

The Coupling of η Mesons to Quarks and Baryons from $D_s^* \rightarrow D_s \pi^0$ Decay

T.A. Lähde* and D.O. Riska†

*Helsinki Institute of Physics and Department of Physics
PL 64, University of Helsinki, 00014 Finland*

Abstract

The known ratio of the branching fractions for $D_s^* \rightarrow D_s \pi^0$ and $D_s^* \rightarrow D_s \gamma$ may be used to extract the coupling of η mesons to strange quarks once the value of the π^0 - η mixing angle is known. This requires that realistic models for the spectra as well as the magnetic dipole (M1) decays of the heavy-light ($Q\bar{q}$) mesons are available. The coupling of η mesons to light quarks may then be estimated using $SU(3)$ flavor symmetry. Applied to the quark model for the baryons, an ηNN pseudovector coupling constant of $f_{\eta NN} = 0.35^{+0.15}_{-0.25}$ is obtained. If the charm quark couples significantly to the η meson, as is suggested by the decay mode $\psi' \rightarrow J/\psi \eta$, then somewhat larger values of $f_{\eta NN}$ can be obtained. These values are sufficiently small to be consistent with phenomenological analysis of photoproduction of the η on the nucleon and the reaction $pp \rightarrow pp\eta$.

*talalde@pcu.helsinki.fi

†riska@pcu.helsinki.fi

1 Introduction

The flavor symmetry breaking $D_s^* \rightarrow D_s \pi^0$ decay mode is mainly due to a small isoscalar η meson component in the physical π^0 meson as only the η meson can couple to the strange quark in the strange-charm mesons [1, 2]. As the magnitude of the π^0 - η mixing angle has been at the focus of a considerable amount of theoretical and phenomenological analysis, its value is now rather well known [3, 4]. The expression given by chiral perturbation theory to lowest order [5] is, for small θ_m ,

$$\theta_m = \frac{\sqrt{3} m_d - m_u}{4 m_s - m_0} \simeq 0.010 \quad (1)$$

where m_0 is the average of the u and d current quark masses. To this should be added the contribution from π^0 - η' mixing, which is smaller because of the larger mass of the η' , and which may increase the value of θ_m by at most about 30% [3, 6]. The resulting effective π^0 - η mixing angle is therefore about ~ 0.012 .

Combination of the notion of the spontaneously broken approximate chiral symmetry of QCD with $SU(3)$ flavor symmetry implies that the coupling of the baryons to the octet of light pseudoscalar mesons (π, K, η) takes the form

$$\mathcal{L}_{NNM} = i \frac{g_A}{2f_m} \bar{\psi}_N \gamma_5 \gamma_\mu \partial_\mu m_a \lambda_a \psi_N, \quad (2)$$

where ψ_N denotes the baryon octet field, m_a the meson octet field, λ_a the $SU(3)$ Gell-Mann matrices, f_m the decay constants of the mesons, and g_A the axial coupling constant of the baryons. The current experimental value of g_A as determined from low energy nuclear beta decays is $g_A = 1.2573$ [7], while the phenomenological value is $g_A = 1.267 \pm 0.004$ [8]. If these values are employed together with the Goldberger-Treiman relations for nucleons, then "effective" pseudovector coupling constants may be defined as

$$f_{\pi NN} = \frac{m_\pi}{2f_\pi} g_A \simeq 0.95, \quad f_{\eta NN} = \frac{1}{\sqrt{3}} \frac{m_\eta}{2f_\eta} g_A \simeq 1.79. \quad (3)$$

The above value for the πNN coupling constant agrees completely with that used in the most recent realistic phenomenological nucleon-nucleon interaction models, the parameters of which have been determined by fits to nucleon-nucleon scattering data [9, 10]. Such fits do not, however, constrain the value of $f_{\eta NN}$ very well, and therefore its value remains uncertain. Different realistic phenomenological potential models give values between ~ 0.5 and ~ 3.6 for this coupling [11], while analysis of data on photoproduction of the η on the nucleon indicate that the value for $f_{\eta NN}$ should not exceed 0.64 [12], a value which also seems to be consistent with data on η production near threshold in proton-proton collisions [13].

The point of the present paper is to show that it is possible, given the value of the π^0 - η mixing angle, to estimate the strength of the coupling of the η meson to strange quarks from the empirically known ratio of the branching fractions for $D_s^* \rightarrow D_s \pi^0$ and $D_s^* \rightarrow D_s \gamma$. This requires that realistic models for the respective hadronic matrix elements and $Q\bar{q}$ wavefunctions are at hand. From the ηss coupling obtained, the corresponding ηqq coupling may be estimated using flavor $SU(3)$ symmetry. One may then, by standard quark model relations, obtain useful and constraining information on the η -baryon, and in particular, the η -nucleon coupling $f_{\eta NN}$. The main source of error in this analysis is the rather large uncertainty in the empirical determination of the π^0 and γ branching fractions for the D_s^* meson. This error is likely to be much larger than that introduced by $SU(3)$ breaking effects into the current analysis.

If one considers the coupling of the octet of light pseudoscalar mesons to the light (u, d, s) quarks, one obtains analogously to eq. (2) the couplings

$$\mathcal{L}_{qqM} = i \frac{g_A^q}{2f_m} \bar{\psi}_q \gamma_5 \gamma_\mu \partial_\mu m_a \lambda_a \psi_q, \quad (4)$$

where g_A^q denotes the axial coupling constant for constituent quarks. For the pions and the η meson, the empirical values of the decay constants are $f_\pi = 93$ MeV and $f_\eta = 112$ MeV, respectively, so at least in this case the $SU(3)$ flavor symmetry is broken only at the 10% level. Combination of the chiral coupling (4) with the representation

$$m_a \lambda_a = \sqrt{2} \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\ \bar{K}^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta^0 \end{bmatrix}, \quad \psi_q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad (5)$$

leads to the following definitions for the quark-level pseudovector coupling constants f_{mqq} , which are analogous to those given by eq. (3):

$$f_{\pi qq} = \frac{m_\pi}{2f_\pi} g_A^q, \quad f_{\eta qq} = \frac{m_\eta}{2\sqrt{3}f_\eta} g_A^q, \quad f_{\eta ss} = -\frac{m_\eta}{\sqrt{3}f_\eta} g_A^q. \quad (6)$$

The above relations then suggest that the magnitude of the coupling of η mesons to u, d quarks should be one-half that of the η coupling to strange quarks, independently of the η meson mass. The numerical value of g_A^q is not very well known, although typical values fall in the range 0.87 - 1 [14, 15]. Calculations of the pionic decay widths of the D^* mesons show that value range to be realistic [16, 17]. In the static quark model the meson-quark coupling constants eq. (6) are related to the meson-nucleon coupling constants given by eq. (3) as (see e.g. ref. [17])

$$f_{\pi NN} = \frac{5}{3} f_{\pi qq}, \quad f_{\eta NN} = f_{\eta qq}. \quad (7)$$

Thus for $g_A \sim 1.26$ one would predict a value of $g_A^q \simeq 0.75$, which may be considered as a lower bound for the constituent quark axial coupling [17]. If the relation (6) is employed with values of g_A^q in the range 0.87 - 1.0 [14, 15], then the corresponding values of $f_{\eta ss}$ would fall between -2.5 and -2.8. Note that the value of g_A^q is conventionally extracted from the axial (spatial) current coupling, as the value of the effective axial charge coupling of confined constituent quarks depends on the form of the confining interaction [16].

The empirical observation of the charmonium decay modes $\psi' \rightarrow J/\psi \pi^0$ and $\psi' \rightarrow J/\psi \eta$ indicates that charm quarks also couple to the π^0 , and may potentially influence the $D_s^* \rightarrow D_s \pi^0$ decay. The fairly large empirical width for $\psi' \rightarrow J/\psi \eta$ of ~ 7.5 keV which corresponds to a branching fraction of $\sim 3\%$ suggests that besides $q\bar{q}$ and $s\bar{s}$ the η meson may have a $c\bar{c}$ admixture. While this would allow direct coupling of the η meson to charm quarks, the charm components of the η and η' mesons are phenomenologically, as well as theoretically, constrained to be rather small [8, 18, 19]. The known decay width for $\psi' \rightarrow J/\psi \eta$ [20, 21] and the Hamiltonian model for the $c\bar{c}$ spectrum of ref. [22] may nevertheless be used to explore the consequences of a nonzero ηcc coupling for the decay $D_s^* \rightarrow D_s \pi^0$ and thereby the resulting value for $f_{\eta NN}$. For this purpose the effective ηcc coupling is here taken to have the form (4).

This paper is divided into 5 sections. In section 2, the method for extracting the ηss coupling from the ratio of the empirically known branching fractions for the decays $D_s^* \rightarrow D_s \pi^0$ and $D_s^* \rightarrow D_s \gamma$ is described, along with the models for the π^0 and γ decays of the D_s^* . It is shown that a relativistic treatment of both the γ and π^0 decays of the D_s^* is called for. In section 3 it is shown that the two-quark exchange current contributions to the γ decay amplitude, which are associated with the scalar confining and vector one-gluon exchange (OGE) components of the $Q\bar{q}$ interaction, provide a way of obtaining a realistic value for the matrix element for γ decay, which is not possible in the impulse approximation. These two-quark contributions appear here because of the explicit elimination of negative energy components in the Blankenbecler-Sugar (BSLT) reduction of the Bethe-Salpeter equation. Section 4 contains the numerical results for the π^0 and γ matrix elements, and an estimated upper bound for the ηcc coupling. Section 5 contains a concluding discussion of the results obtained for the coupling of η mesons to quarks and nucleons.

2 The $D_s^* \rightarrow D_s \pi^0$ and $D_s^* \rightarrow D_s \gamma$ Decays

2.1 Extraction of the ηss coupling

Application of the relations (6) together with eq. (4) yields a coupling of η mesons to strange quarks in terms of $f_{\eta ss}$. The coupling of the π^0 meson to the strange quark may then be expressed in terms of the "effective" mixing angle θ_m as

$$\mathcal{L}_{ss\pi^0} = i \frac{f_{\eta ss}}{m_\eta} \theta_m \bar{\psi}_s \gamma_5 \gamma_\mu \partial_\mu \phi_{\pi^0} \psi_s, \quad (8)$$

where $\sin(\theta_m)$ has been replaced by θ_m , which is a very good approximation for values of $\theta_m \sim 0.015$ rad. In eq. (8), θ_m now corresponds to the sum of the π^0 - η and π^0 - η' contributions as discussed in the previous section.

The differential decay width of an excited heavy-light ($Q\bar{q}$) meson may be expressed in the form

$$\frac{d\Gamma}{d\Omega} = \frac{q}{8\pi^2} \frac{M_f}{M_i} |T_{fi}|^2, \quad (9)$$

where q denotes the momentum of the emitted particle, T_{fi} is the hadronic decay amplitude for the process in question, and M_f/M_i is a normalization factor for the quarkonium states analogous to that employed in ref. [17]. The width for the process $D_s^* \rightarrow D_s \pi^0$ may then be obtained directly from the expression for $D^{\pm*} \rightarrow D^\pm \pi^0$ given in ref. [16] by the replacement $g_A^q/2f_\pi \rightarrow f_{\eta ss}\theta_m/m_\eta$, giving

$$\Gamma(D_s^* \rightarrow D_s \pi^0) = \frac{1}{6\pi} \frac{M_{D_s}}{M_{D_s^*}} \frac{f_{\eta ss}^2}{m_\eta^2} \theta_m^2 q_\pi^3 |\mathcal{M}_\pi|^2, \quad (10)$$

if the π^0 emission takes place at the strange quark. Here \mathcal{M}_π is a radial matrix element for pion emission. Similarly, using eq. (9) the decay width for the radiative M1 transition $D_s^* \rightarrow D_s \gamma$ may be written in the form [23]

$$\Gamma(D_s^* \rightarrow D_s \gamma) = \frac{16}{3} \frac{M_{D_s}}{M_{D_s^*}} \alpha q_\gamma^3 |\mathcal{M}_\gamma|^2, \quad (11)$$

where α is the fine structure constant. In analogy with eq. (10), \mathcal{M}_γ is a radial matrix element for M1 decay, which includes the inverse quark mass factor (see below). By means of eqs. (10) and (11), the ratio of the π^0 and γ decay widths of the D_s^* meson is then obtained as

$$\frac{\Gamma_\pi}{\Gamma_\gamma} = \frac{8}{9\pi} \frac{f_{\eta ss}^2 \theta_m^2}{m_\eta^2 \alpha} \left(\frac{q_\pi}{q_\gamma} \right)^3 \left(\frac{|\mathcal{M}_\pi|}{|\mathcal{M}_\gamma|} \right)^2. \quad (12)$$

Note that in the above equation, the dimension of $|\mathcal{M}_\gamma|$ is $[\text{MeV}]^{-1}$. Through use of the empirical ratio of pion and photon momenta [20] known to be approximately 139/48 and the η meson mass of 547 MeV one may solve for the coupling constant $f_{\eta ss}$ to get

$$f_{\eta ss}^2 = \theta_m^{-2} \frac{\Gamma_\pi}{\Gamma_\gamma} \left(\frac{|\mathcal{M}_\gamma|}{|\mathcal{M}_\pi|} \right)^2 \cdot 4.814 \text{ fm}^{-2}. \quad (13)$$

As the ratio of the π^0 and γ decay rates is experimentally known, albeit with quite large errors, to be 0.062 ± 0.028 [20], it is, given the rather well known value of θ_m , possible to obtain an estimate for the coupling constant $f_{\eta ss}$. This requires a model for the matrix elements in eq. (13), as discussed below.

Eq. (13) was obtained by assuming that the π^0 decay takes place only at the strange quark. However, the observed $\psi' \rightarrow J/\psi \eta$ decay shows that the η meson also couples to charm quarks, which implies that π^0 emission by the charm quark may also take place in the D_s^* meson in addition to π^0 emission by the strange quark through $\pi^0 - \eta$ mixing. If the pion emission at the charm quark is taken to proceed through a coupling of the type (8), then the effect of π^0 emission by the charm quark may be taken into account by replacement of eq. (13) with the expression

$$R = \theta_m^{-2} \frac{\Gamma_\pi}{\Gamma_\gamma} |\mathcal{M}_\gamma|^2 \cdot 4.814 \text{ fm}^{-2}, \quad (14)$$

where R is defined as

$$R = |f_{\eta ss} \mathcal{M}_\pi^s + f_{\eta cc} \mathcal{M}_\pi^c|^2. \quad (15)$$

Here \mathcal{M}_π^c denotes the radial matrix element for pion emission by the charm quark. Because of the small momentum of the emitted π^0 and the heaviness of the charm quark, this matrix element will be very close to 1. The phenomenological implications of a nonzero contribution to π^0 decay by the charm quark will be explored in section 5.

The coupling constant $f_{\eta cc}$ can only be estimated from the empirically observed decay $\psi' \rightarrow J/\psi \eta$ which, although somewhat suppressed by the orthogonality of the ψ' and J/ψ wavefunctions, is known [20] to have a rather large width of ~ 7.5 keV corresponding to a branching fraction of about 3%. However, reliable estimation of the coupling constant $f_{\eta cc}$ from the decay $\psi' \rightarrow J/\psi \eta$ is difficult for several reasons. The first is that the matrix element for $\psi' \rightarrow J/\psi \eta$ is very small in the nonrelativistic approximation because of the orthogonality of the wavefunctions. In a relativistic calculation this matrix element is typically increased by an order of magnitude. Furthermore, the axial charge component from a coupling of the type (8), along with the associated two-quark effects, may also contribute significantly to such a decay [16]. Thus if the axial current component of eq. (8) is used with the replacement $f_{\eta ss} \theta_m \rightarrow f_{\eta cc}$, then the value for $f_{\eta cc}$ obtained by fitting the empirical width for η decay of the ψ' should be expected to represent an upper bound only. The expression for the width so obtained is

$$\Gamma(\psi' \rightarrow J/\psi \eta) = \frac{4}{3\pi} \frac{M_{J/\psi}}{M_{\psi'}} \frac{f_{\eta cc}^2}{m_\eta^2} q_\eta^3 |\mathcal{M}_\eta|^2. \quad (16)$$

Here \mathcal{M}_η is the matrix element for η decay, which is given in the next section together with the matrix element \mathcal{M}_π . Note that in eq. (16), the difference of a factor 8 as compared with eq. (10) arises from the product of a factor 2 from the spin sum for a triplet-triplet transition and a factor 4 from the sum of the quark and antiquark contributions to η decay. In section 4 it is shown that evaluation of eq. (16) leads to an upper bound for $f_{\eta cc}$ of about ~ 0.8 .

2.2 Matrix elements for pseudoscalar emission

When the strange and charm constituent quarks are treated non-relativistically, the radial matrix element for π^0 decay, \mathcal{M}_π , is

$$\mathcal{M}_\pi^{\text{NR}} = \int_0^\infty dr u^2(r) j_0\left(\frac{q_\pi r}{2}\right) \simeq 1 - \frac{\langle r^2 \rangle q_\pi^2}{24} + \mathcal{O}(q_\pi^4), \quad (17)$$

where $u(r)$ is the reduced radial wavefunction of $c\bar{s}$ system. Because of the smallness of q_π (48 MeV) and $\langle r^2 \rangle$ ($\simeq 0.2 \text{ fm}^2$), the numerical value of the matrix element $\mathcal{M}_\pi^{\text{NR}}$ is very close to unity. If the constituent strange and charm quarks are treated as Dirac particles, then the matrix element (17) for π^0 emission by the strange constituent quark is modified to

$$\begin{aligned} \mathcal{M}_\pi^{\text{Rel}} &= \frac{1}{\pi} \int_0^\infty dr' r' u(r') \int_0^\infty dr r u(r) \int_0^\infty dP P^2 \int_{-1}^1 dz F_s^\pi(P, q_\pi, z) \\ & j_0\left(r' \sqrt{P^2 + \frac{q_\pi^2}{16} + \frac{Pq_\pi z}{2}}\right) j_0\left(r \sqrt{P^2 + \frac{q_\pi^2}{16} - \frac{Pq_\pi z}{2}}\right), \end{aligned} \quad (18)$$

where the function F_s^π , which describes the strange quark contribution to pion emission by the D_s^* , which is a 1^3S_1 state, to the D_s ground state from the Lagrangian (8) may be expressed as [16]

$$F_s^\pi(P, q_\pi, z) = \sqrt{\frac{(E' + m_s)(E + m_s)}{4EE'}} \left(1 - \frac{P^2 - q_\pi^2/4}{3(E' + m_s)(E + m_s)}\right). \quad (19)$$

Here the energy factors of the strange quark are defined as $E = \sqrt{m_s^2 + P^2 + q_\pi^2/4 - Pq_\pi z}$ and $E' = \sqrt{m_s^2 + P^2 + q_\pi^2/4 + Pq_\pi z}$ respectively. In the above expressions, \vec{P} is defined as $\vec{P} = (\vec{p}' + \vec{p})/2$ in terms of the relative momenta in the center-of-momentum system. In realistic models for the heavy-light mesons [16, 23] and the baryons [24, 25] typical values of the constituent masses for the strange and charm quarks fall in the range $m_s = 400 \text{ MeV} - 600 \text{ MeV}$ and $m_c = 1.3 - 1.6 \text{ GeV}$, respectively. Because of the small constituent mass of the strange quark the value of the expression (19) is expected to deviate significantly from the nonrelativistic limit (17).

Because of the small value of the momentum of the π^0 , one may to a very good approximation, as in eq. (17), set q_π to zero in the expression (19), which then reduces to

$$F_s^\pi(P) = \lim_{q \rightarrow 0} F_s^\pi(P, q_\pi, z) = \frac{1}{3} + \frac{2}{3} \frac{m_s}{E}. \quad (20)$$

In this approximation the relativistic matrix element for π^0 decay may then be expressed as

$$\mathcal{M}_\pi^{\text{Rel}} = \frac{2}{\pi} \int_0^\infty dr' r' u(r') \int_0^\infty dr r u(r) \int_0^\infty dP P^2 F_s^\pi(P) j_0(r'P) j_0(rP). \quad (21)$$

Eq. (20) thus indicates that in the ultrarelativistic limit, the matrix element for π^0 decay assumes the value $1/3$. For $m_s = 560 \text{ MeV}$, the value of the matrix element (21) is ~ 0.8 , which shows that the strange constituent quark has to be treated relativistically. In addition, two-quark mechanisms associated with intermediate negative energy quarks were shown in ref. [16] to give a small contribution to the matrix element for pion decay. These effects will be discussed in more detail in section 3.

In order to obtain the matrix element for pion emission by the charm quark in the D_s^* meson, it is sufficient to make the substitution $m_s \rightarrow m_c$ in all equations of this subsection. On the other hand, the matrix element for the decay $\psi' \rightarrow J/\psi \eta$ takes the form

$$\begin{aligned} \mathcal{M}_\eta^{\text{Rel}} &= \frac{1}{\pi} \int_0^\infty dr' r' u_{J/\psi}(r') \int_0^\infty dr r u_{\psi'}(r) \int_0^\infty dP P^2 \int_{-1}^1 dz F_c^\eta(P, q_\eta, z) \\ &\quad j_0 \left(r' \sqrt{P^2 + \frac{q_\eta^2}{16} + \frac{Pq_\eta z}{2}} \right) j_0 \left(r \sqrt{P^2 + \frac{q_\eta^2}{16} - \frac{Pq_\eta z}{2}} \right), \end{aligned} \quad (22)$$

where $u_{J/\psi}$ and $u_{\psi'}$ denote the reduced radial wavefunctions for the J/ψ and ψ' states, respectively. The factor F_c^η may be obtained by substitution of $m_s \rightarrow m_c$ and $q_\pi \rightarrow q_\eta$ in eq. (19).

2.3 Matrix element for γ decay

In the nonrelativistic approximation the matrix element for radiative M1 decay of a $c\bar{s}$ meson in eq. (13) is [23]

$$\mathcal{M}_\gamma^{\text{NR}} = \frac{1}{12} \left[\frac{2}{m_c} - \frac{1}{m_s} \right]. \quad (23)$$

Although the photon momentum q_γ is somewhat larger (139 MeV) than q_π , it is still small enough so that the M1 approximation may be considered valid. If the strange and charm constituent quarks are treated as Dirac particles, then the canonical boosts lead to relativistic modifications of the spin-flip magnetic moment operators [23]. The relativistic matrix element for M1 decay may thus, in analogy with eqs. (18) and (21), be written as

$$\mathcal{M}_\gamma^{\text{Rel}} = \frac{2}{\pi} \int_0^\infty dr' r' u(r') \int_0^\infty dr r u(r) \int_0^\infty dP P^2 F_s^\gamma(P) j_0(r'P) j_0(rP), \quad (24)$$

where the factor F_s^γ is defined as

$$F_s^\gamma(P) = \frac{1}{12} \left[\frac{2}{m_c} f_c^\gamma(P) - \frac{1}{m_s} f_s^\gamma(P) \right]. \quad (25)$$

The functions $f_i^\gamma(P)$ represent correction factors to the static spin-flip magnetic moment operators, and may be expressed as

$$f_i^\gamma(P) = \frac{m_i}{3E_i} \left[2 + \frac{m_i}{E_i} \right] \quad (26)$$

for the charm and strange constituent quarks. Here the energy factors are defined as $E_i = \sqrt{P^2 + m_i^2}$. From eq. (23) it is evident that there is destructive interference between the contributions from the heavy and light quark currents to the M1 decay rate of a heavy-light meson. Insertion of standard values of $m_c \sim 1500$ MeV and $m_s \sim 500$ MeV for the constituent quark masses reveals that the light constituent quark contribution is somewhat larger than that of the heavy quark, which leads to an overall negative value for the γ decay matrix element, cf. Table 4. However, if the relativistic form, eq. (24), is employed then the charm and strange quark contributions very nearly cancel each other. This occurs because of the larger relativistic suppression of the strange quark contribution. The end result is, that in the relativistic impulse approximation the width for M1 decay of a charged heavy-light meson is very small [23].

This result is, however, unrealistic, since two-quark mechanisms associated with intermediate negative energy quarks will contribute significantly to the matrix elements for M1 decay. These mechanisms, which are illustrated by the Feynman diagrams in Fig. 1 appear because of the explicit elimination of negative energy components in the Blankenbecler-Sugar reduction of the Bethe-Salpeter equation. The resulting two-quark magnetic moment operators have been calculated for equal constituent masses in ref. [27]. It has been shown in ref. [29] that the two-quark contribution associated with the scalar confining interaction is required for obtaining agreement with experiment for the M1 decays in the charmonium ($c\bar{c}$) system. Without that contribution, the width for $\psi' \rightarrow J/\psi \gamma$ would be overpredicted by a factor ~ 3 .

However, the situation is much more complicated for the heavy-light mesons because of the uncertain structure of $Q\bar{q}$ interaction. Relativistic effects are also likely to be substantial for the two-quark operators because of the low masses of the light constituent quarks. It will be shown in the next two sections that once the two-quark effects associated with a $Q\bar{q}$ interaction formed of scalar confining and OGE components are taken into account, then the matrix elements for M1 decay of the $D^{\pm*}$ and $D_s^{\pm*}$ may be restored to the same order of magnitude as the non-relativistic estimate in eq. (23). The form of these two-quark operators will be outlined in section 3, and the numerical results are given in section 4.

3 Two-quark contributions to D_s^* decay

3.1 Two-quark contributions to γ decay

In the reduction of the Bethe-Salpeter equation for the $s\bar{c}$ system to a Blankenbecler-Sugar equation, the negative energy components are eliminated in the impulse approximation. These do however contribute to the current matrix elements of the $s\bar{c}$ system as transition matrix elements, as illustrated in Fig. 1. In the Blankenbecler-Sugar equation framework these transition matrix elements have to be included as explicit two-quark current operators [30]. These two-quark matrix elements have been shown to give a large correction to the spin-flip matrix element of the single quark current [29].

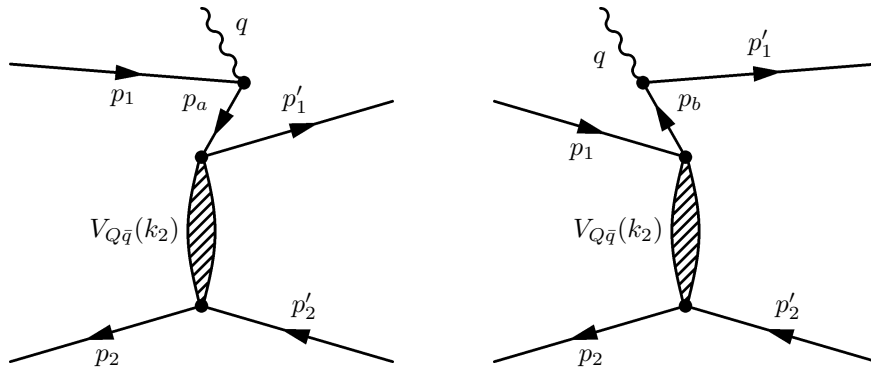


Figure 1: Negative energy Born diagrams for photon emission by a $c\bar{s}$ meson. The diagrams shown correspond to both time orderings of the γ emission from the charm quark. Two similar diagrams describe γ emission by the strange antiquark. The quark momenta are defined according to $p_a = p_1 + q$ and $p_b = p'_1 - q$, while k_2 is the momentum transferred to the strange antiquark. Note that only the negative energy component of the intermediate quark propagator is to be retained.

The qualitative effect of this two-quark current may be understood in terms of a shift of the constituent quark mass by the scalar confining interaction $m \rightarrow m + V_c(r)$. Since the constituent mass appears in the denominator of the magnetic moment operator, this mass shift will lead to a reduction of the corresponding single quark contribution to the M1 decay width. However, because of the cancellation of the relativistic single quark current matrix element (24) for the $c\bar{s}$ system outlined in section 2.3, the two quark currents give the main contribution to the transition rate for $D_s^* \rightarrow D_s\gamma$ [23]. For the scalar confining interaction, that current may be expressed in the form

$$\vec{j}_c(\vec{q}, \vec{k}_1, \vec{k}_2) = -e \left(\frac{Q_1^* \vec{P}_1}{m_1^2} + \frac{Q_2^* \vec{P}_2}{m_2^2} + \frac{i}{2} (\vec{\sigma}_1 + \vec{\sigma}_2) \times \vec{q} \left[\frac{Q_1^*}{2m_1^2} + \frac{Q_2^*}{2m_2^2} \right] + \frac{i}{2} (\vec{\sigma}_1 - \vec{\sigma}_2) \times \vec{q} \left[\frac{Q_1^*}{2m_1^2} - \frac{Q_2^*}{2m_2^2} \right] \right), \quad (27)$$

where \vec{q} is the momentum of the emitted photon, which corresponds to $\vec{p}' - \vec{p}$ in case of the single quark amplitude. In eq. (27), the variables Q_1^* and Q_2^* are defined as $Q_1^* = V_c(k_2)Q_1$ and $Q_2^* = V_c(k_1)Q_2$, where $V_c(k_n)$ is the (formal) Fourier transform of the scalar confining interaction. The momentum variables \vec{P}_1 and \vec{P}_2 correspond to the expressions $(\vec{p}_1 + \vec{p}'_1)/2$ and $(\vec{p}_2 + \vec{p}'_2)/2$ respectively, while \vec{k}_n denotes the momentum transferred to quark n according to Fig. 1. The analogous expression for the two-quark current induced by the vector coupled OGE interaction is of the form

$$\vec{j}_g(\vec{q}, \vec{k}_1, \vec{k}_2) = -e \left(Q_1^* \left[\frac{i\vec{\sigma}_1 \times \vec{k}_2}{2m_1^2} + \frac{2\vec{P}_2 + i\vec{\sigma}_2 \times \vec{k}_2}{2m_1 m_2} \right] + Q_2^* \left[\frac{i\vec{\sigma}_2 \times \vec{k}_1}{2m_2^2} + \frac{2\vec{P}_1 + i\vec{\sigma}_1 \times \vec{k}_1}{2m_1 m_2} \right] \right), \quad (28)$$

with $Q_1^* = V_g(\vec{k}_2)Q_1$ and $Q_2^* = V_g(\vec{k}_1)Q_2$. Here $V_g(\vec{k})$ denotes the momentum-space form of the OGE interaction. The corresponding magnetic moment operators may then be computed from eqs. (27) and (28) according to

$$\vec{\mu} \equiv -\frac{i}{2} \lim_{q \rightarrow 0} \left[\vec{\nabla}_q \times \vec{j}(\vec{q}, \vec{k}_1, \vec{k}_2) \right], \quad (29)$$

where the momentum transfer variables are given by $\vec{k}_1 = \vec{q}/2 - \vec{k}$ and $\vec{k}_2 = \vec{q}/2 + \vec{k}$. Upon Fourier transformation, the resulting magnetic moment operators may, for transitions between S -wave states, be written in the form

$$\vec{\mu}_c = -\frac{eV_c(r)}{4} \left\{ \left[\frac{Q_1}{m_1^2} - \frac{Q_2}{m_2^2} \right] (\vec{\sigma}_1 - \vec{\sigma}_2) + \left[\frac{Q_1}{m_1^2} + \frac{Q_2}{m_2^2} \right] (\vec{\sigma}_1 + \vec{\sigma}_2) \right\} \quad (30)$$

for the scalar confining interaction, and

$$\vec{\mu}_g = -\frac{eV_g(r)}{8} \left\{ \left[\frac{Q_1}{m_1^2} - \frac{Q_2}{m_2^2} - \frac{Q_1 - Q_2}{m_1 m_2} \right] (\vec{\sigma}_1 - \vec{\sigma}_2) + \left[\frac{Q_1}{m_1^2} + \frac{Q_2}{m_2^2} + \frac{Q_1 + Q_2}{m_1 m_2} \right] (\vec{\sigma}_1 + \vec{\sigma}_2) \right\} \quad (31)$$

for the one-gluon exchange interaction. In the limit $m_1 = m_2$, the above magnetic moments reduce to those given in ref. [27]. Note that for the case of equal constituent quark masses, the one-gluon exchange magnetic moment operator has no spin-flip term and consequently does not contribute to M1 decay if $m_1 = m_2$. The terms in eqs. (30) and (31) that are symmetric in the quark spins affect only the magnetic moments of the heavy-light mesons and give no contribution to the M1 decay widths.

The spin-flip matrix elements for radiative M1 decay of a $c\bar{s}$ meson that should be added to the relativistic single quark matrix element (24) are thus

$$\mathcal{M}_\gamma^{\text{Conf}} = - \int_0^\infty dr u^2(r) \frac{V_c(r)}{12} \left[\frac{2}{m_c^2} - \frac{1}{m_s^2} \right], \quad (32)$$

in case of the scalar confining interaction, and

$$\mathcal{M}_\gamma^{\text{Oge}} = - \int_0^\infty dr u^2(r) \frac{V_g(r)}{24} \left[\frac{2}{m_c^2} - \frac{1}{m_s^2} - \frac{1}{m_c m_s} \right] \quad (33)$$

for the one-gluon exchange interaction. In the above expressions, the appropriate values of the quark charge operators have been entered. For the scalar confining interaction, the form $V_c(r) = cr - b$ will be used, where the parameters c and b are taken to be those obtained in the potential model of ref. [23]. The one-gluon exchange potential $V_g(r)$ is taken to be of the form

$$V_g(r) = -\frac{4}{3} \frac{2}{\pi} \int_0^\infty dk \alpha_s(\vec{k}^2) j_0(kr), \quad (34)$$

which is known as the Richardson potential [32] and features the running QCD coupling α_s which is parameterized in terms of the QCD scale Λ_{QCD} and a dynamical gluon mass m_g , which in ref. [23] were obtained as 280 MeV and 240 MeV, respectively:

$$\alpha_s(k^2) = \frac{12\pi}{27} \frac{1}{\ln[(k^2 + 4m_g^2)/\Lambda_{\text{QCD}}^2]}. \quad (35)$$

Note that if α_s is taken to be constant, then the above form reduces to the standard Coulombic one-gluon exchange potential of perturbative QCD.

3.2 Two-quark contributions to π^0 decay

In ref. [16] it was shown that a two-quark mechanism analogous to that for M1 decay shown in Fig. 1, which is due to coupling of the emitted pion to an intermediate negative energy quark, which interacts with the heavy spectator quark by the $Q\bar{q}$ interaction may contribute to the amplitude for pion decay. If the decay amplitude for emitted pions is written in the form

$$T_\pi = i\vec{q}_\pi \cdot \vec{A}, \quad (36)$$

then the axial current \vec{A} may be decomposed into single- and two-quark contributions according to $\vec{A} = \vec{A}_s + \vec{A}_{\text{ex}}$. The two-quark contributions, which have been calculated for equal constituent masses in ref. [26], are in this case however proportional to m_q^{-3} in the nonrelativistic limit, which means that they are of much less importance than the two-quark contributions to the magnetic moment operator. Furthermore, it has been shown in ref. [16] that the axial exchange current contribution associated with the scalar confining interaction is much reduced if relativistic effects are considered, because of the low mass of the strange constituent quark.

The end result is, that two-quark effects due to the scalar confining and one-gluon exchange interactions will only modify the relativistic matrix element (21) at the $\sim 5 - 10\%$ level. As the uncertainties in the value of the mixing angle θ_m and the empirically determined ratio of γ and π^0 decay are much greater, there is no need to include the two-quark contributions to the amplitude for π^0 decay at this time.

4 Numerical Results for $D_s^* \rightarrow D_s \gamma$ and $D_s^* \rightarrow D_s \pi^0$

In order to obtain a description of the decay $D_s^* \rightarrow D_s \gamma$ that is as realistic as possible, the model for M1 decay presented here should be checked against the empirical results on the corresponding decays of the non-strange D mesons. As the decay width for $D^{\pm*} \rightarrow D^{\pm} \gamma$ is known directly and that for $D^{0*} \rightarrow D^0 \gamma$ can be estimated from the empirical branching ratios and model calculations of the pionic decays of the D mesons, it is possible to calibrate the mass of the light constituent quark so that an optimal description of the M1 decay of the D^{\pm} is obtained. Together with the potential model calculation in ref. [23], this information can then be used to obtain a realistic estimate of the strange quark constituent mass. In order to calculate the M1 decay rate $D^{\pm*} \rightarrow D^{\pm} \gamma$, it is sufficient to make the substitution $m_s \rightarrow m_q$ in all matrix elements, but for $D^{0*} \rightarrow D^0 \gamma$, one should substitute $m_s^{-1} \rightarrow -2m_q^{-1}$ in eq. (25), and $m_s^{-2} \rightarrow -2m_q^{-2}$, $1/m_c m_s \rightarrow 4/m_c m_q$ in eqs. (32) and (33). The obtained results for the M1 widths of the non-strange D mesons are given for different light constituent quark masses in Tables 1 and 2.

m_q	NRIA	RIA	RIA + Conf + Oge
450 MeV	0.58 keV	$9.4 \cdot 10^{-3}$ keV	1.09 keV
420 MeV	0.79 keV	$1.5 \cdot 10^{-2}$ keV	1.43 keV
390 MeV	1.07 keV	$2.2 \cdot 10^{-2}$ keV	1.90 keV

Table 1: Numerical results for the M1 transition $D^{\pm*} \rightarrow D^{\pm} \gamma$, for a charm quark mass of 1580 MeV [23]. The column NRIA (non-relativistic impulse approximation) corresponds to eq. (23), and the relativistic impulse approximation RIA to eq. (24). In the rightmost column, the two-quark contributions from eqs. (32) and (33) have been added. In the contribution from the confining interaction, the parameters $c = 1120$ MeV/fm and $b = 320$ MeV have been employed [23].

m_q	NRIA	RIA	RIA + Conf + Oge
450 MeV	21.1 keV	8.86 keV	8.95 keV
420 MeV	23.5 keV	9.18 keV	9.89 keV
390 MeV	26.4 keV	9.52 keV	11.1 keV

Table 2: Numerical results for the M1 transition $D^{0*} \rightarrow D^0 \gamma$, for a charm quark mass of 1580 MeV [23]. The columns NRIA (non-relativistic impulse approximation) and RIA are as for Table 1, although with the appropriate modifications due to the different quark charge operators. The same modifications apply to the rightmost column, where the two-quark contributions from eqs. (30) and (31) have been added. In the contribution from the confining interaction, the parameters $c = 1120$ MeV/fm and $b = 320$ MeV have been employed [23].

It is instructive to compare the results presented in Table 1 with the current experimental data for the $D^{\pm*}$. Until recently, only the relative branching ratios for π and γ decay were known [20]. However, a first empirical measurement of the total width of the $D^{\pm*}$ has recently been published by the CLEO collaboration [31]. The reported result is $\Gamma(D^{\pm*}) = 96 \pm 4 \pm 22$ keV, where the latter error represents the systematic uncertainties. Taking into account the reported [20] branching ratio of $1.6 \pm 0.4\%$ for radiative decay, one obtains 1.5 ± 0.6 keV for the transition $D^{\pm*} \rightarrow D^{\pm}\gamma$. Here most of the uncertainty stems from the systematic errors of the experimental result. However, it is evident that the results of Table 1 reproduce this result well for a range of values of the light constituent quark mass m_q . The value $m_q = 450$ MeV corresponds to the potential model of ref. [23], while the value $m_q = 420$ MeV has been suggested by refs. [33, 34]. From Table 1 it is thus seen that a light constituent quark mass of 420 MeV produces a width for radiative decay which is close to 1.5 keV. That value of the width is also favored by the analysis of ref. [35].

m_q	NRIA	RIA	RIA + Conf + Oge
560 MeV	0.18 keV	$2.6 \cdot 10^{-4}$ keV	0.38 keV
530 MeV	0.26 keV	$3.9 \cdot 10^{-5}$ keV	0.49 keV
500 MeV	0.36 keV	$8.6 \cdot 10^{-4}$ keV	0.64 keV

Table 3: Numerical results for the M1 transition $D_s^{\pm*} \rightarrow D_s^{\pm}\gamma$, for a charm quark mass of 1580 MeV [23]. The column NRIA (non-relativistic impulse approximation) corresponds to eq. (23), and the relativistic impulse approximation RIA to eq. (24). In the rightmost column, the two-quark contributions from eqs. (32) and (33) have been added. In the contribution from the confining interaction, the parameters $c = 1120$ MeV/fm and $b = 260$ MeV have been employed [23]. Note that the RIA contribution becomes exactly zero for a strange quark mass of ~ 540 MeV.

Even though the total width of the neutral D^{0*} meson has not yet been determined [20], considerable information about the expected width for $D^{0*} \rightarrow D^0\gamma$ may be extracted from the reported branching fraction of $38.1 \pm 2.9\%$ [20], since the corresponding width for pion decay can be constrained by means of the empirically determined width of the $D^{\pm*}$ and model calculations of the pionic decays of D mesons [16, 17, 36]. This is seen directly if one first notes that the branching fraction of $D^{\pm*} \rightarrow D^0\pi^{\pm}$ is reported as $67.7 \pm 0.5\%$ [20], which implies a width for this decay mode of $\sim 65 \pm 14$ keV. From the model calculation of ref. [16] one finds that this corresponds to a width of $\sim 40 \pm 10$ keV for $D^{0*} \rightarrow D^0\pi^0$. As the relative branching fractions for π^0 and γ decay of the D^{0*} are well known [20], the best estimate for the width of $D^{0*} \rightarrow D^0\gamma$ is ~ 25 keV. There remains, however, still a considerable uncertainty of $\sim \pm 10$ keV from the systematic errors in the empirical measurement of $\Gamma(D^{\pm*})$.

A width for $D^{0*} \rightarrow D^0\gamma$ of around 20-30 keV is also preferred by ref. [35]. The result of the model calculation in Table 2 is smaller by about a factor ~ 2 as the OGE contribution is weaker in this case as compared to the contribution from the scalar confining interaction. Moreover, as a relativistic treatment of the two-quark currents will effectively lead to replacement of the quark masses by energy-dependent factors, it may be argued that relativistic effects will generally lead to a weakening of the two-quark contributions to the matrix element for M1 decay. It is thus entirely possible that a fully relativistic treatment will render the two-quark contributions too weak to account for the experimental data on M1 decay of the D^{\pm} meson as well.

This conclusion is in line with that reached in ref. [35], where it was found that the introduction of a large anomalous magnetic moment is required in order to fit the observed M1 decay widths of the heavy-light mesons. Another interesting possibility is that the instanton induced interaction for $Q\bar{q}$ systems given in refs. [33, 34] may also contribute a significant two-quark current. The interaction of refs. [33, 34], which was found to be short-ranged, negative and attractive, has scalar coupling to the light constituent quark. One may thus conclude that this interaction will add up constructively with the OGE contribution from eq. (33). Thus the instanton induced interaction will have an overall favorable effect on the widths for M1 decay, particularly for the D^{0*} , which may be inferred from the matrix elements given in Table 4.

As it appears to be possible to obtain a realistic description of the radiative M1 decay of the D^\pm meson using the model presented in this paper, then a prediction based on the potential model [23] for the M1 decay $D_s^{\pm*} \rightarrow D_s^\pm \gamma$ and the associated matrix element \mathcal{M}_γ can be made. That information is needed as input for eq. (13) in order to obtain an estimate for $f_{\eta_{ss}}$. These results are given in Tables 3 and 4.

	$D^{\pm*} \rightarrow D^\pm \gamma$	$D^{0*} \rightarrow D^0 \gamma$	$D_s^{\pm*} \rightarrow D_s^\pm \gamma$
$\mathcal{M}_\gamma^{\text{NR}}$	$-1.83 \cdot 10^{-2}$	$9.91 \cdot 10^{-2}$	$-8.55 \cdot 10^{-3}$
$\mathcal{M}_\gamma^{\text{Rel}}$	$-2.55 \cdot 10^{-3}$	$6.20 \cdot 10^{-2}$	$3.24 \cdot 10^{-4}$
$\mathcal{M}_\gamma^{\text{Conf}}$	$1.23 \cdot 10^{-2}$	$-3.07 \cdot 10^{-2}$	$7.25 \cdot 10^{-3}$
$\mathcal{M}_\gamma^{\text{Oge}}$	$-3.45 \cdot 10^{-2}$	$3.31 \cdot 10^{-2}$	$-1.98 \cdot 10^{-2}$
$\mathcal{M}_\gamma^{\text{Tot}}$	$-2.48 \cdot 10^{-2}$	$6.44 \cdot 10^{-2}$	$-1.22 \cdot 10^{-2}$

Table 4: Matrix elements for the M1 decays considered in Tables 1, 2 and 3, in units of [fm]. The matrix elements correspond to a charm quark mass of 1580 MeV, a light constituent quark mass of 420 MeV and a strange constituent quark mass of 560 MeV. The matrix element $\mathcal{M}_\gamma^{\text{Tot}}$ shows the sum of the 'Rel' + 'Conf' + 'Oge' contributions, which in case of the decay $D_s^{\pm*} \rightarrow D_s^\pm \gamma$ is used for the determination of the coupling constant $f_{\eta_{ss}}$.

In order to estimate the coupling constant $f_{\eta_{ss}}$, the matrix element (21) for π^0 decay of the D_s^* meson also needs to be evaluated. The value of that matrix element for a strange constituent quark mass of 560 MeV is 0.794, which together with the total matrix element for γ decay of the D_s^* may then be used with eq. (13) to obtain a value for $f_{\eta_{ss}}$. If the charm quark also couples significantly to the η meson, then the matrix element for pion emission by the charm quark in eq. (15) needs to be evaluated as well. This can be done by the substitution $m_s \rightarrow m_c$ in eq. (21). For a charm quark mass of 1580 MeV, the result $\mathcal{M}_\pi^c = 0.949$ is obtained. Thus, in the evaluation of eqs. (14) and (15), the following values for the matrix elements for γ and π^0 decay will be used:

$$\boxed{\mathcal{M}_\gamma = -1.22 \cdot 10^{-2} \text{ fm}}$$

for the γ decay matrix element, and

$$\boxed{\mathcal{M}_\pi^s = 0.794 \quad \mathcal{M}_\pi^c = 0.949}$$

for the π^0 decay matrix elements. Note that in all the calculations of this section, the masses of the initial and final quarkonium states as well as the momenta of the emitted photons and π^0 mesons have been taken to equal those given in ref. [20].

4.1 Numerical Results for $\psi' \rightarrow J/\psi \eta$

The evaluation of eq. (16) involves the computation of the matrix element \mathcal{M}_η given by eq. (22). The ψ' and J/ψ wavefunctions are here taken to be those computed in the potential model of ref. [22]. Because of the somewhat larger value of q_η (200 MeV) [20], and the orthogonality of the ψ' and J/ψ radial wavefunctions, it is undesirable to make the approximation $q_\eta = 0$ in this case. The resulting relativistic matrix element for the decay $\psi' \rightarrow J/\psi \eta$ is obtained as $\mathcal{M}_\eta = -3.53 \cdot 10^{-2}$, and gives the desired width of ~ 7.5 keV if the coupling constant $f_{\eta cc}$ is taken to be ~ 0.8 . The sign of $f_{\eta cc}$ is of course not determined by this computation.

As there exists both theoretical and phenomenological arguments [8] against a significant $c\bar{c}$ admixture in the η meson, then this value for $f_{\eta cc}$ may appear uncomfortably large. Indeed, it will be shown in the next section that a value of $f_{\eta cc}$ around ~ 0.8 is, in view of the $D_s^* \rightarrow D_s \pi^0$ decay, about one half of the so obtained value for $f_{\eta ss}$. However, as neither the axial charge component nor the associated two-quark contributions to the amplitude for η decay of the ψ' have been considered, the value of 0.8 for $f_{\eta cc}$ should be considered as an upper limit only. Nevertheless, it will be shown in the next section that the inclusion of a pion-charm quark coupling will serve to increase the obtained value for $f_{\eta ss}$, thus bringing it closer to the phenomenologically preferred values [12, 13].

5 Discussion

Assuming that in the π^0 decay of the D_s^* , the pion couples mostly to the strange constituent quark, eq. (13) may be used directly together with the matrix elements for π^0 and γ decay given in the previous section to estimate the magnitude of $f_{\eta ss}$. Insertion of the matrix elements and taking $\theta_m = 0.012$ yields $R = 0.3085$, giving $|f_{\eta ss}| = 0.70$. Taking into account the uncertainties in the mixing angle and the empirical decay widths for π^0 and γ decay, the best estimate is

$$\boxed{f_{\eta ss} = -0.7^{+0.5}_{-0.3}}$$

In the above result, the negative sign is suggested by the relations in eq. (6). The static quark model then implies, through eq. (7), that the magnitude of the corresponding pseudovector η -nucleon coupling constant $f_{\eta NN}$ should be one half of this value. Thus one obtains the following value for the η -nucleon coupling:

$$\boxed{f_{\eta NN} = 0.35^{+0.15}_{-0.25}}$$

This result should be compared with the value for $f_{\eta NN}$ (or the equivalent pseudoscalar coupling constant $g_{\eta NN} = (2m_N/m_\eta)f_{\eta NN}$, which has been determined by phenomenological model fits to photoproduction of η mesons on the nucleon [12]. The latter value for $f_{\eta NN}$ is 0.64. This value has also been found to be realistic in calculations of the cross section for $pp \rightarrow pp\eta$ near threshold [13]. Although the result

obtained above for $f_{\eta NN}$ has quite large uncertainties which are mostly of empirical origin, it still appears to be significantly smaller. A larger value for $f_{\eta NN}$ could of course be obtained by decreasing the mixing angle θ_m .

Additional modifications to the current estimation of $f_{\eta NN}$ may of course also arise from $SU(3)$ breaking effects in eq. (6), which relates $f_{\eta qq}$ to $f_{\eta ss}$. Likewise, relativistic effects may also modify the static quark model result (7) which relates $f_{\eta qq}$ to $f_{\eta NN}$. However, in view of the large experimental uncertainty in the determination of the branching fractions for γ and π^0 decay of the D_s^* , there appears to be little motivation to include such effects at this time. Another possibility is that the matrix element for γ decay as given by Table 4 is too small. However, as the computed width for $D_s^* \rightarrow D_s \gamma$ is already somewhat larger than that given in ref. [35], this possibility may be considered unlikely.

Another option worth considering, in view of the fact that the empirical detection of a considerable branching fraction for $\psi' \rightarrow J/\psi \eta$ indicates that the η meson also couples to the charm quark, is that the coupling of the η meson to the charm quark would be strong enough to significantly influence the $D_s^* \rightarrow D_s \gamma$ decay. For nonzero values of the coupling $f_{\eta cc}$, the coupling constant $f_{\eta ss}$ may thus be determined from eq. (15) according to

$$f_{\eta ss} = \frac{\sqrt{R} - f_{\eta cc} \mathcal{M}_\pi^c}{\mathcal{M}_\pi^s}, \quad (37)$$

where the different possibilities are $\sqrt{R} = \pm 0.5554$ and $f_{\eta cc} = \pm 0.8$. The most realistic of these appears to be $\sqrt{R} = -0.5554$, $f_{\eta cc} = +0.8$, giving $f_{\eta ss} = -1.66$. In this case a nonzero and positive $f_{\eta cc}$ leads to a larger and negative value for $f_{\eta ss}$, allowing for better agreement with the values of $f_{\eta NN}$ suggested above. The value for $f_{\eta NN}$ that corresponds to $f_{\eta cc} = +0.8$ turns out to be

$$f_{\eta NN} = 0.83_{-0.35}^{+0.25}, \quad (38)$$

which is closer to the phenomenological estimates given by refs. [12] and [13]. Although the value $+0.8$ for $f_{\eta cc}$ is, in view of the discussion in the previous section, probably much too large, there still remains a distinct possibility that the charm quark contributes significantly to the width for $D_s^* \rightarrow D_s \pi^0$. In this case it is found that values around $f_{\eta ss} \simeq -1.7$ are favored by an eta-charm coupling of $\sim +0.8$.

It is useful to compare the results obtained from the present phenomenological analysis with the predictions of eq. (6). With the value $g_A^q = 0.87$ for the axial coupling constant of the constituent quarks, eq. (6) yields the value $f_{\pi qq} = 0.65$ for the pion-quark coupling, which is close to the value $f_{\pi qq} = 3/5 f_{\pi NN} \simeq 0.57$ suggested by the static quark model (cf. (3)). The values for $f_{\eta qq}$ and $f_{\eta ss}$ so obtained are 1.25 and -2.5 respectively. The values for $f_{\eta ss}$ obtained here from the ratio of the empirical branching fractions for $D_s^* \rightarrow D_s \pi^0$ and $D_s^* \rightarrow D_s \gamma$, are much smaller than this quark model value. In the completely $SU(3)$ flavor symmetric limit, in which the pion and η are degenerate in mass, the value for $f_{\eta ss}$ would however be equal to $-2/\sqrt{3} f_{\pi qq}$, which is only -0.75. The results from various modern phenomenological analyses are typically about twice this value.

Naive application of $SU(3)$ flavor symmetry at the level of a chiral symmetry violating pseudoscalar coupling model for pions and η mesons to baryons gives the result $g_{\eta NN} = 1/\sqrt{3} g_{\pi NN}$. With the (now) standard value of 12.8 for $g_{\pi NN}$ this relation would give the value 7.4 for $g_{\eta NN}$. Such a large value would correspond to $f_{\eta NN} = 2.2$. While values of that magnitude are ruled out by η meson photoproduction, as well as, as shown here, by the π^0 decay of the D_s^* meson, they were employed in early realistic boson exchange models for the nucleon-nucleon interaction.

Acknowledgments

We are indebted to Dr.S. Wycech for instructive discussions. DOR thanks the W.K. Kellogg Radiation Laboratory of the California Institute of Technology for its hospitality during the completion of this work. Research supported in part by the Academy of Finland by grants 43982 and 54038.

References

- [1] P. Cho and M. Wise, Phys. Rev. **D49:6228** (1994), hep-ph/9401301
- [2] I.W. Stewart, Nucl. Phys. **B529:62** (1998), hep-ph/9803227
- [3] A. Magiera and H. Machner, Nucl. Phys. **A674:515** (2000)
- [4] T. Meissner and E.M. Henley, Phys. Rev. **C55:3093** (1997), nucl-th/9601030
- [5] J. Gasser and H. Leutwyler, Nucl. Phys. **B250:465** (1985)
- [6] S.A. Coon and M.D. Scadron, Phys. Rev. **C26:562** (1982)
- [7] S. Weinberg, *The Quantum Theory of Fields, Vol. II*, p.187 (Cambridge University Press, 1996)
- [8] Th. Feldmann, Int. J. Mod. Phys. **A15:159** (2000), hep-ph/9907491
- [9] V.G.J. Stoks, R. Timmermans and J.J. de Swart, Phys. Rev. **C47:512** (1993), nucl-th/9211007
- [10] R.B. Wiringa, V.G.J. Stoks and R. Schiavilla, Phys. Rev. **C51:38** (1995), nucl-th/9408016
- [11] D.O. Riska, nucl-th/0204016
- [12] L.Tiator, C. Bennhold and S.S. Kamalov, Nucl. Phys. **A580:455** (1994), nucl-th/9404013
- [13] M.T. Peña, H. Garcilazo and D.O. Riska, Nucl. Phys. **A683:322** (2001), hep-ph/0006011
- [14] S. Weinberg, Phys. Rev. Lett. **65:1181** (1990)
- [15] S. Weinberg, Phys. Rev. Lett. **67:3473** (1991)
- [16] K.O.E. Henriksson *et al.*, Nucl. Phys. **A686:355** (2001), hep-ph/0009095
- [17] J.L. Goity and W. Roberts, Phys. Rev. **D60:034001** (1999), hep-ph/9809312
- [18] M. Franz, P.V. Pobylitsa, M.V. Polyakov and K. Goeke, Phys. Lett. **B454:335** (1999), hep-ph/9810343
- [19] M. Franz, M.V. Polyakov and K. Goeke, Phys. Rev. **D62:074024** (2000), hep-ph/0002240
- [20] D.E. Groom *et al.*, Eur. Phys. J. **C15:1** (2000) (Particle Data Group)
- [21] Ll. Ametller, C. Ayala and A. Bramon, Phys. Lett. **B141:414** (1984)
- [22] T.A. Lähde and D.O. Riska, Nucl. Phys. **A707/3-4:345** (in press), hep-ph/0112131

- [23] T.A. Lähde, C.J. Nyfält and D.O. Riska, Nucl. Phys. **A674:141** (2000), hep-ph/9908485
- [24] L.Ya. Glozman and D.O. Riska, Phys. Rept. **268:263** (1996), hep-ph/9505422
- [25] L.Ya. Glozman and D.O. Riska, Nucl. Phys. **A603:326** (1996), hep-ph/9509269
erratum *ibid.* **A620:510** (1997)
- [26] K. Tsushima and D.O. Riska, Nucl. Phys. **A549:313** (1992)
- [27] K. Tsushima, D.O. Riska and P.G. Blunden, Nucl. Phys. **A559:543** (1993)
- [28] R. Blankenbecler and R. Sugar, Phys. Rev. **142:1051**(1966)
- [29] T.A. Lähde, C.J. Nyfält and D.O. Riska, Nucl. Phys. **A645:587** (1999), hep-ph/9808438
- [30] F. Coester and D.O. Riska, Ann. Phys. **234:141** (1994)
- [31] A. Anastassov *et al.* (CLEO Coll.) Phys. Rev. **D65:032003** (2002), hep-ex/0108043
- [32] C. Davies, hep-ph/9710394
- [33] S. Chernyshev, M.A. Nowak and I. Zahed, Phys. Lett. **B350:238** (1995), hep-ph/9409207
- [34] S. Chernyshev, M.A. Nowak and I. Zahed, Phys. Rev. **D53:5176** (1996), hep-ph/9510326
- [35] J.L. Goity and W. Roberts, Phys. Rev. **D64:094007** (2001), hep-ph/0012314
- [36] M. Di Pierro and E. Eichten, Phys. Rev. **D64:114004** (2001), hep-ph/0104208