here $C_1(s, Q^2, s')$ is an unknown function, regular in s' in the neighbourhood of the real axis for $4M^2 \leq s' < \infty$. Now let us connect the *in*- and *out*-bases in the variable s . Taking into account the explicit form of G_i (75), (76) we obtain the boundary value Riemann–Hilbert problem for the function C_1 in the variable s with the solution:

$$\begin{aligned} C_1(s - i\varepsilon, Q^2, s') B(s - i\varepsilon) - C_1(s + i\varepsilon, Q^2, s') B(s + i\varepsilon) \\ = \left[g_0(s, Q^2, s') B(s') + \tilde{G}(s, Q^2, s') \right] \Delta(s), \end{aligned} \quad (77)$$

$$\begin{aligned} B(s) C_1(s, Q^2, s') = -\frac{1}{2\pi i} \int_{4M^2}^{\infty} ds''' \\ \times \frac{\left[g_0(s''', Q^2, s') B(s') + \tilde{G}(s''', Q^2, s') \right] \Delta(s''')}{s - s'''} \\ + C(s, Q^2, s'). \end{aligned} \quad (78)$$

The unknown function $C(s, Q^2, s')$ is regular in the neighbourhood of the real axis for $4M^2 \leq s, s' < \infty$ in s' as well as in s . Now let us consider the matrix element of

the total current (69), (71) between *in*-states. Substituting (75), (76), (78) in (71) we obtain finally the following form for the form factor of the total current (69) in *in*-basis:

$$\begin{aligned} G(s, Q^2, s') = g_0(s, Q^2, s') \\ - \frac{1}{2\pi i B(s' + i\varepsilon)} \int_{4M^2}^{\infty} ds'' \frac{g_0(s, Q^2, s'') \Delta(s'')}{s' - s'' + i\varepsilon} \\ - \frac{1}{2\pi i B(s - i\varepsilon)} \int_{4M^2}^{\infty} ds''' \frac{g_0(s''', Q^2, s') \Delta(s''')}{s - s''' - i\varepsilon} \\ - \frac{1}{4\pi^2 B(s - i\varepsilon) B(s' + i\varepsilon)} \\ \times \int_{4M^2}^{\infty} ds''' \int_{4M^2}^{\infty} ds'' \frac{g_0(s''', Q^2, s'') \Delta(s''') \Delta(s'')}{(s - s''' - i\varepsilon)(s' - s'' + i\varepsilon)} \\ + \frac{C(s, Q^2, s')}{B(s - i\varepsilon) B(s' + i\varepsilon)}. \end{aligned} \quad (79)$$

The Eq.(79) provides the correct analytic properties of the form factor obtained in QFT approach [43]. In particular, this form contains the anomal branch points known from the dispersion approach to composite systems (e.g., to the deuteron). The Eq.(79) can be used to obtain the form factor of the constituent bound state for the case of the S -state of relative motion. Now it is necessary to perform the analytic continuation of (79) in the variables s, s' to the bound state point $s = s' = M_c^2$ (M_c is the bound state mass) and to take the residues in the poles. As the result we obtain the bound-system form factor directly in terms of the S -scattering phase shift for constituents:

$$F_c(Q^2) = \Gamma^2 \int_{4M^2}^{\infty} ds ds' \frac{g_0(s, Q^2, s') \Delta(s) \Delta(s')}{(s - M_c^2)(s' - M_c^2)}. \quad (80)$$

Now the constant Γ^2 is determined by the condition $F_c(0) = e_c$ and indirectly takes into account the contributions of the so called unphysical cuts. The Jost-function discontinuities can be written in terms of experimental scattering phase shift. The free two-particle form factor for our model has the form (40). $F_c(Q^2)$ is the functional generated by the generalized function $g_0(s, Q^2, s')$ on the test functions $\Delta(s)/(s - M_c^2)$. The described formalism was applied to the deuteron in Ref. [46] and gave a good agreement with experimental data.

Let us note, that the form (80) obtained through the modified dispersion approach is in close analogy to the

forms (55), (62) obtained in the frame of IF RHD. This analogy can be made even more obvious using the results of the Ref. [45] where neutron-proton system was considered in nonrelativistic case. In [45] it is shown that if the deuteron electrodisintegration amplitude satisfies Mandelstam representation, then the wave function of the system has the well fixed form and can be expressed in terms of np -scattering phases. This wave function satisfies a dispersion relation. The analytical properties used during the derivation of this relation are the same for the large group of phenomenological potentials. So the obtained dispersion relation can be used to find explicit form for real two-nucleon systems wave functions. As one can see from the solution structure, such a reconstruction of wave function is stable both in the usual sense and in the sense of the large energy phases influence. Finally, the bound state wave function $u(r)$ is of the form:

$$u(r) = \tilde{\Gamma} \int_{-\infty}^{\infty} dx \frac{\Delta(x)}{(x - \kappa)} [\sin(xr)]. \quad (81)$$

(See [45] for details.) Here κ^2 is the deuteron binding energy and the nonphysical cuts contribution enters the normalization constant $\tilde{\Gamma}$.

In nonrelativistic case the Jost-function discontinuity multiplied by the pole term in (80) gives the nonrelativistic wave function up to nonphysical cuts contribution [45]. When these contributions are taken into account the equations (62) and (80) do coincide.

So, the relativistic Eq.(55) can be motivated (at least for composite systems which have the scattering states) in the frame of modified dispersion approach, that is on the usual level of correctness for obtaining the analytical properties of the form factors of composite systems in the frame of QFT.

IV. THE ELECTROWEAK STRUCTURE OF PION

Now we apply the method of previous sections to the calculation of the electroweak structure of pion. There exists a lot of experimental data on pion, so the effectiveness of the method can be checked by the comparison with the data (see, e.g., [16] and references therein).

A. The electromagnetic form factor of pion

There are many facts that make it interesting to consider the pion in the frame of the formalism developed in the previous sections. First, the pion is an important

object in the particle physics and is in the focus of interest for years. Second, the pion consists of light quarks and thus has to be considered in the frame of relativistic approach. Third, at the present time the large program on the pion structure is on line in JLab [50].

We consider the pion as a system of two constituent quarks. The system is described by a phenomenological wave function.

In theoretical treatment of the pion electromagnetic structure one has to make difference between "soft" and "hard" parts of the form factor. The "soft" part which dominates at small and intermediate momentum transfers, needs nonperturbative approaches. The "hard" part which defines the form factor at asymptotically large values of momentum transfers can be calculated from perturbative QCD. However, a controversy still exists concerning the scale of momentum transfers characteristic of the transition from the nonperturbative to the perturbative regime (see, e.g. [51]). It is pointed out by different authors that the existing experimental data are defined by the "soft" part of the form factor and can be described by use of phenomenological wave functions of constituent quarks without involving the perturbative QCD. Usually such calculations were performed in the frame of light-front relativistic quantum mechanics. In the present paper we calculate the "soft" part of the charge form factor of pion in the frame of IF RHD. We obtain a good description of the behaviour of the form factor in the wide region of momentum transfers where the experimental data exist $0 \leq Q^2 \leq 8(GeV/c)^2$ [20].

We pay a special attention to the role of relativistic properties of quark spins for the pion structure. The relativistic spin rotation effect (Wigner rotation) caused by the summation of quark spins gives large contribution to the pion form factor (10 - 20% depending on the value of momentum transfer) [24]. It is interesting that spin rotation effect vanishes as $Q^2 \rightarrow 0$ but gives large contribution ($\sim 30\%$) to the pion charge radius [17], which is defined by the slope of the form factor at $Q^2 = 0$.

In our version of IF RHD the meson form factor asymptotics at large momentum transfer is the same as in perturbative QCD. The asymptotics is now determined by relativistic kinematics only, specifically by the relativistic effect of spin rotation, and does not depend on the choice of the quark wave function, that is of the quark interaction model.

Our version of IF RHD approach gives, in agreement with the experimental data, an adequate description of the pion electromagnetic structure in large region of momentum transfers.

The pion is spinless, so the electromagnetic current matrix element has the form (26) with $p_c \rightarrow p_\pi$, $F_c(Q^2) \rightarrow F_\pi(Q^2)$. In the frame of composite quark model pion is

considered as the bound state of u^- and \bar{d}^- quarks. We suppose that quark masses are equal: $m_u = m_d = M$.

To calculate in MIA the composite-system form factor one needs to construct first the free two-particle form factor (36), (40), (55). Contrary to the simple model of the previous Section now we consider the system of two charged particles with spins 1/2. This gives the following complications. First, the Eq. (32) for the current operator of the free system is now transformed to the form:

$$j_\mu^{(0)}(0) = j_{1\mu} \otimes I_2 \oplus j_{2\mu} \otimes I_1. \quad (82)$$

Here $j_{(1,2)\mu}$ - the electromagnetic currents of particles, $I_{(1,2)}$ - the unity operators in the one-particle state Hilbert spaces. The Eq.(82) can be rewritten in terms of matrix elements:

$$\begin{aligned} & \langle \vec{p}_1, m_1; \vec{p}_2, m_2 | j_\mu^{(0)}(0) | \vec{p}'_1, m'_1; \vec{p}'_2, m'_2 \rangle = \\ & = \langle \vec{p}_2, m_2 | \vec{p}'_2, m'_2 \rangle \langle \vec{p}_1, m_1 | j_{1\mu} | \vec{p}'_1, m'_1 \rangle + (1 \leftrightarrow 2). \end{aligned} \quad (83)$$

Second, the matrix element of one-particle current contains now, contrary to (33), the magnetic form factors of quarks as well as the charge ones. Now the parameterization (the elementary-particle one following [4]) is of the form:

$$\begin{aligned} & \langle \vec{p}, m | j^\mu(0) | \vec{p}', m' \rangle \\ & = \bar{u}_{\vec{p}m} \gamma^\mu u_{\vec{p}'m'} F_1(Q^2) - \bar{u}_{\vec{p}m} \sigma^{\mu\nu} q_\nu u_{\vec{p}'m'} F_2(Q^2), \end{aligned} \quad (84)$$

$u_{\vec{p}m}$ - the Dirac bispinor, γ^μ - Dirac matrix,

$$\sigma^{\mu\nu} = \frac{1}{2}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu), \quad q_\nu = (p - p')_\nu,$$

Using multipole parameterization we can write the one-particle current matrix element in terms of Sachs form factors:

$$G_E(Q^2) = \tilde{F}_1(Q^2) + \frac{\kappa Q^2}{4M^2} \tilde{F}_2(Q^2),$$

$$G_M(Q^2) = \tilde{F}_1(Q^2) + \kappa \tilde{F}_2(Q^2),$$

$$F_1(Q^2) = e \tilde{F}_1(Q^2), \quad F_2(t) = \frac{\kappa}{2M} \tilde{F}_2(Q^2). \quad (85)$$

Here $G_{E,M}$ - Sachs electric and magnetic form factors, respectively, e is the particle charge, κ is the anomalous magnetic moment.

It is convenient to use the canonical parameterization of matrix elements [32]:

$$\langle \vec{p}, m | j_\mu(0) | \vec{p}', m' \rangle$$

$$= \sum_{m''} \langle m | D^j(p, p') | m'' \rangle \langle m'' | f_1(Q^2) K'_\mu + i f_2(Q^2) R_\mu | m' \rangle,$$

$$K'_\mu = (p + p')_\mu, \quad R_\mu = \epsilon_{\mu\nu\lambda\rho} p^\nu p'^\lambda \Gamma^\rho(p'). \quad (86)$$

$\Gamma(p)$ is 4-vector of spin:

$$\vec{\Gamma}(p) = M \vec{j} + \frac{\vec{p}(\vec{p}\vec{j})}{p_0 + M}, \quad \Gamma_0(p) = (\vec{p}\vec{j}).$$

The form factors $f_1(Q^2)$ $f_2(Q^2)$ are the electric and magnetic form factors of particles. They are connected with Sachs form factors [52]:

$$f_1(Q^2) = \frac{2M}{\sqrt{4M^2 + Q^2}} G_E(Q^2),$$

$$f_2(Q^2) = -\frac{4}{M\sqrt{4M^2 + Q^2}} G_M(Q^2). \quad (87)$$

Third, now the CG coefficients are of more complicated form. They are given by (19) with $J = S = l = 0$. Contrary to the previous simple case, now the CG coefficients contain the Wigner rotation matrices.

Finally, the free two-particle form factor for the system of two particles with spin 1/2 and quantum numbers $J = S = l = 0$ is of the form (see also [20]):

$$g_0^{q\bar{q}}(s, Q^2, s') = n_c \frac{(s + s' + Q^2) Q^2}{2\sqrt{(s - 4M^2)(s' - 4M^2)}}$$

$$\times \frac{\theta(s, Q^2, s')}{[\lambda(s, -Q^2, s')]^{3/2}} \frac{1}{\sqrt{1 + Q^2/4M^2}}$$

$$\begin{aligned} & \times \left\{ (s + s' + Q^2)(G_E^u(Q^2) + G_E^{\bar{d}}(Q^2)) \cos(\omega_1 + \omega_2) + \right. \\ & \left. + \frac{1}{M} \xi(s, Q^2, s')(G_M^u(Q^2) + G_M^{\bar{d}}(Q^2)) \sin(\omega_1 + \omega_2) \right\}, \end{aligned} \quad (88)$$

Here

$$\xi(s, Q^2, s') = \sqrt{ss'Q^2 - M^2\lambda(s, -Q^2, s')},$$

n_c is the number of quark colours, ω_1 ω_2 - the Wigner rotation parameters:

$$\omega_1 = \arctan \frac{\xi(s, Q^2, s')}{M \left[(\sqrt{s} + \sqrt{s'})^2 + Q^2 \right] + \sqrt{ss'}(\sqrt{s} + \sqrt{s'})},$$

$$\omega_2 = \arctan \frac{\alpha(s, s') \xi(s, Q^2, s')}{M(s + s' + Q^2) \alpha(s, s') + \sqrt{ss'}(4M^2 + Q^2)}, \quad (89)$$

with $\alpha(s, s') = 2M + \sqrt{s} + \sqrt{s'}$, and $G_{E,M}^{u,d}(Q^2)$ are Sachs form factors for quarks. The θ - function in (88) is the same as in (40).

An interesting effect follows from (88): due to the relativistic Wigner spin rotation effect the pion charge form factor contains the contribution of quark magnetic form factors.

The pion charge form factor can be calculated using (55), with (88) for the free two-particle form factor:

$$F_\pi(Q^2) = \int d\sqrt{s} d\sqrt{s'} \varphi(s) g_0^{q\bar{q}}(s, Q^2, s') \varphi(s'). \quad (90)$$

B. The lepton decay constant of pion

Let us calculate now the lepton decay constant of pion in the frame of our approach. The interest to such a calculation is threefold. First, this constant is measured in experiment with great accuracy [53], so that it can be a test for the model, and give the limits for parameters of models. Second, it is interesting to describe the electromagnetic form factors and the weak decay constant in the frame of one and the same approach: the decay constant indirectly, through the parameters of the model, defines the behavior of form factors at large values of momentum transfers Q^2 . Third, it is interesting to estimate relativistic effects in the lepton decay.

The lepton decay constant f_π is defined by the electroweak-current matrix element [16]:

$$\langle 0 | j_\mu(0) | p_\pi \rangle = i f_\pi p_{\pi \mu} \frac{1}{(2\pi)^{3/2}}. \quad (91)$$

p_π - 4-momentum of meson. Let us decompose the l.h.s. of (91) in the basis (9). Using the explicit form of the meson wave function (21) one can obtain for (91):

$$\begin{aligned} & \int \frac{N_c}{N_{CG}} d\sqrt{s} \langle 0 | j_\mu(0) | \vec{p}_\pi, \sqrt{s} \rangle \varphi(s) \\ &= i f_\pi p_{\pi \mu} \frac{1}{(2\pi)^{3/2}}. \end{aligned} \quad (92)$$

As in Section II (Eq.(49)) one can divide the integrand in (92) into two parts: the covariant part (smooth ordinary function) and the invariant part.

$$\frac{N_c}{N_{CG}} \langle 0 | j_\mu(0) | \vec{p}_\pi, \sqrt{s} \rangle = i G(s) B_\mu(s) \frac{1}{(2\pi)^{3/2}}. \quad (93)$$

The invariant form factor $G(s)$ is a generalized function. In the same way as in calculating (54) of the previous section, we now obtain the lepton decay constant of pion in the form

$$\int d\sqrt{s} G(s) \varphi(s) = f_\pi. \quad (94)$$

In general, the form factor $G(s)$ can be calculated in the frame of the standard model for electroweak interactions. However, in this paper we limit ourselves by 4-fermion interaction. We take for $G(s)$ the form factor which parameterizes the decay of free two-quark system:

$$\langle 0 | j_\mu^{(0)}(0) | \vec{P}, \sqrt{s} \rangle = i G_0(s) P_\mu \frac{1}{(2\pi)^{3/2}}. \quad (95)$$

The explicit form (95) is written by analogy to (36) not taking into account the current conservation law. The form (95) is quite similar to (91) but instead of the constant f_π the form factor depending on invariant variables is written. To calculate $G_0(s)$ let us decompose (95) in the one-particle basis (8). Now we obtain for (95):

$$\begin{aligned} & i G_0(s) P_\mu \frac{1}{(2\pi)^{3/2}} \\ &= \sum_{m_1, m_2, i_c} \int \frac{d\vec{p}_1}{2p_{10}} \frac{d\vec{p}_2}{2p_{20}} \langle 0 | j_{\mu i_c}^{(0)} | \vec{p}_1, m_1; \vec{p}_2, m_2 \rangle \\ & \quad \times \langle \vec{p}_1, m_1; \vec{p}_2, m_2 | \vec{P}, \sqrt{s} \rangle. \end{aligned} \quad (96)$$

$i_c = 1, 2, 3$, the sum over i_c is the sum over the colours. The CG coefficients are known (19). The current matrix element in the basis (8) can be written in the standard way in terms of the lepton decay current matrix element [16]:

$$\begin{aligned} & \langle 0 | j_\mu^{(0)} | \vec{p}_1, m_1; \vec{p}_2, m_2 \rangle \\ &= \frac{1}{(2\pi)^3} \bar{v}(\vec{p}_2, m_2) \gamma_\mu (1 + \gamma^5) u(\vec{p}_1, m_1). \end{aligned} \quad (97)$$

We integrate in (96) in the coordinate frame with $\vec{P} = 0$. Finally, we obtain:

$$G_0(s) = \frac{n_c}{2\sqrt{2}\pi P_0} (p_0 + M) \left[1 - \frac{k^2}{(p_0 + M)^2} \right], \quad (98)$$

$$p_0 = \sqrt{k^2 + M^2}.$$

Substituting (98) in the Eq.(94) we obtain the result which has the following form if written in invariant variables:

$$f_\pi = \frac{2M n_c}{2\sqrt{2}\pi} \int d\sqrt{s} \frac{1}{\sqrt{s}} \varphi(s). \quad (99)$$

Let us notice that the Eq.(99) coincides with that obtained in the frame of light-front dynamics [16]. However, although all forms of RHD are unitary equivalent [10], nevertheless after the physical approximations are made in more complicated cases the results, e.g. for form factors, can be different. This is possibly due to the fact that the unitary operators connecting different forms of RHD are interaction dependent [10] and so the RHD forms realize one and the same approximation in different ways.

Let us remark that the nonrelativistic limit of the Eq.(99) gives the standard form in terms of coordinate space wave function at zero value.

C. The results of calculations

To calculate the electroweak structure of pion using (90), (88), (99), (22) the following meson wave functions were utilized:

1. A gaussian or harmonic oscillator (HO) wave function

$$u(k) = N_{HO} \exp(-k^2/2b^2). \quad (100)$$

2. A power-law (PL) wave function

$$u(k) = N_{PL} (k^2/b^2 + 1)^{-n}, \quad n = 2, 3. \quad (101)$$

3. The wave function with linear confinement from Ref. [54]:

$$u(r) = N_T \exp(-\alpha r^{3/2} - \beta r), \quad \alpha = \frac{2}{3} \sqrt{M a},$$

$$\beta = \frac{M}{2} b. \quad (102)$$

a, b – parameters of linear and Coulomb parts of potential respectively.

In the Ref. [20] in the calculation of pion electromagnetic structure we supposed the quarks to be point-like. The results of [20] can be considered as preliminary results. However, one has to take into account the structure of constituent quarks [55], in particular, the anomalous magnetic moment. As anomalous magnetic moments are connected with finite size of quark, one has to take into account the explicit form of quark form factors entering (88) and the pion charge form factor (90). As in [19] let us use the following forms for quark form factors:

$$G_E^q(Q^2) = e_q f(Q^2),$$

$$G_M^q(Q^2) = (e_q + \kappa_q) f(Q^2). \quad (103)$$

Here e_q – the quark charge, κ_q – the quark anomalous magnetic moment (in natural units). To obtain the explicit form of the function $f(Q^2)$ let us consider the asymptotics of pion charge form factor as $Q^2 \rightarrow \infty, M \rightarrow 0$.

To obtain the asymptotic behavior let us first make the asymptotic estimation of the integrals in (90) in the point-like quark approximation ($f(Q^2) = 1, \kappa = 0$ in(103)). Omitting the details of calculation (given in [56]) we write the final result for the asymptotics in the form:

$$F_\pi(Q^2) \sim Q^{-2}. \quad (104)$$

The asymptotics does not depend on the actual form of the wave function and coincides with that obtained in QCD. The actual form we obtain, e.g. for (100) is:

$$F_\pi(Q^2) \sim 32\sqrt{2} \frac{[\Gamma(\frac{5}{4})]^2 b^2}{\sqrt{\pi} Q^2}. \quad (105)$$

It is worth to compare the form (105) with the detailed QCD result [57]:

$$F_\pi(Q^2) = \frac{8\pi\alpha_s f_\pi^2}{Q^2}. \quad (106)$$

If $\alpha_s/\pi \sim 0.1$ then (105) and (106) coincide at $b \sim 0.1$. So the asymptotics (104) is quite realistic.

In the case of non-point-like quarks we obtain another asymptotics because the form factor depends upon the momentum transfer. It is known that QCD gives logarithmic corrections to (106). To agree with this QCD corrected asymptotics we can, for example, choose the following form for $f(Q^2)$:

$$f_q(Q^2) = \frac{1}{1 + \ln(1 + \langle r_q^2 \rangle Q^2/6)}. \quad (107)$$

Here $\langle r_q^2 \rangle$ is the MSR of the constituent quark which can be considered as the model parameter. Let us fix it (as in [19]) to be: $\langle r_q^2 \rangle \simeq 0.3/M^2$.

For the constituent quark mass in pion we use the value which is usually used in the calculations in RHD: $M = 0.25$ GeV.

The quark anomalous magnetic moments can be taken from [55]: $\kappa_u = 0.029, \kappa_d = -0.059$.

We choose the parameters b in (100), (101) and a in (102) in such a way as to fit the pion MSR: $\langle r_\pi^2 \rangle = (0.432 \pm 0.016) \text{ Fm}^2$ [58]. We choose this way to fix the model parameters because the pion MSR is defined by the form factor at small values of Q^2 , that is the range where potential models work well.

The fit of the pion MSR gives the following parameters of the wave functions: in the model (100) $b = 0.2784$ GeV, model (101) at $n = 2$ $b = 0.3394$ GeV, model (101) at $n = 3$ $b = 0.5150$ GeV, model (102) $b = (4/3)\alpha_s$, $\alpha_s = 0.59$ at light meson mass scale, $a = 0.0567$ GeV².

The results of calculation are presented on Figs.3 and 4.

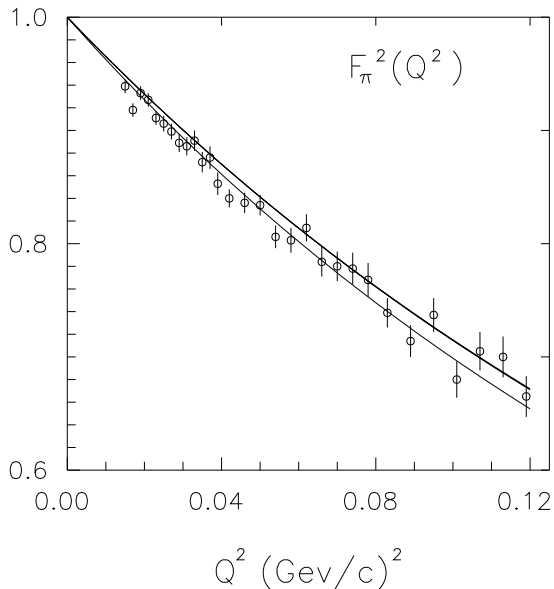


FIG. 3. The square of the pion form factor at small values of momentum transfers for different models.

The square of the pion form factor at small values of momentum transfers for different models (100) – (102) is presented on Fig.3. Results of calculation in the models (100), (101) at $n = 3$ and (102) coincide very closely.

The calculations of product $Q^2 F_\pi(Q^2)$ at high momentum transfers for different models (100) – (102) are presented on Fig.4. Legend is following: 1 – harmonic oscillator wave function (100), 2 – power-law wave function (101) at $n = 2$, 3 – power-law wave function (101) at $n = 3$, and wave function from model with linear confinement (102) (these curves coincide very closely).

All the models for the interaction (100), (101), (102) give a good description of the existing experimental data⁴.

The dependence of the results on the actual model is much less pronounced that in the case of point-like quarks [20].

The lepton decay constants calculated following Eq.

(99) with different wave functions have the following values: $f_\pi = 0.1210$ GeV in the model (100), $f_\pi = 0.1327$ GeV in the model (101) with $n = 2$, $f_\pi = 0.1282$ GeV in the model (101) with $n = 3$, and $f_\pi = 0.1290$ GeV in the model (102). Let us emphasize that we have used no fitting parameters to calculate the lepton decay constant. Nevertheless, the obtained values are very close to the experimental value: $f_{\pi exp} = 0.1307 \pm 0.0005$ GeV [53].

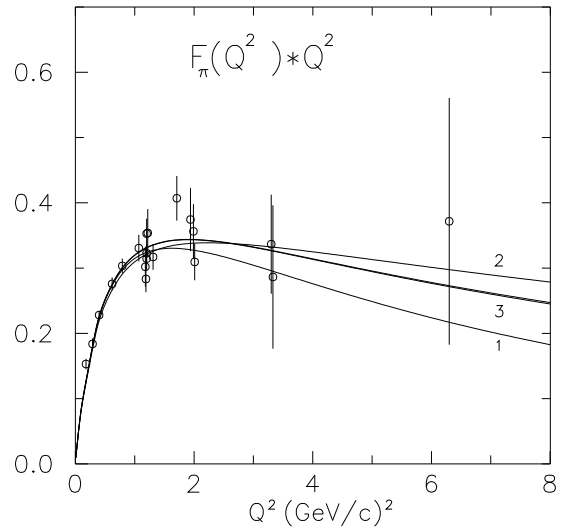


FIG. 4. Electromagnetic form factor, $Q^2 F_\pi(Q^2)$, at high momentum transfers.

Now let us compare the numerical results for the pion form factor obtained in MIA (90) with that of the traditional IA. Let us choose for the comparison, for example, the null-component of the current.

To obtain the pion form factor in IA we proceed in the same way as while obtaining (59) of the preceding Section. Now, however,

- 1) the decomposition (26) of the IA matrix current element over the state set (8) is realized following (83),
- 2) the parameterization of the one-particle matrix element is given by (86), (87) (instead of (33)),
- 3) the CG coefficient (19) in (58) are for pion quantum numbers.

Acting in the same way as while obtaining (59), and using the null-component of the current matrix element, we can write the pion form factor in IA in the following form:

$$F_\pi(Q^2) = \frac{M_\pi}{4} \frac{\sqrt{2(2M_\pi^2 + Q^2)}}{4M_\pi^2 + Q^2} \frac{n_c}{\sqrt{1 + Q^2/4M^2}} \times \int \sqrt{\frac{s}{s'}} \frac{d\sqrt{s} d\sqrt{s'}}{\sqrt{(s - 4M^2)(s' - 4M^2)}}$$

⁴TheJLab new results [59] are discussed in connection with our approach in [60]

$$\begin{aligned}
& \times \frac{(s + s' + Q^2)^3 Q^2}{[\lambda(s, -Q^2, s')]^{3/2}} \frac{1}{(s s')^{1/4}} \frac{1}{\sqrt{s'}(s + Q^2)} \varphi(s) \varphi(s') \\
& \times \left\{ (s + s' + Q^2) \left[G_E^u(Q^2) + G_E^{\bar{d}}(Q^2) \right] \cos(\omega_1 + \omega_2) \right. \\
& \left. + \frac{1}{M} \xi(s, Q^2, s') \left[G_M^u(Q^2) + G_M^{\bar{d}}(Q^2) \right] \sin(\omega_1 + \omega_2) \right\}.
\end{aligned} \tag{108}$$

Here $M_\pi = 139.5702 \pm 0.0004$ MeV [53] is mass of pion.

The normalization condition $F_\pi(0) = 1$ is satisfied for the form factor (108) if the wave functions (22) satisfy (24).

To compare the numerical results given by the Eqs.(90), (88) with that given by (108) let us calculate the pion form factor using the wave function (100) with the parameters of the calculations presented in Figs.3 and 4. The results are shown in the Fig.5. The results obtained with the use of the parameterization (55), (88) differ essentially from that obtained without such parameterization (108). The form factor calculated in our approach describes the existing experimental data adequately.

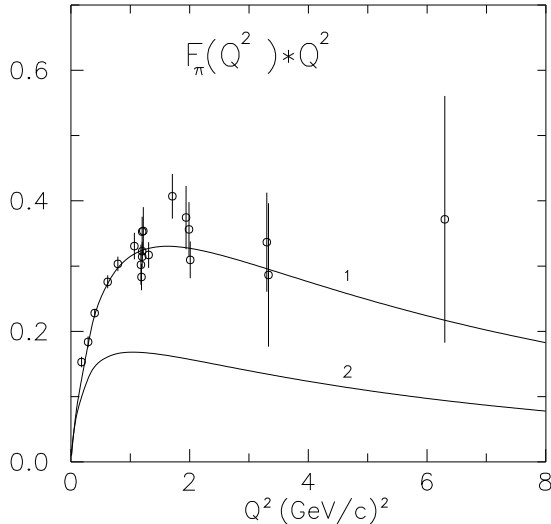


FIG. 5. $Q^2 F(Q^2)$ for MIA (1) and for IA (2). Results of calculation with wave function (100). Parameters are the same as in Fig.3.

Let us emphasize once again that the form factor obtained in MIA does not depend on the choice of coordinate frame. This is an important advantage of our relativistic MIA.

V. CONCLUSION

Let us summarize the results.

1. A new approach to the electromagnetic properties of two-particle composite systems is developed. The approach is based on IF RHD.
2. The main novel feature of this approach is the new method of construction of the matrix element of the electroweak current operator. The electroweak current matrix element satisfies the relativistic covariance conditions and in the case of the electromagnetic current also the conservation law automatically.
3. The method of the construction of the current operator matrix element consists of the extraction of the invariant part – the reduced matrix element on the Lorentz group (form factor) – and the covariant part defining the transformation properties of the current. The form factors contain all the dynamical information about transition. The properties of the system as well as the approximations used are formulated in terms of form factors, which in general have to be considered as generalized functions.
4. The approach makes it possible to formulate relativistic impulse approximation (modified impulse approximation – MIA) in such a way that the Lorentz-covariance of the current is ensured. In the electromagnetic case the current conservation law is ensured, too.
5. The results of the calculations are unambiguous: they do not depend on the choice of the coordinate frame and on the choice of "good" components of the current as it takes place in the standard form of light-front dynamics.
6. The formalism enables one to solve in part the problem of connection of RHD and QFT by comparison of RHD with the dispersion approach. In this paper RHD is compared with a modified approach where dispersion-relation integrals over composite-particle mass are used.
7. The effectiveness of the approach is demonstrated by the calculation of the electroweak structure of the pion. Our approach gives good results for the pion electromagnetic form factor in the whole range of momentum transfers available for experiments at present time.

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