# INTRODUCTION TO LOW $x$ PHYSICS AND DIFFRACTION [a]

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The basic concepts relevant for the theoretical description of deep inelastic scattering within the QCD improved parton model are introduced. Recent developments in low $x$ DIS and in deep inelastic diffraction are briefly summarised. This includes discussion of the BFKL dynamics including the subleading effects and of the saturation model. The dedicated measurements, which probe the QCD pomeron are also discussed.

The aim of this talk is to discuss the following issues:

1. Low $x$ physics in QCD.

    (a) BFKL equation.

    (b) Saturation model.

2. Deep inelastic diffraction.

We shall discuss the low $x$ physics in QCD on the example of deep inelastic ep scattering, i.e. the process:

$$\mathrm{e}(p_{\mathrm{e}}) \ + \ \mathrm{p}(p) \rightarrow \mathrm{e}(p'_{\mathrm{e}}) + X \ . \tag{1}$$

The conventional kinematical variables for the description of this process are

$$s = (p_{\mathrm{e}} + p)^2, \quad q = p_{\mathrm{e}} - p'_{\mathrm{e}}, \quad Q^2 = -q^2, \quad W^2 = (q + p)^2 \tag{2}$$

$$y = \frac{pq}{p_{\mathrm{e}}p}, \quad x = \frac{Q^2}{2pq} \ . \tag{3}$$

The ep inelastic scattering is controlled by the virtual photon exchange mechanism and the total $\gamma^*$p cross-section is closely related to the structure function $F_2$

$$\sigma_{\gamma^*\mathrm{p}} = \frac{4\pi^2 \alpha_{\mathrm{em}}}{Q^2} F_2 \ .$$

It should be observed that the small $x$ behaviour of $F_2$ is related to the large $W^2$, i.e. Regge limit of $\sigma_{\gamma^*p}$.

The ep DIS is conventionally described within the QCD improved parton model. In this model, the structure function $F_2$ is directly related to the quark and antiquark distributions in the nucleon:

$$F_2(x, Q^2) = x \sum_f e_f^2 [q_f(x, Q^2) + \bar{q}_f(x, Q^2)] + O(\alpha_s) \ . \tag{4}$$

The parton (i.e. quark and gluon) distributions satisfy the DGLAP equations, which in LO have the following structure:

$$Q^2 \frac{dq_i(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} (P_{qq}^{(0)} \otimes q_i + P_{qg}^{(0)} \otimes g)$$

$$Q^2 \frac{dg(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} [P_{gq}^{(0)} \otimes \sum_i (q_i + \bar{q}_i) + P_{gg} \otimes g] \ . \tag{5}$$

Beyond LO (i.e. at NLO + ...) we have:

$$\frac{\alpha_s(Q^2)}{2\pi} P_{ij}^{(0)} \rightarrow$$

$$\frac{\alpha_s(Q^2)}{2\pi} P_{ij}^{(0)} + \left( \frac{\alpha_s(Q^2)}{2\pi} \right)^2 P_{ij}^{(1)} + .... \tag{6}$$

At low $x$, the dominant role is played by the gluons. That follows from the singular behaviour of the splitting function $P_{gg}(z)$ for $z \rightarrow 0$,

$$P_{gg}^{(0)}(z) \sim \frac{2N_c}{z} \ .$$

The small $x$ behaviour of the parton distributions depends upon the structure of their small $x$ behaviour at the reference scale $Q_0^2$, i.e.

$$xp_i(x, Q_0^2) \sim x^{-\lambda} \rightarrow xp_i(x, Q^2) \sim x^{-\lambda}$$

$$\lambda > 0 \tag{7}$$

$$xp_i(x, Q_0^2) \sim const \rightarrow xp_i(x, Q^2) \sim \exp[2\sqrt{\xi(Q^2) \ln(1/x)}] \ , \tag{8}$$

where

$$\xi(Q^2) = \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{N_c \alpha_s q^2}{\pi} \ .$$

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In both cases, $xg$, $x(q + \bar{q})$, $F_2$... etc. are found to increase in the limit $x \to 0$.

The LO and NLO, the DGLAP formalism which sums the leading and next-to-leading powers of $\ln(Q^2/Q_0^2)$ is incomplete at low $x$. In this region one has to resum (leading and subleading) powers of $\ln(1/x)$. Small-$x$ resummation of leading (+subleading) powers of $\ln(1/x)$ generates the QCD pomeron.

Diagramatically, the QCD Pomeron corresponds to gluon ladder exchange. The basic dynamical quantity in this case is the unintegrated gluon distribution $f(x, \hat{k}^2)$ where $\hat{k}^2$ denotes the square of the transverse momentum of the gluon. The unintegrated gluon distribution satisfies the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation, which, in the leading $\ln(1/x)$ approximation, has the following form:[1,2]

$$f(x, \hat{k}^2) = f^0(x, \hat{k}^2) + \frac{3\alpha_s}{\pi} K \otimes f , \qquad (9)$$

where

$$K \otimes f = \int_x^1 \frac{\mathrm{d}z}{z} \int \frac{\mathrm{d}^2\hat{q}}{\pi \hat{q}^2} [f(z, (\hat{k} + \hat{q})^2) - \Theta(\hat{k}^2 - \hat{q}^2) f(z, \hat{k})] . \qquad (10)$$

The conventional (integrated) gluon ditribution is given by:

$$xg(x, Q^2) = \int^{Q^2} \frac{\mathrm{d}\hat{k}^2}{\hat{k}^2} f(x, \hat{k}^2) . \qquad (11)$$

The following properties of the BFKL dynamics should be mentioned:

1. Diffusion of transverse momentum along the chain which should reflect itself in the hadronic final state.

2. Characteristic rise with decreasing $x$.

   In LO $f \sim x^{-\lambda}, \lambda = 4\ln(2)3\alpha_s/\pi$

3. Large subleading effects. Their major part is understood and is under control.

4. The BFKL equation embodies (part of the) LO DGLAP evolution.

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One of the most important recent theoretical developments in the low $x$ physics has been the completion of the calculation of the NLO $\ln(1/x)$ effects.[3,4] Those effects were found to be very important and in particular they were found to reduce significantly the QCD pomeron intercept, which in the NLO approximation is given by

$$\lambda = 4\ln(2)\bar{\alpha}_s(1 - 6.3\bar{\alpha}_s) \; , \tag{12}$$

where

$$\bar{\alpha}_s = \frac{3\alpha_s}{\pi} \; .$$

It is, therefore, obvious that resummation of subleading $\ln(1/x)$ beyond NLO is needed. It has been found, however, that the dominant part of the subleading corrections is generated by the phase-space limitations. They can be taken into account exactly, i.e. beyond NLO.[4,5,6]

The observable quantities, as the structure function $F_2$ are obtained from the unintegrated gluon distributions through the $k_{\rm t}$ factorisation:[7]

$$F_2 \sim F^{\gamma g} \otimes f \; , \tag{13}$$

where '$\otimes$' denotes in this case convolution in transverse and longitidinal momenta. At leading twist, the $k_{\rm t}$ factorisation theorem can be recast into conventional collinear factorisation form:

$$Q^2\frac{\partial F_2}{\partial Q^2} \sim \frac{\alpha_s}{2\pi}P_{\rm qg}^{\rm resummed}(\alpha_s) \otimes xg^{\rm BFKL} \; , \tag{14}$$

where the leading (and possibly also subleading) $\ln(1/x)$ effects are included to all orders in $xg^{\rm BFKL}$ and in the splitting function $P_{\rm qg}^{\rm resummed}$. They include in particular (part of) conventional DGLAP NNLO effects. Small-$x$ resummation in $P_{\rm qg}^{\rm resummed}$ has important implications for the extraction of $xg$ from the scaling violations.[8] The recent NNLO analysis of the DGLAP equations which embodies those effects shows that the gluon distributions in NNLO approximation are significantly smaller at small $x$ than those obtained within the NLO framework.[9]

It is possible to obtain a very economical description of the $F_2$ HERA data within the BFKL - $k_{\rm t}$ factorisation framework.[6] One can, however, get an equally good description of the data staying within the conventional NLO DGLAP formalism. The measurement of the structure function alone is, therefore, not a sensitive discriminator of the underlying dynamics. In order to

probe the details of the QCD pomeron, it is particularily useful to study the high energy processes characterised by two comparable scales $Q_1^2$ and $Q_2^2$ at the 'ends' of the gluon ladder which corresponds to the QCD pomeron. In this kinematical configuration, the conventional LO DGLAP evolution from the scale $Q_1^2$ to $Q_2^2$ is suppressed and the corresponding cross-sections are sensitive to the diffusion of the transverse momenta along the chain, which is a chacteristic feature of the BFKL dynamics. The following dedicated measurements are particularily useful for this purpose:

- Two-jet production in high energy hadronic collisions with $k_{\mathrm{T}1}^2 \sim k_{\mathrm{T}2}^2$.[10]

- Forward jet $(k_{j\mathrm{T}}^2 \sim Q^2)$ (or forward $\pi^0$) production in ep DIS.[11]

- Doubly tagged $e^+e^-$ events which are related to the $\gamma^*(Q_1^2)\gamma^*(Q_2^2)$ total cross-section.[12,13,14]

Indefinite increase of parton distributions $xp(x, Q^2)$ with decreasing $x$ cannot hold forever. The QCD improved parton model based upon linear evolution equations has to break down when

$$\frac{xp(x, Q^2)}{Q^2} \sim \pi R^2 \ ,$$

where $R$ denotes the (transverse) radius describing the size of the region within which the partons are concentrated. In the small $x$ region, the linear evolution equations have to be modified by the non-linear screening corrections which eventually lead to parton saturation.[2,15] A semi-phenomenological approach to saturation has recently been developed within the colour dipole model by K. Golec-Biernat and M. Wüsthoff.[16,17] This formulation, that has proved to be phenomenologically very successful, utilises the picture in which the high energy $\gamma^*$p total cross-section is driven by the interaction of the q$\bar{\mathrm{q}}$ colour dipole into which the virtual photon fluctuates,[15] i.e.

$$\sigma_{\gamma^*\mathrm{p}}(Q^2, x) \sim \int \mathrm{d}z \mathrm{d}r^2 |\Psi(r, Q, z)|^2 \sigma_{\mathrm{q}\bar{\mathrm{q}}}(r, x) \ . \tag{15}$$

In equ. (15), $\Psi(r, Q, z)$ denotes the wave function of the virtual photon, $\sigma_{\mathrm{q}\bar{\mathrm{q}}}(r, x)$ is the total cross-section describing the interaction of the q$\bar{\mathrm{q}}$ dipole with the proton target, $r$ is the transverse size of the dipole and $z$ is the momentum fraction of the virtual photon carried by a quark (antiquark). In the leading $\ln(1/x)$ approximation, the dipole picture corresponds to the $k_\mathrm{t}$ factorisation

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formula (partially) transformed into the coordinate representation.[18] In the formulation of the model discussed in ref. [16], we have

$$\sigma_{q\bar{q}}(r, x) = \sigma_0[1 - \exp(-r^2/R_0^2(x))] \ ,$$

where the saturation radius $R_0(x)$ is a decreasing function of the parameter $x$ and is parametrised in the following form:

$$R_0^2(x) \sim x^\lambda \ . \tag{16}$$

The parameter $\sigma_0$ denotes the magnitude of the dipole cross-section in the large $r$ limit. The behaviour of the $\gamma^*$p cross-section (15) in the region of large (small) values of $Q^2$ is linked with the properties of the dipole cross-section $\sigma_{q\bar{q}}(r, x)$ for small (large) values of the dipole size $r$. To be precise, we have:[16]

$$r^2 \ll R_0^2(x) \leftrightarrow Q^2 \gg 1/R_0^2(x) \ ,$$

where

$$\sigma_{q\bar{q}}(r, x) \sim r^2/R_0^2(x) \ , \tag{17}$$

that gives

$$\sigma_{\gamma^*p}(Q^2, x) \sim \frac{1}{Q^2 R_0^2(x)} \tag{18}$$

and

$$r^2 > R_0^2(x) \leftrightarrow Q^2 < 1/R_0^2(x) \ ,$$

where

$$\sigma_{q\bar{q}}(r, x) \sim \sigma_0 \ , \tag{19}$$

that gives

$$\sigma_{\gamma^*p}(Q^2, x) \sim \ln[Q^2 R_0^2(x)] \ . \tag{20}$$

The latter behaviour corresponds to the saturation of the cross-section. The remarkable property of the saturation model is geometric scaling of $\sigma_{\gamma^*p}(Q^2, x)$,[19] which means that at low values of $x$ this cross-section becomes the function of only one dimensionless variable, i.e.

$$\sigma_{\gamma^*p}(Q^2, x) \rightarrow \Phi(\tau) \ , \tag{21}$$

where

$$\tau = Q^2 R_0^2(x) \ . \tag{22}$$

The geometric scaling (21) is very well supported by experimental data.[19]

Deep inelastic diffraction in ep inelastic scattering is a process:

$$e(p_e) + p(p) \rightarrow e'(p'_e) + X + p'(p') , \qquad (23)$$

where there is a large rapidity gap between the recoil proton (or excited proton) and the hadronic system $X$.[20,21] To be precise, process (23) reflects the diffractive disssociation of the virtual photon. Diffractive dissociation is described by the following kinematical variables:

$$\beta = \frac{Q^2}{2(p - p')q} \qquad (24)$$

$$x_P = \frac{x}{\beta} \qquad (25)$$

$$t = (p - p')^2. \qquad (26)$$

Assuming that diffraction dissociation is dominated by the pomeron exchange and that the pomeron is described by a Regge pole, one gets the following factorizable expression for the diffractive structure function:[22]

$$\frac{\partial F_2^{\text{diff}}}{\partial x_P \partial t} = f(x_P, t) F_2^P(\beta, Q^2, t) \qquad (27)$$

where the "flux factor" $f(x_P, t)$ is given by the following formula :

$$f(x_P, t) = N \frac{B^2(t)}{16\pi} x_P^{1 - 2\alpha_P(t)} , \qquad (28)$$

with $B(t)$ describing the pomeron coupling to a proton and $N$ being the normalisation factor. The function $F_2^P(\beta, Q^2, t)$ is the pomeron structure function, which, in the (QCD improved) parton model, is related in a standard way to the quark and antiquark distribution functions in a pomeron:

$$F_2^P(\beta, Q^2, t) = \beta \sum e_i^2 [q_i^P(\beta, Q^2, t) + \bar{q}_i^P(\beta, Q^2, t)] , \qquad (29)$$

with $q_i^P(\beta, Q^2, t) = \bar{q}_i^P(\beta, Q^2, t)$. The variable $\beta$, which is the Bjorken scaling variable appropriate for deep inelastic lepton-pomeron "scattering", has the meaning of the momentum fraction of the pomeron carried by the probed quark (antiquark). The quark and gluon distributions in a pomeron are assumed to obey the standard Altarelli-Parisi evolution equations: The deep inelastic diffraction may therefore probe the quark-gluon content of the Pomeron.

The deep inelastic diffraction is sensitive to the interplay between the soft and hard pomerons.[23] It turns out that the effective pomeron intercept extracted from the diffractive data is higher than that of the soft pomeron , i.e. (effective) $\alpha_P^{\text{eff}}(0) \sim 1.2$. This implies an important contribution of the hard pomeron exchange. One also finds important higher twist contribution to the diffractive structure functions.

Important diffractive processes which can probe the hard pomeron are the following ones:[21]

- $\gamma^* + p \to V + p$

- $(\gamma, \gamma^*) + p \to J/\Psi + p$

- diffractive jet production .

One of the still open problems of the theory of hard diffraction processes is the violation of the QCD (collinear) factorisation in hadronic collisions.[24] Finally, let us point out that within the saturation model the total diffractive cross-section is given by:

$$\sigma_{\gamma^* p}^{\text{diff}}(Q^2, x) \sim \int \mathrm{d}z \mathrm{d}r^2 |\Psi(r, Q, z)|^2 \sigma_{q\bar{q}}^2(r, x) \ . \tag{30}$$

The fact that $\sigma_{\gamma^* p}^{\text{diff}}(Q^2, x)$ is given in terms of $\sigma_{q\bar{q}}^2(r, x)$ implies that it is sensitive to the contribution from the large $r$ region.

To sum up, we have introduced in this talk the basic concepts of low $x$ physics and of hard diffraction. We have also summarised some of the most recent developments including NLO BFKL, the saturation model, etc. Theoretical QCD predictions for low $x$ phenomena have been intensively studied both at HERA and Tevatron[25] as well as at LEP.[13,14] Most of those predictions are also extremely relevant for the measurements at future colliders, which will open up hitherto unexplored regime(s).

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