

Minimal scalar sector of 3-3-1 models without exotic electric charges

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Abstract

We study the minimal set of higgs scalars, for models based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ which do not contain particles with exotic electric charges. We show that only two higgs $SU(3)_L$ triplets are needed in order to properly break the symmetry. The exact tree-level scalar mass matrices resulting from symmetry breaking are calculated at the minimum of the most general scalar potential, and the gauge bosons are obtained, together with their coupling to the physical scalar fields. We show how the scalar sector introduced is enough to produce masses for fermions in a particular model which is an E_6 subgroup. By using experimental results we constraint the scale of new physics to be above 1.3 TeV.

1 Introduction

The Standard Model (SM) based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ [1] can be extended in several different ways: first by adding new fermion fields (adding a right-handed neutrino field constitute its simplest extension and has profound consequences, as for example the implementation of the see-saw mechanism, and the enlarging of the possible number of local gauge abelian symmetries that can be gauged simultaneously); second, by augmenting the scalar

sector to more than one higgs representation, and third by enlarging the local gauge group. In this last direction $SU(3)_L \otimes U(1)_X$ as a flavor group has been studied previously by many authors in the literature [2]-[9] who have explored possible fermion and higgs-boson representation assignments. From now on, models based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ are going to be called 331 models.

There are in the literature several 331 models; the most popular one, the Pleitez-Frampton model [2], is far from being the simplest construction. Not only its scalar sector is quite complicated and messy (three triplets and one sextet [3]), but its physical spectrum is plagued with particles with exotic electric charges, namely: double charged gauge and higgs bosons and exotic quarks with electric charges $5/3$ and $-4/3$. Other 331 models in the literature are just introduced or merely sketched in a few papers [4, 5, 6, 7], with a detailed phenomenological analysis of them still lacking. In particular, there is not published papers related to the study of the scalar sector for those other models.

All possible 331 models without exotic electric charges in their gauge boson sector and in their spin 1/2 fermion content are presented in Ref. [8], where it is shown that there are just a few anomaly free models for one or three families which share in common all of them the same gauge-boson content and, as we are going to show next, they may share a common scalar sector too. This scalar sector does not contain particles with exotic electric charges either.

Our study is organized as follows: in section two we analyze the electric charge operator in the context of 331 models. In section three we present all the possible 331 models without exotic electric charges for one and three families. In section four we study the (common) scalar sector for all those models, including the analysis of its mass spectrum. In section five we analyze the gauge boson structure common to all the models considered. In section six we present the couplings between the neutral scalar fields in the model and the SM gauge bosons, and in the last section we present our conclusions. An appendix at the end shows how the higgs scalars used to break the symmetry, can also be used to produce a consistent mass spectrum for the fermion fields, in the particular model which is an E_6 subgroup [6].

2 Charge content of 331 models

In what follows we assume that the electroweak group is $SU(3)_L \otimes U(1)_X \supset SU(2)_L \otimes U(1)_Y$. We also assume that the left-handed quarks (color triplets) and left-handed leptons (color singlets) transform under the two fundamental representations of $SU(3)_L$ (the 3 and 3^*) and that $SU(3)_c$ is vectorlike as in the SM.

The most general electric charge operator in $SU(3)_L \otimes U(1)_X$ is a linear combination of the three diagonal generators of the gauge group

$$Q = aT_{3L} + \frac{2}{\sqrt{3}}bT_{8L} + XI_3, \quad (1)$$

where $T_{iL} = \lambda_{iL}/2$, being λ_{iL} the Gell-Mann matrices for $SU(3)_L$ normalized as $\text{Tr}(\lambda_i \lambda_j) = 2\delta_{ij}$, $I_3 = Dg(1, 1, 1)$ is the diagonal 3×3 unit matrix, and a and b are arbitrary parameters to be determined anon. The X values are fixed by anomaly cancelation [8, 9] and an eventual coefficient for XI_3 can be absorbed in the hypercharge definition.

If we assume that the usual isospin $SU(2)_L$ of the SM is such that $SU(2)_L \subset SU(3)_L$, then $a = 1$ and we have just a one parameter set of models, all of them characterized by the value of b . So, Eq.(1) allows for an infinite number of models in the context of the 331 gauge structure, each one associated to a particular value of the parameter b , with characteristic signatures that make each one quite different from each other.

There are a total of 17 gauge bosons in the gauge group under consideration, they are: one gauge field B^μ associated with $U(1)_X$, the 8 gluon fields associated with $SU(3)_c$ which remain massless after breaking the symmetry, and other 8 associated with $SU(3)_L$ and that we may write in the following way:

$$\frac{1}{2}\lambda_{\alpha L}A_\mu^\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} D_{1\mu}^0 & W_\mu^+ & K_\mu^{(1/2+b)} \\ W_\mu^- & D_{2\mu}^0 & K_\mu^{-(1/2-b)} \\ K_\mu^{-(1/2+b)} & \bar{K}_\mu^{1/2-b} & D_{3\mu}^0 \end{pmatrix}$$

where $D_{1\mu}^0 = A_{3\mu}/\sqrt{2} + A_{8\mu}/\sqrt{6}$, $D_{2\mu}^0 = -A_{3\mu}/\sqrt{2} + A_{8\mu}/\sqrt{6}$, and $D_{3\mu}^0 = -2A_{8\mu}/\sqrt{6}$. The upper indices of the gauge bosons in the former expression stand for the electric charge of the corresponding particle, some of them functions of

the b parameter as they should be [9]. Notice that the gauge bosons have integer electric charges only for $b = \pm 1/2, \pm 3/2, \pm 5/2, \dots, \pm(2n+1)/2, n = 0, 1, 2, 3, \dots$. A deeper analysis shows that each negative b value can be related to the positive one just by taking the complex conjugate in the covariant derivative, which in turn is equivalent to replace $3 \leftrightarrow 3^*$ in the fermion content of each particular model.

Our first conclusion is thus that if we want to avoid exotic electric charges in the gauge sector of our theory, then b must be equal to $1/2$, which is also the condition for excluding exotic electric charges in the fermion sector [9].

Now, contrary to the SM where only the abelian $U(1)_Y$ factor is anomalous, in the 331 theory both, $SU(3)_L$ and $U(1)_X$ are anomalous ($SU(3)_c$ is vectorlike as in the SM). So, special combination of multiplets must be used in each particular model in order to cancel the several possible anomalies, and end with physical acceptable models. The triangle anomalies we must take care of are: $[SU(3)_L]^3$, $[SU(3)_c]^2U(1)_X$, $[SU(3)_L]^2U(1)_X$, $[grav]^2U(1)_X$ (the gravitational anomaly), and $[U(1)_X]^3$.

In order to present specific examples let us see how the charge operator in Eq.(1) acts on the representations 3 and 3^* of $SU(3)_L$:

$$Q[3] = Dg.(\frac{1}{2} + \frac{b}{3} + X, -\frac{1}{2} + \frac{b}{3} + X, -\frac{2b}{3} + X)$$

$$Q[3^*] = Dg.(-\frac{1}{2} - \frac{b}{3} + X, \frac{1}{2} - \frac{b}{3} + X, \frac{2b}{3} + X).$$

Notice from this expressions that, if we accommodate the known left-handed quark and lepton isodoublets in the two upper components of 3 and 3^* (or 3^* and 3), and forbid the presence of exotic electric charges in the possible models, then the electric charge of the third component in those representations must be equal either to the charge of the first or second component, which in turn implies $b = \pm 1/2$. Since the negative value is equivalent to the positive one, $b = 1/2$ is a necessary and sufficient condition in order to exclude exotic electric charges in the fermion sector too.

As an example of the former discussion let us take $b = 3/2$, then $Q[3] = Dg.(1+X, X, X-1)$ and $Q[3^*] = Dg.(X-1, X, 1+X)$. Then the following multiplets are associated with the respective $(SU(3)_c, SU(3)_L, U(1)_X)$ quantum numbers:

$(e^-, \nu_e, e^+)_L^T \sim (1, 3^*, 0)$; $(u, d, j)_L^T \sim (3, 3, -1/3)$ and $(d, u, k)_L^T \sim (3, 3^*, 2/3)$, where j and k are isosinglet exotic quarks of electric charges $-4/3$ and $5/3$ respectively. This multiplet structure is the basis of the Pleitez-Frampton model [2] for which the anomaly-free arrangement for the three families is given by:

$$\begin{aligned}
\psi_L^a &= (e^a, \nu^a, e^{ca})_L^T \sim (1, 3^*, 0), \\
q_L^i &= (u^i, d^i, j^i)_L^T \sim (3, 3, -1/3), \\
q_L^1 &= (d^1, u^1, k)_L^T \sim (3, 3^*, 2/3), \\
u_L^{ca} &\sim (3^*, 1, -2/3), \quad d_L^{ca} \sim (3^*, 1, 1/3), \\
k_L^a &\sim (3^*, 1, -5/3), \quad j_L^{ci} \sim (3^*, 1, -4/3),
\end{aligned}$$

where the upper c symbol stands for charge conjugation, $a = 1, 2, 3$ is a family index and $i = 2, 3$ is related to two of the three families (in the 331 basis). As can be seen, there are six triplets of $SU(3)_L$ and six anti-triplets, which ensures cancelation of the $[SU(3)_L]^3$ anomaly. A power counting shows that the other four anomalies also vanish.

3 Models without exotic electric charges

As discussed before, after fixing $a = 1$, the value $b = 1/2$ is a necessary condition in order to avoid particles with exotic electric charges in models based on the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ gauge structure. For that particular value let us start first defining the following closed set of fermions (closed in the sense that they include the antiparticles of the charged particles):

- $S_1 = [(\nu_\alpha, \alpha^-, E_\alpha^-); \alpha^+; E_\alpha^+]$ with quantum numbers $[(1, 3, -2/3); (1, 1, 1); (1, 1, 1)]$.
- $S_2 = [(\alpha^-, \nu_\alpha, N_\alpha^0); \alpha^+]$ with quantum numbers $[(1, 3^*, -1/3); (1, 1, 1)]$.
- $S_3 = [(d, u, U); d^c; u^c; U^c]$ with quantum numbers $(3, 3^*, 1/3); (3^*, 1, 1/3); (3^*, 1, -2/3)$ and $(3^*, 1, -2/3)$, respectively.

- $S_4 = [(u, d, D); u^c; d^c; D^c]$ with quantum numbers $(3, 3, 0); (3^*, 1, -2/3); (3^*, 1, 1/3)$ and $(3^*, 1, 1/3)$, respectively.
- $S_5 = [(e^-, \nu_e, N_1^0); (E^-, N_2^0, N_3^0); (N_4^0, E^+, e^+)]$ with quantum numbers $(1, 3^*, -1/3); (1, 3^*, -1/3)$ and $(1, 3^*, 2/3)$, respectively.
- $S_6 = [(\nu_e, e^-, E_1^-); (E_2^+, N_1^0, N_2^0); (N_3^0, E_2^-, E_3^-); e^+; E_1^+; E_3^+]$ with quantum numbers $[(1, 3, -2/3); (1, 3, 1/3); (1, 3, -2/3); (1, 1, 1); (1, 1, 1); (1, 1, 1)]$.

Where the quantum numbers in parenthesis refer to $(SU(3)_c, SU(3)_L, U(1)_X)$ representations.

The several anomalies for the former six sets are presented in the following Table.

TABLE I. Anomalies for S_i .

| Anomalies | S_1 | S_2 | S_3 | S_4 | S_5 | S_6 |
|----------------------|-------|-------|-------|-------|-------|-------|
| $[SU(3)_c]^2 U(1)_X$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $[SU(3)_L]^2 U(1)_X$ | -2/3 | -1/3 | 1 | 0 | 0 | -1 |
| $[grav]^2 U(1)_X$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $[U(1)_X]^3$ | 10/9 | 8/9 | -12/9 | -6/9 | 6/9 | 12/9 |
| $[SU(3)_L]^3$ | 1 | -1 | -3 | 3 | -3 | 3 |

Table I allows us to build anomaly-free models without exotic electric charges, for one, two, three, four or more families. Let us extract out of the Table the possible models for one and three families:

3.1 One family models

There are just two anomaly-free one family structures that can be extracted from the Table. They are:

Model A: $(S_4 + S_5)$. This models is associated with an E_6 subgroup and has been partially analyzed in Ref. [6]. (see also the appendix at the end of this paper).

Model B: $(S_3 + S_6)$. This models is associated with an $SU(6)_L \otimes U(1)_X$ subgroup and has been partially analyzed in Ref. [7]

3.2 Three family models

Model C: $(3S_2 + S_3 + 2S_4)$. This model deals with the following multiplets associated with the given quantum numbers: $(u, d, D)_L^T \sim (3, 3, 0)$, $(e^-, \nu_e, N^0)_L^T \sim (1, 3^*, -1/3)$ and $(d, u, U)_L^T \sim (3, 3^*, 1/3)$, where D and U are exotic quarks with electric charges $-1/3$ and $2/3$ respectively. With such a gauge structure the three family anomaly-free model is given by:

$$\begin{aligned}
\psi_L^{\prime a} &= (e^{-a}, \nu^a, N^{0a})_L^T \sim (1, 3^*, -1/3), \\
e_L^{\prime a} &\sim (1, 1, 1), \\
q_L^{\prime i} &= (u^i, d^i, D^i)_L^T \sim (3, 3, 0), \\
q_L^{\prime 1} &= (d^1, u^1, U)_L^T \sim (3, 3^*, 1/3), \\
u_L^{ca} &\sim (3^*, 1, -2/3), \quad d_L^{ca} \sim (3^*, 1, 1/3), \\
U_L^c &\sim (3^*, 1, -2/3), \quad D_L^{ci} \sim (3^*, 1, 1/3),
\end{aligned}$$

where $a = 1, 2, 3$ is a family index and $i = 1, 2$ is related to two of the three families. This models has been analyzed in the literature in Ref. [4]. If needed, this model can be augmented with an undetermined number of neutral Weyl states $N_L^{0j} \sim (1, 1, 0)$, $j = 1, 2, \dots$ without violating the anomaly cancelation.

Model D: $(3S_1 + 2S_3 + S_4)$. It makes use of the same multiplets used in the previous model arranged in a different way, plus a new lepton multiplet $(\nu_e, e^-, E^-)_L^T \sim (1, 3, -2/3)$. The family structure of this new anomaly-free model is given by:

$$\begin{aligned}
\psi_L^{\prime\prime a} &= (\nu^a, e^a, E^a)_L^T \sim (1, 3, -2/3), \\
e_L^{ca} &\sim (1, 1, 1), \quad E_L^{ca} \sim (1, 1, 1), \\
q_L^{\prime\prime 1} &= (u^1, d^1, D)_L^T \sim (3, 3, 0), \\
q_L^{\prime\prime i} &= (d^i, u^i, U^i)_L^T \sim (3, 3^*, 1/3), \\
u_L^{ca} &\sim (3^*, 1, -2/3), \quad d_L^{ca} \sim (3^*, 1, 1/3), \\
U_L^c &\sim (3^*, 1, 1/3), \quad U_L^{ci} \sim (3^*, 1, 2/3).
\end{aligned}$$

This model has been analyzed in the literature in Ref. [5].

Model E: $(S_1 + S_2 + S_3 + 2S_4 + S_5)$. **Model F:** $(S_1 + S_2 + 2S_3 + S_4 + S_6)$.

Besides the former four three family models, other four, carbon copy of the two one family models **A,B** can also be constructed. They are:

Model G: $3(S_4 + S_5)$. **Model H:** $3(S_3 + S_6)$. **Model I:** $2(S_4 + S_5) + (S_3 + S_6)$.
Model J: $(S_4 + S_5) + 2(S_3 + S_6)$.

For a total of eight different three-family models, each one with a different fermion field content. Notice in particular that in models **E** and **F** each one of the three families is treated differently. As far as we know the last six models have not been studied in the literature so far.

If we wish we may construct also two, four, five, etc. family models (a two family model is given for example by $(S_1 + S_2 + S_3 + S_4)$), but we believe all those models are not realistic at all.

4 The scalar sector

If we pretend to use the simplest $SU(3)_L$ representations in order to break the symmetry, at least two complex scalar triplets, equivalent to twelve real scalar fields, are required. For $b = 1/2$ there are just two higgs scalars (together with their complex conjugates) which may develop nonzero Vacuum Expectation Values (VEV); they are $\phi_1(1, 3^*, -1/3)^T = (\phi_1^-, \phi_1^0, \phi_1^+)^T$ with VEV $\langle \phi_1 \rangle^T = (0, v_1, V)$ and $\phi_2(1, 3^*, 2/3)^T = (\phi_2^0, \phi_2^+, \phi_2'^+)^T$ with VEV $\langle \phi_2 \rangle^T = (v_2, 0, 0)$. As we will see ahead, to reach consistency with phenomenology we must have the hierarchy $V > v_1 \sim v_2$.

Our aim is to break the symmetry in one single step

$$SU(3)_c \otimes SU(3)_L \otimes U(1)_X \longrightarrow SU(3)_c \otimes U(1)_Q$$

which implies the existence of eight Goldstone bosons included in the scalar sector of the theory [10]. For the sake of simplicity we assume that the VEV are real. This means that the CP violation through the scalar exchange is not considered in this work. Now, for convenience in reading we rewrite the expansion of the scalar fields which acquire VEV as:

$$\phi_1^0 = V + \frac{H_{\phi_1}^0 + iA_{\phi_1}^0}{\sqrt{2}} \quad \phi_1^{\prime 0} = v_1 + \frac{H'_{\phi_1}{}^0 + iA'_{\phi_1}{}^0}{\sqrt{2}} \quad \phi_2^0 = v_2 + \frac{H_{\phi_2}^0 + iA_{\phi_2}^0}{\sqrt{2}}. \quad (2)$$

In the literature, a real part H is called a CP-even scalar and an imaginary part A a CP-odd scalar or pseudoscalar field.

Now, the most general potential which includes ϕ_1 and ϕ_2 can then be written in the following form:

$$V(\phi_1, \phi_2) = \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1). \quad (3)$$

Requiring that in the shifted potential $V(\phi_1, \phi_2)$, the linear terms in fields must be absent, we get in the tree-level approximation the following constraint equations:

$$\begin{aligned} \mu_1^2 + 2\lambda_1(v_1^2 + V^2) + \lambda_3 v_2^2 &= 0 \\ \mu_2^2 + \lambda_3(v_1^2 + V^2) + 2\lambda_2 v_2^2 &= 0. \end{aligned} \quad (4)$$

The analysis to the former equations shows that they are related to a minimum of the scalar potential with the value

$$V_{min} = -v_2^4 \lambda_2 - (v_1^2 + V^2)[(v_1^2 + V^2)\lambda_1 + v_2^2 \lambda_3] = V(v_1, v_2, V), \quad (5)$$

where $V(v_1 = 0, v_2, V) > V(v_1 \neq 0, v_2, V)$, implying that $v_1 \neq 0$ is preferred.

Substituting Eqs.(2) and (4) in Eq.(3) we get the following mass matrices:

4.1 Spectrum in the scalar neutral sector

In the $(H_{\phi_1}^0, H_{\phi_2}^0, H_{\phi_1}^{\prime 0})$ basis, the square mass matrix can be calculated using $M_{ij}^2 = 2 \frac{\partial^2 V(\phi_1, \phi_2)}{\partial H_{\phi_i}^0 \partial H_{\phi_j}^0}$. After imposing the constraints in Eq.(4) we get:

$$M_H^2 = 2 \begin{pmatrix} 2\lambda_1 V^2 & \lambda_3 v_2 V & 2\lambda_1 v_1 V \\ \lambda_3 v_2 V & 2\lambda_2 v_2^2 & \lambda_3 v_1 v_2 \\ 2\lambda_1 v_1 V & \lambda_3 v_1 v_2 & 2\lambda_1 v_1^2 \end{pmatrix}, \quad (6)$$

which has zero determinant, providing us with a Goldstone boson G_1 and two physical massive neutral scalar fields H_1 and H_2 with masses

$$M_{H_1, H_2}^2 = 2(v_1^2 + V^2)\lambda_1 + 2v_2^2 \lambda_2 \pm 2\sqrt{[(v_1^2 + V^2)\lambda_1 + v_2^2 \lambda_2]^2 + v_2^2 (v_1^2 + V^2)(\lambda_3^2 - 4\lambda_1 \lambda_2)}, \quad (7)$$

where real lambdas produce positive masses for the scalars only if $\lambda_1 > 0$ and $4\lambda_1\lambda_2 > \lambda_3^2$ (which implies $\lambda_2 > 0$).

We may see from the former equations that in the limit $V > v_1 \sim v_2$, and for lambdas of order one, there is a neutral higgs scalar with a mass of order V and other one with a mass of the order of $v_1 \sim v_2$, which may be identified with the SM scalar as we will see ahead.

The physical fields are related to the scalars in the weak basis by the lineal transformation:

$$\begin{pmatrix} H_{\phi_1}^0 \\ H_{\phi_2}^0 \\ H_{\phi_1'}^0 \end{pmatrix} = \begin{pmatrix} \frac{v_2 V}{S_1} & \frac{v_2 V}{S_2} & -\frac{v_1}{\sqrt{v_1^2 + V^2}} \\ \frac{M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1}{2S_1\lambda_3} & -\frac{(M_{H_1}^2 - 4v_2^2\lambda_2)}{2S_2\lambda_3} & 0 \\ \frac{v_1 v_2}{S_1} & \frac{v_1 v_2}{S_2} & \frac{V}{\sqrt{v_1^2 + V^2}} \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ G_1 \end{pmatrix}, \quad (8)$$

where we have defined $S_1 = \sqrt{v_2^2(v_1^2 + V^2) + (M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1)^2/4\lambda_3^2}$ and $S_2 = \sqrt{v_2^2(v_1^2 + V^2) + (M_{H_1}^2 - 4v_2^2\lambda_2)^2/4\lambda_3^2}$.

4.2 Spectrum in the pseudoscalar neutral sector

The analysis shows that $V(\phi_1, \phi_2)$ in Eq.(3), when expanded around the most general vacuum given by Eqs.(2) and using the constraints in Eq.(4), does not contain pseudoscalar fields $A_{\phi_i}^0$. This allows us to identify another three Goldstone bosons $G_2 = A_{\phi_1}^0$, $G_3 = A_{\phi_2}^0$ and $G_4 = A_{\phi_1'}^0$.

4.3 Spectrum in the charged scalar sector

In the basis $(\phi_1^+, \phi_2^+, \phi_2'^+)$ the square mass matrix is given by

$$M_+^2 = 2\lambda_4 \begin{pmatrix} v_2^2 & v_1 v_2 & v_2 V \\ v_1 v_2 & v_1^2 & v_1 V \\ v_2 V & v_1 V & V^2 \end{pmatrix}, \quad (9)$$

which has two eigenvalues equal to zero equivalent to four Goldstone bosons (G_5^\pm, G_6^\pm) and two physical charged higgs scalars with large masses given by $\lambda_4(v_1^2 + v_2^2 + V^2)$, with the new constraint $\lambda_4 > 0$.

Our analysis shows that, after symmetry breaking, the original twelve degrees of freedom in the scalar sector have become eight Goldstone bosons (four electrically neutral and four charged), and four physical scalar fields, two neutrals (one of them the SM higgs scalar) and two charged ones. The eight Goldstone bosons must be swallow up by eight gauge fields as we will see in the next section.

5 The Gauge boson sector

For $b = 1/2$, the nine gauge bosons in $SU(3)_L \otimes U(1)_X$ when acting on left-handed triplets can be arranged in the following convenient way:

$$\mathcal{A}_\mu = \frac{1}{2}g\lambda_{\alpha L}A_\mu^\alpha + g'XB_\mu I_3 = \frac{g}{\sqrt{2}} \begin{pmatrix} Y_{1\mu}^0 & W_\mu^+ & K_\mu^+ \\ W_\mu^- & Y_{2\mu}^0 & K_\mu^0 \\ K_\mu^- & \bar{K}_\mu^0 & Y_{3\mu}^0 \end{pmatrix},$$

where $Y_{1\mu}^0 = A_{3\mu}/\sqrt{2} + A_{8\mu}/\sqrt{6} + \sqrt{2}(g'/g)XB_\mu$, $Y_{2\mu}^0 = -A_{3\mu}/\sqrt{2} + A_{8\mu}/\sqrt{6} + \sqrt{2}(g'/g)XB_\mu$, and $Y_{3\mu}^0 = -2A_{8\mu}/\sqrt{6} + \sqrt{2}(g'/g)XB_\mu$; X being the hypercharge value of the given left-handed triplet (for example $-1/3$ and $2/3$ for ϕ_1 and ϕ_2 respectively).

After breaking the symmetry with $\langle\phi_i\rangle$, $i = 1, 2$, and using for the covariant derivative for triplets $D^\mu = \partial^\mu - i\mathcal{A}^\mu$, we get the following mass terms in the gauge boson sector:

5.1 Spectrum in the charged gauge boson sector

In the basis (K_μ^\pm, W_μ^\pm) the square mass matrix produced is

$$M_\pm^2 = \frac{g^2}{2} \begin{pmatrix} (V^2 + v_2^2) & v_1V \\ v_1V & (v_1^2 + v_2^2) \end{pmatrix}. \quad (10)$$

The former symmetric matrix give us the masses $M_{W'}^2 = g^2v_2^2/2$ and $M_{K'}^2 = g^2(v_1^2 + v_2^2 + V^2)/2$, related to the physical fields $W'_\mu = \eta(v_1K_\mu - VW_\mu)$, and $K'_\mu = \eta(VK_\mu + v_1W_\mu)$ associated with the known charged weak current W'^\pm_μ , and with a new one K'^\pm_μ predicted in the context of this model ($\eta^{-2} = v_1^2 + V^2$ is a normalization factor). From the experimental value $M_{W'} = 80.419 \pm 0.056$ GeV [12] we obtain $v_2 \simeq 174$ GeV as in the SM.

5.2 Spectrum in the neutral gauge boson sector

For the five electrically neutral gauge bosons we get first, that the imaginary part of $K_\mu^0 = (K_{\mu R}^0 + iK_{\mu I}^0)/\sqrt{2}$ decouples from the other four electrically neutral gauge bosons, acquiring a mass $M_{K_I^0}^2 = g^2(v_1^2 + V^2)/2$. Then, in the basis $(B^\mu, A_3^\mu, A_8^\mu, K_R^{0\mu})$, the following squared mass matrix is obtained:

$$M_0^2 = \begin{pmatrix} \frac{g'^2}{9}(v_1^2 + V^2 + 4v_2^2) & -\frac{gg'}{6}(v_1^2 + 2v_2^2) & -\frac{gg'}{3\sqrt{3}}(V^2 + v_2^2 - v_1^2/2) & gg'v_1V/3 \\ -\frac{gg'}{6}(v_1^2 + 2v_2^2) & g^2(v_1^2 + v_2^2)/4 & \frac{g^2}{4\sqrt{3}}(v_2^2 - v_1^2) & -g^2v_1V/4 \\ -\frac{gg'}{3\sqrt{3}}(V^2 + v_2^2 - v_1^2/2) & \frac{g^2}{4\sqrt{3}}(v_2^2 - v_1^2) & \frac{g^2}{12}(v_1^2 + v_2^2 + 4V^2) & -g^2v_1V/(4\sqrt{3}) \\ gg'v_1V/3 & -g^2v_1V/4 & -g^2v_1V/(4\sqrt{3}) & g^2(v_1^2 + V^2)/4 \end{pmatrix} \quad (11)$$

This matrix has determinant equal to zero which implies that there is a zero eigenvalue associated to the photon field with eigenvector

$$A^\mu = S_W A_3^\mu + C_W \left[\frac{T_W}{\sqrt{3}} A_8^\mu + (1 - T_W^2/3)^{1/2} B^\mu \right], \quad (12)$$

where $S_W = \sqrt{3}g'/\sqrt{3g^2 + 4g'^2}$ and C_W are the sine and cosine of the electroweak mixing angle ($T_W = S_W/C_W$). Orthogonal to the photon field A^μ we may define other two fields

$$\begin{aligned} Z^\mu &= C_W A_3^\mu - S_W \left[\frac{T_W}{\sqrt{3}} A_8^\mu + (1 - T_W^2/3)^{1/2} B^\mu \right] \\ Z'^\mu &= -(1 - T_W^2/3)^{1/2} A_8^\mu + \frac{T_W}{\sqrt{3}} B^\mu, \end{aligned} \quad (13)$$

where Z^μ corresponds to the neutral current of the SM and Z'^μ is a new weak neutral current predicted for these models.

We may also identify the gauge boson Y^μ associated with the SM hypercharge in $U(1)_Y$ as:

$$Y^\mu = \left[\frac{T_W}{\sqrt{3}} A_8^\mu + (1 - T_W^2/3)^{1/2} B^\mu \right].$$

In the basis $(Z'^\mu, Z^\mu, K_R^{0\mu})$ the mass matrix for the neutral sector reduces to:

$$\frac{g^2}{4C_W^2} \begin{pmatrix} \delta^2(v_1^2 C_{2W}^2 + v_2^2 + 4V^2 C_W^4) & \delta(v_1^2 C_{2W} - v_2^2) & \delta C_W v_1 V \\ \delta(v_1^2 C_{2W} - v_2^2) & v_1^2 + v_2^2 & -C_W v_1 V \\ \delta C_W v_1 V & -C_W v_1 V & C_W^2(v_1^2 + V^2) \end{pmatrix}, \quad (14)$$

where $C_{2W} = C_W^2 - S_W^2$ and $\delta = (4C_W^2 - 1)^{-1/2}$. The eigenvectors and eigenvalues of this matrix are the physical fields and their masses. In the approximation $v_1 = v_2 \equiv v \ll V$ and using $q \equiv v^2/V^2$ as an expansion parameter we get up, to first order in q , the following eigenvalues:

$$\begin{aligned} M_{Z_1}^2 &\approx \frac{1}{2} g^2 C_W^{-2} v^2 (1 - q T_W^4), \\ M_{Z_2}^2 &\approx \frac{g^2 V^2}{1 + 2C_{2W}} [1 + C_{2W} - q(S_{2W}^2 + C_{2W}^{-1})/2C_W^2], \\ M_{K_R^0}^2 &\approx g^2 V^2 [1 + q(1 + C_{2W}^{-1})]. \end{aligned}$$

So we have a neutral current associated to a gauge boson Z_1^0 , related to a mass scale $v \simeq 174$ GeV, which may be identified with the known experimental neutral current as we will see in what follows, and two new electrically neutral currents associated to a large mass scale $V \gg v$.

The former is the way how the eight would be Goldstone bosons are absorbed by the longitudinal components of the eight massive gauge bosons ($W'^\pm, K'^\pm, K_I^0, K_R^0, Z_1^0$ and Z_2^0) as expected.

From the expressions for $M_{W'}$ and M_{Z_1} we obtain $\rho_0 = M_{W'}^2/(M_{Z_1}^2 C_W^2) \approx 1 + T_W^4 q^2$, and the global fit for $\rho_0 = 1.0012_{-0.0014}^{+0.0023}$ [11] provides us with the lower limit $V \geq 1.3$ TeV (where we are using for $S_W^2 = 0.23113$ [12]). This result justifies the existence of the expansion parameter $q \leq 0.01$ which sets the scale of new physics, together with the hierarchy $V > v_1 \sim v_2$.

6 Higgs-SM gauge boson couplings

In order to identify the considered above Higgs bosons with the one in the SM, in this section we present the couplings of the two neutral scalar fields H_1 and H_2 from section 4 with the physical gauge bosons W'^\pm and Z_1^0 ; then we take the

limit $V \gg v = v_1 = v_2$ which produces the couplings of the physical scalars H_1 and H_2 with the SM gauge bosons W^\pm and Z^0 .

When the algebra gets done we obtain the following trilinear couplings, provided $\lambda_3 < 0$:

$$\begin{aligned}
g(W'W'H_1) &= \frac{g^2 v_2 [M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1]}{2\sqrt{2}S_1\lambda_3} \xrightarrow{v \gg v} \frac{g^2 v_2^2 \lambda_3}{2\sqrt{2}\lambda_1 V} \\
g(W'W'H_2) &= \frac{g^2 v_2 (4v_2^2 \lambda_2 - M_{H_1}^2)}{2\sqrt{2}S_2\lambda_3} \xrightarrow{v \gg v} \frac{g^2 v_2}{\sqrt{2}} \\
g(Z_1^0 Z_1^0 H_1) &= \frac{g^2 v_1}{S_1} \left[\frac{M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1}{4\sqrt{2}C_W^2 \lambda_3} + q \frac{v_1^2 (\lambda_1 - \lambda_3 S_W^2)}{8\sqrt{2}C_W^2 \lambda_1} + \dots \right] \\
&\xrightarrow{v \gg v} \frac{g^2 v_1^2 \lambda_3}{4\sqrt{2}\lambda_1 C_W^2 V} \\
g(Z_1^0 Z_1^0 H_2) &= \frac{g^2 v_1 V^2}{S_2} \left[-\frac{\lambda_1}{\sqrt{2}\lambda_3 C_W^2} + q \frac{4\lambda_1 \lambda_2 - \lambda_3^2 + 2\lambda_1^2 (T_W^2 C_W^{-2} - 2)}{4\sqrt{2}C_W^2 \lambda_1 \lambda_3} + \dots \right] \\
&\xrightarrow{v \gg v} \frac{g^2 v_1}{2\sqrt{2}C_W^2},
\end{aligned}$$

where $g(W'W'H_i^0)$, $i = 1, 2$ are exact expressions and $g(Z_1^0 Z_1^0 H_i)$ are expansions in the parameter q up to first order.

The quartic couplings are determined to be:

$$\begin{aligned}
g(W'W'H_1 H_1) &= \frac{g^2 [M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1]^2}{16S_1^2 \lambda_3^2} \xrightarrow{v \gg v} \frac{g^2 v_2^2 \lambda_3^2}{16\lambda_1^2 V^2} \\
g(W'W'H_2 H_2) &= \frac{g^2 (M_{H_1}^2 - 4v_2^2 \lambda_2)^2}{16S_2^2 \lambda_3^2} \xrightarrow{v \gg v} \frac{g^2}{4} \\
g(Z_1^0 Z_1^0 H_1 H_1) &= \frac{g^2}{S_1^2} \left[\frac{[M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1]^2}{32C_W^2 \lambda_3^2} + q \frac{2v_1^4 \lambda_3^2 - S_W^2 [M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1]^2}{64C_W^6 \lambda_3^2} + \dots \right] \\
&\xrightarrow{v \gg v} \frac{g^2 v_1^2 \lambda_3^2}{32\lambda_1^2 C_W^2 V^2} \\
g(Z_1^0 Z_1^0 H_2 H_2) &= \frac{g^2 V^4}{S_2^2} \left[\frac{\lambda_1^2}{2\lambda_3^2 C_W^2} + q \frac{\lambda_3^2 - 4\lambda_1 \lambda_2 + \lambda_1^2 (4 - C_W^{-2} T_W^2)}{4C_W^2 \lambda_3^2} + \dots \right] \\
&\xrightarrow{v \gg v} \frac{g^2}{8C_W^2}
\end{aligned}$$

where as before $g(W'W'H_i^0H_i^0)$, $i = 1, 2$ are exact expressions and $g(Z_1^0Z_1^0H_i^0H_i^0)$ are expansions in the parameter q up to first order.

As can be seen, in the limit $V > v_1 \sim v_2$ the couplings $g(W'W'H_2)$, $g(Z_1^0Z_1^0H_2)$, $g(W'W'H_2H_2)$ and $g(Z_1^0Z_1^0H_2H_2)$ coincide with those in the SM as far as $\lambda_3 < 0$. This gives additional support to the hierarchy $V > v_1 \sim v_2$.

Summarizing, from the couplings of the SM gauge bosons with the physical Higgs scalars we can conclude, as anticipated before, that the scalar H_2 can be identified with the SM neutral Higgs particle, and that Z_1^0 can be associated with the known neutral current of the SM (more support to this last statement is presented in the Appendix).

7 Conclusions

In this paper we have studied in detail the minimal scalar sector of some models based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$. By restricting the field representations to particles without exotic electric charges we end up with ten different models, two one family models and eight models for three families. The two one family models are studied in the papers in Refs. [6, 7], but enough attention was not paid to the scalar sector in the analysis done. As far as we know, most of the three family models are new in the literature, but models **C** and **D**, which has been partially analyzed in Refs. [4] and [5] respectively.

We have also considered the mass spectrum eigenstates of the most general scalar potential specialized for the 331 models without exotic electric charges, with two Higgs triplets with the most general VEV possible. It is shown that in the considered models there is just one light neutral Higgs scalar which can be identified with the SM Higgs scalar; there are besides three more heavy scalars, one charged and its charge conjugate and one extra neutral one.

The two triplets of $SU(3)_L$ scalars with the most general VEV possible produces a consistent fermion mass spectrum at least for one of the models in the literature and the scale of the new physics predicted by the class of models analyzed in this paper lies above 1.3 TeV as shown in the main text. This scale is consistent with the analysis done in other papers [6, 7] using a different phenomenological analysis.

Finally notice that our analysis allows us to constraint all the parameters in the scalar potential; that is, our model is a consistent one as far as $\lambda_1 > 0$, $\lambda_2 > 0$, $4\lambda_1\lambda_2 > \lambda_3^2$, $\lambda_3 < 0$ and $\lambda_4 > 0$.

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A Appendix

In this appendix we show how the fermion fields of a particular model acquire masses with the Higgs scalars and VEV introduced in the main text. The analysis is model dependent, so let us use the one family model **A**, for which the fermion multiplets are [6] $\chi_L^T = (u, d, D)_L \sim (3, 3, 0)$; $u_L^c \sim (3^*, 1, -2/3)$; $d_L^c \sim (3^*, 1, 1/3)$, $D_L^c \sim (3^*, 1, 1/3)$; $\psi_{1L}^T = (e^-, \nu_e, N_1^0)_L \sim (1, 3^*, -1/3)$, $\psi_{2L}^T = (E^-, N_2^0, N_3^0)_L \sim (1, 3^*, -1/3)$, and $\psi_{3L}^T = (N_4^0, E^+, e^+)_L \sim (1, 3^*, 2/3)$. As shown in Ref. [6], this structure corresponds to an E_6 subgroup.

A.1 Bare Masses for fermion fields

The most general Yukawa Lagrangian that the Higgs scalars in Section 4 produce for the fermion fields in this model, can be written as $\mathcal{L}_Y = \mathcal{L}_Y^Q + \mathcal{L}_Y^l$, with

$$\begin{aligned}\mathcal{L}_Y^Q &= \chi_L^T C(h_u \phi_2 u_L^c + h_D \phi_1 D_L^c + h_d \phi_1 d_L^c) + h.c., \\ \mathcal{L}_Y^l &= \epsilon_{abc} [\psi_{1L}^a C(h_1 \psi_{2L}^b \phi_2^c + h_2 \psi_{3L}^b \phi_1^c) + \psi_{2L}^a C h_3 \psi_{3L}^b \phi_1^c] + h.c.,\end{aligned}$$

where h_η , $\eta = u, d, D, 1, 2, 3$ are Yukawa couplings of order one; a, b, c are $SU(3)_L$ tensor indices and C is the charge conjugation operator.

Using for $\langle \phi_i \rangle$, $i = 1, 2$ the VEV in section 4 we get $m_u = h_u v_2$ for the mass of the up type-quark and for the down sector in the basis (d, D) we get the mass matrix

$$M_d = \begin{pmatrix} h_d v_1 & h_d V \\ h_D v_1 & h_D V \end{pmatrix}; \quad (15)$$

now, looking for the eigenvalues of $M_d M_d^\dagger$, we get $\sqrt{(h_d^2 + h_D^2)(v_1^2 + V^2)}$ and zero. Notice that for $h_u = 1$ and assuming for example that we are referring to the third family, we obtain the correct mass for the top quark (remember from Section 5 that $v_2 \simeq 174$ GeV), the bottom quark remains massless at zero level, and there is an exotic Bottom quark with a very large mass. Since there is no way to distinguish between d_L^c and D_L^c in the Yukawa Lagrangian it is just natural to impose the discrete symmetry $h_d = h_D \equiv h$.

For the charged lepton sector the mass eigenvalues are 0 and $\sqrt{(h_2^2 + h_3^2)(v_1^2 + V^2)}$, with similar consequences as in the down quark sector, where again it is natural to impose the symmetry $h_2 = h_3 \equiv h'$.

The analysis of the neutral lepton sector is more elaborated; at zero level and in the basis $(\nu, N_1, N_2, N_3, N_4)$ we get the mass matrix:

$$M_N = \begin{pmatrix} 0 & 0 & 0 & h_1 v_2 & -h_2 V \\ 0 & 0 & -h_1 v_2 & 0 & h_2 v_1 \\ 0 & -h_1 v_2 & 0 & 0 & -h_3 V \\ h_1 v_2 & 0 & 0 & 0 & h_3 v_1 \\ -h_2 V & h_2 v_1 & -h_3 V & h_3 v_1 & 0 \end{pmatrix}, \quad (16)$$

with eigenvalues 0, $\pm h_1 v_2$ and $\pm \sqrt{h_1^2 v_2^2 + (h_2^2 + h_3^2)(V^2 + v_1^2)}$, which implies a Majorana neutrino of zero mass and two Dirac neutral particles with masses one of them at the electroweak mass scale and the other one at the TeV scale.

So, at zero level the charged exotic particles get large masses of order $V > 1.3$ TeV, the top quark and a Dirac neutral particle get masses of order $v_2 \sim 174$ GeV, there is a Dirac neutral particle with a mass of order V, and the bottom quark, charged lepton and a Majorana neutrino remain massless. In what follows we will see that they pick up a radiative mass in the context of the model studied here.

A.2 Currents

The interactions among the charged gauge fields in Section 5 with the fermions of **Model A** are [6]:

$$\begin{aligned}
H^{CC} &= \frac{g}{\sqrt{2}}[W_\mu^+(\bar{u}_L\gamma^\mu d_L - \bar{\nu}_{eL}\gamma^\mu e_L - \bar{N}_{2L}^0\gamma^\mu E_L^- - \bar{E}_L^+\gamma^\mu N_{4L}^0) \\
&+ K_\mu^+(\bar{u}_L\gamma^\mu D_L - \bar{N}_{1L}^0\gamma^\mu e_L - \bar{N}_{3L}^0\gamma^\mu E_L^- - \bar{e}_L^+\gamma^\mu N_{4L}^0) \\
&+ K_\mu^0(\bar{d}_L\gamma^\mu D_L - \bar{N}_{1L}^0\gamma^\mu \nu_{eL} - \bar{N}_{3L}^0\gamma^\mu N_{2L}^0 - \bar{e}_L^+\gamma^\mu E_L^+)] + h.c.,
\end{aligned}$$

where the first two terms constitute the charged weak current of the SM, and K^\pm , K^0 and \bar{K}^0 are related to new charged currents which violate weak isospin.

The algebra also shows that the neutral currents $J_\mu(EM)$, $J_\mu(Z)$ and $J_\mu(Z')$, associated with the Hamiltonian $H^0 = eA^\mu J_\mu(EM) + \frac{g}{C_W}Z^\mu J_\mu(Z) + \frac{g'}{\sqrt{3}}Z'^\mu J_\mu(Z')$ (where A_μ is the photon field in Eq.(12) and Z_μ and Z'_μ are the neutral gauge bosons introduced in Eq.(13)) are:

$$J_\mu(EM) = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}(\bar{d}\gamma_\mu d + \bar{D}\gamma_\mu D) - \bar{e}^-\gamma_\mu e^- - \bar{E}^-\gamma_\mu E^-, \quad (17)$$

$$J_\mu(Z) = J_{\mu,L}(Z) - S_W^2 J_\mu(EM), \quad (18)$$

$$J_\mu(Z') = T_W J_\mu(EM) - J_{\mu,L}(Z'), \quad (19)$$

where $e = gS_W = g'C_W\sqrt{(1 - T_W^2/3)} > 0$ is the electric charge, $J_\mu(EM)$ is the (vectorlike) electromagnetic current, and the two neutral left-handed currents are given by:

$$\begin{aligned}
J_{\mu,L}(Z) &= \bar{u}_L\gamma_\mu u_L - \bar{d}_L\gamma_\mu d_L + \bar{\nu}_{eL}\gamma_\mu \nu_{eL} - \bar{e}_L^-\gamma_\mu e_L^- + \bar{N}_2^0\gamma_\mu N_2^0 - \bar{E}^-\gamma_\mu E^- \\
&= \sum_f T_{3f}\bar{f}_L\gamma_\mu f_L,
\end{aligned} \quad (20)$$

$$\begin{aligned}
J_{\mu,L}(Z') &= S_{2W}^{-1}(\bar{u}_L\gamma_\mu u_L - \bar{e}_L^-\gamma_\mu e_L^- - \bar{E}_L^-\gamma_\mu E_L^- - \bar{N}_{4L}^0\gamma_\mu N_{4L}^0) \\
&+ T_{2W}^{-1}(\bar{d}_L\gamma_\mu d_L - \bar{E}_L^+\gamma_\mu E_L^+ - \bar{\nu}_{eL}\gamma_\mu \nu_{eL} - \bar{N}_{2L}^0\gamma_\mu N_{2L}^0) \\
&- T_W^{-1}(\bar{D}_L\gamma_\mu D_L - \bar{e}_L^+\gamma_\mu e_L^+ - \bar{N}_{1L}^0\gamma_\mu N_{1L}^0 - \bar{N}_{3L}^0\gamma_\mu N_{3L}^0),
\end{aligned} \quad (21)$$

where $S_{2W} = 2S_W C_W$, $T_{2W} = S_{2W}/C_{2W}$, $\bar{N}_2^0\gamma_\mu N_2^0 = \bar{N}_{2L}^0\gamma_\mu N_{2L}^0 + \bar{N}_{2R}^0\gamma_\mu N_{2R}^0 = \bar{N}_{2L}^0\gamma_\mu N_{2L}^0 - \bar{N}_{2L}^{0c}\gamma_\mu N_{2L}^{0c} = \bar{N}_{2L}^0\gamma_\mu N_{2L}^0 - \bar{N}_{4L}^0\gamma_\mu N_{4L}^0$, similarly $\bar{E}\gamma_\mu E = \bar{E}_L^-\gamma_\mu E_L^- -$

$\bar{E}_L^+ \gamma_\mu E_L^+$ and $T_{3f} = Dg(1/2, -1/2, 0)$ is the third component of the weak isospin acting on the representation 3 of $SU(3)_L$ (the negative when acting on $\bar{3}$). Notice that $J_\mu(EM)$ and $J_\mu(Z)$ are just the generalization of the electromagnetic and neutral weak currents of the SM, as they should be, implying that Z_μ can be identified as the neutral gauge boson of the SM.

A.3 Radiative masses for fermion fields

Using the currents in the previous section and the off diagonal entries in matrix in Eq.(14), we may draw the four diagrams in Fig. 1 which allow for non diagonal entries in the mass matrix for the down quark sector of the form $(\Delta_D D_L d_R + h.c.)$ and $(\Delta_d d_L D_R + h.c.)$ respectively, which in turn produce a radiative mass for the ordinary down quark. Notice that due to the presence of $K_R^{0\mu}$ in the graphs, mass entries of the form $d_L d_R$ and $D_L D_R$ are not present.

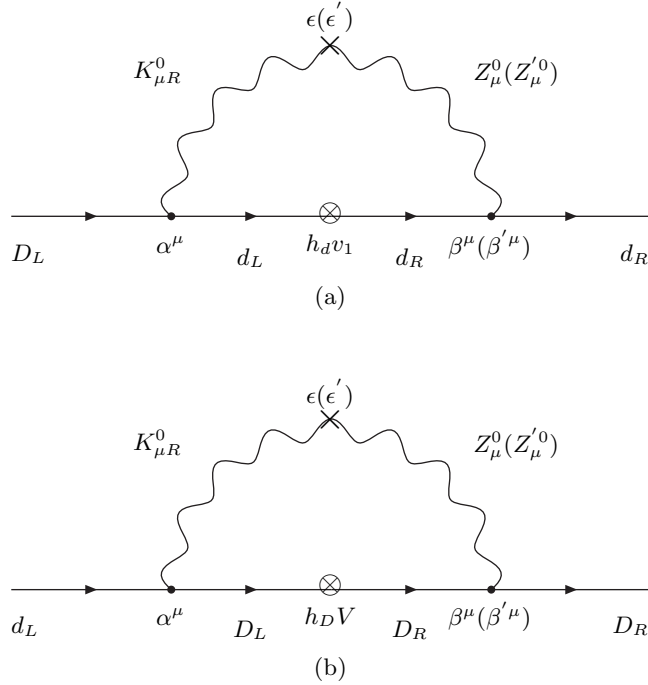


Fig.1. Four one-loop diagrams contributing to the radiative generation of the ordinary down quark mass. For the meaning of α , β and ϵ see the main text.

The equations in this paper imply for the diagrams in Fig. 1 that: $\alpha_\mu = g\gamma_\mu/2$, $\beta_\mu = g\gamma_\mu S_W T_W/3$, $\beta'_\mu = -g'\gamma_\mu T_W/\sqrt{27}$, $\epsilon = -C_W v_1 V$ and $\epsilon' = C_W v_1 V/\sqrt{4C_W^2 - 1}$.

In a similar way we achieve radiative masses for the charged lepton and for the Majorana neutrino. The detailed analysis for these leptons will be presented elsewhere.

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