Fragmentation Production of Ω_{ccc} and Ω_{bbb} Baryons

M.A. Gomshi Nobary

Department of Physics, Faculty of Science, Razi University,Kermanshah, Iran.

The Center for Theoretical Physics and Mathematics, A.E.O.I., Roosbeh Building, P.O. Box 11365-8486 Tehran, Iran.

E-mail:mnobary@aeoi.org.ir

Abstract

The Λ baryons with a single heavy flavor which transfer the quark polarization, have been studied both theoretically and experimentally. The Ξ's with two heavy constituents are well treated in quark-diquark model. In this work we study the production of triply heavy baryons in the perturbative QCD regime and calculate the fragmentation functions for Ω_{ccc} and Ω_{bbb} in the c and b quark fragmentation, respectively. We then obtain the total fragmentation probability and the average fragmentation parameter for each case.

Keywords: Fragmentation; Heavy Quark; Perturbative QCD

1 Introduction

The quark model of hadrons has proved to be successful in describing hadrons and their properties. In the heavy quark sector it predicts hadrons having c, b and t quarks as constituents. However, the discovery of the top quark [1] and the determination of its lifetime [2] made it clear that it cannot participate in strong interactions and therefore only the c and b flavors are left to take part in the hadron production interplay.

Recently meson states constituting heavy flavor have received considerable attention. Specially B_c and B_c^* states with $\bar{b}c$ quark content have been in focus both theoretically [3] and experimentally [4] in the last few years. It is established that the fragmentation functions describing their production mechanism are calculable in perturbative QCD [3] and hence the total fragmentation probabilities and production cross sections are calculated in due course.

Baryon states with heavy flavor fall into three categories. States containing one heavy flavor such as Λ_c and Λ_b are interesting states due to the fact that they carry the original heavy flavor polarization. They are presently being studied experimentally [5]. The second category involves baryons with two heavy flavor like the states Ξ_{cc}, Ξ_{bb} and Ξ_{bc} [6]. They are treated within the approximate quark-diquark model [7]. The model treats the production of the so called diquark perturbatively similar to the states such as B_c. Then, it can be proved that the formation of a baryon out of the diquark is almost the same as the fragmentation of an antiquark into a meson. In this way one obtains the fragmentation functions, the total production probabilities and other relevant parameters which specify their properties. In the third category, we have baryons with three heavy constituents. If we follow the scheme used in the case of heavy mesons and assume that their fragmentation functions are calculable in the perturbative regime, then we can calculate Feynman diagrams like the one in figure 1 to obtain the fragmentation functions. There are eight such diagrams in the lowest order contributing triply heavy baryons, i.e. Ω_{ccc}, Ω_{ccb}, Ω_{cbb} and Ω_{bbb} production [8].

In this paper our aim is to calculate the fragmentation of the Ω_{ccc} and Ω_{bbb} baryons in the lowest order perturbative regime and obtain their fragmentation functions in an exact analytical form.

2 Kinematics

We consider the fragmentation of a heavy quark Q into a QQQ system with three identical flavor. This procedure is illustrated in Figure 1. We have used an infinite momentum frame in which all of the particles are

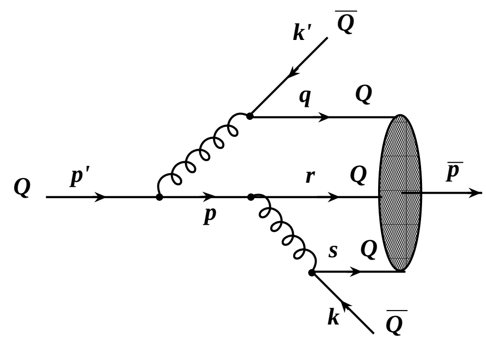

Fig. 1. Feynman diagram illustrating the lowest order fragmentation of a heavy quark, Q, into a Ω_{QQQ} baryon. The four momenta are labelled.

moved in the forward direction, i.e. the longitudinal direction along the z axes, where the QQQ moves. We let the original quark keep its transverse momentum. Furthermore, we assume that the two antiquark jets move almost in the same direction. This assumption is justified due to the fact that the very high momentum of the initial heavy quark will predominantly be carried in the forward direction. Due to momentum conservation, the total transverse momentum of the two jets will be identical to the transverse momentum of the initial quark. In this context the four momenta of the particles will assume the following form

$$\begin{aligned} p'_\mu &= [p'_\circ, \mathbf{p'}_T, p'_L],\ q_\mu = [q_\circ, \mathbf{0}, q_L],\ r_\mu = [r_\circ, \mathbf{0}, r_L] \\ s_\mu &= [s_\circ, \mathbf{0}, s_L],\ k_\mu = [k_\circ, \mathbf{k}_T, k_L],\ k'_\mu = [k'_\circ, \mathbf{k'}_T, k'_L]. \end{aligned} \tag{1}$$

We have used the fragmentation parameter, z, as defined in the literature, i.e.,

$$z = \frac{(E + p_\parallel)_B}{(E + p_\parallel)_Q} = \frac{E_B}{E_Q}. \tag{2}$$

The last step follows form application of the infinite momentum frame.

Therefore, the final state particle energies are parameterized as follows

$$\overline{p}_\circ = zp'_\circ, \tag{3}$$

where $\overline{p}_\circ = r_\circ + q_\circ + s_\circ$ is the energy of the baryon. Therefore

$$r_\circ = x_1 zp'_\circ, \quad q_\circ = x_2 zp'_\circ, \quad s_\circ = x_3 zp'_\circ. \tag{4}$$

Here the x's are the energy ratios carried by the constituents. Since the constituents are identical and fly together, it is found that $x_1 = x_2 = x_3 = 1/3$. This is consistent with our argument about the wave function for such states in the next section. We also have assumed that the two anti-quarks which initiate the two jets have equal energies, i.e.

$$k_\circ = k'_\circ = \frac{1}{2}(1-z)p'_\circ. \tag{5}$$

On the other hand due to our discussion about transverse momentum, we have

$$\mathbf{k}_T = \mathbf{k'}_T = \frac{1}{2}\mathbf{p'}_T. \tag{6}$$

We will discuss this later assumption in the final section.

3 Calculation of the Fragmentation Functions for Ω_{QQQ}

We are now ready to calculate the diagram shown in figure 1. The fragmentation of a heavy quark Q into a heavy baryon Ω_{QQQ} is obtained by squaring the total amplitude and integrating over final state phase space,

$$D_Q^B(z,\mu_\circ) = \frac{1}{2}\sum_s \int |T_B|^2 \delta^3(\overline{\mathbf{p}} + \mathbf{k} + \mathbf{k'} - \mathbf{p'}) \mathrm{d}^3\overline{\mathrm{p}}\mathrm{d}^3\mathrm{k}\mathrm{d}^3\mathrm{k}', \tag{7}$$

where T_B is the amplitude of the baryon production which involves the hard scattering amplitude T_H and the non-perturbative smearing of the bound state. The average over initial spin states and the sum over final spin states are performed. The heavy hadron production amplitude is composed of a partonic part, which can be calculated using perturbative QCD, and a non-perturbative part, which describes the transition of free quarks into the final state hadron. In the framework of non-relativistic quark model, this non-perturbative part could be accounted for through the wave function which is calculable using potential models. Since at present there is no known information concerning such wave functions, we have assumed a delta function type wave function for them. This assumption guarantees that the constituents will fly parallel and have no transverse momentum with respect to their direction of motion. This is also consistent with our assumptions in section 2. The hard scattering amplitude which is obtained by perturbative calculations of the tree diagram in figure 1, may be put in the following form [9],

$$T_H = \frac{24\pi^2\alpha_s^2 m^4 C_F}{\sqrt{2p'_\circ \overline{p}_\circ k_\circ k'_\circ}} \frac{\Gamma}{g_1(z)g_2(z)g_3(z)(\overline{p}_\circ + k_\circ + k'_\circ - p'_\circ)}. \tag{8}$$

Here $\alpha_s = g^2/4\pi$ is the strong interaction coupling constant and Γ indicates that part of the amplitude which embeds spinors and gamma matrices. The $1/g$'s are the propagators of the two gluons and the intermediate fermion respectively.

To absorb the soft behavior of the bound state into hard scattering amplitude we have used the scheme introduced in [7]. The probability amplitude at large momentum transfer factories into a convolution of the hard-scattering amplitude T_H, and baryon-distribution amplitude ϕ_M [10], i.e.,

$$T_B(k_i, p_i) = \int [dx] T_H(k_i, p_i, x_i)\phi_B(x_i, q'^2), \tag{9}$$

where T_H is given by (8) and ϕ_B is the probability amplitude to find quarks co-linear up to a scale q'^2 in the baryonic bound state. In (9), x_i's are the momentum fractions carried by the constituent quarks and

$[dx] = dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)$. In view of our early discussion in this section, we propose the following expression for the probability amplitude

$$\phi_B = f_B\, \delta\Big\{x_i - \frac{m_i}{m_B}\Big\}, \tag{10}$$

where m_B is the baryon mass and f_B refers to the characteristics of the baryon bound state and is similar to the meson bound state where the decay constant f_M is introduced. Putting this expression and (8) in (9) and carrying out the necessary integrations, we find

$$T_B = \frac{24\pi^2\alpha_s^2 m^4 f_B C_F}{\sqrt{2p'_\circ \overline{p}_\circ k'_\circ k_\circ}} \frac{\Gamma}{g_1(z)g_2(z)g_3(z)(\overline{p}_\circ + k_\circ + k'_\circ - p'_\circ)}. \tag{11}$$

Now we are able to obtain the fragmentation function in (7) as

$$\begin{aligned} D(z,\mu) = & \frac{(48\pi^2\alpha_s^2 m^4 f_B C_F)^2}{8} \\ & \times \int \frac{\frac{1}{2}\sum_s \overline{\Gamma}\Gamma \delta^3(\overline{\mathbf{p}} + \mathbf{k} + \mathbf{k}' - \mathbf{p}')\mathrm{d}^3\overline{\mathrm{p}}\mathrm{d}^3\mathrm{k}\mathrm{d}^3\mathrm{k}'}{\overline{p}_\circ p'_\circ k_\circ k'_\circ [g_1(z)g_2(z)g_3(z)(\overline{p}_\circ + k_\circ + k'_\circ - p'_\circ)]^2}. \end{aligned} \tag{12}$$

Spin sum-average of $\overline{\Gamma}\Gamma$ for Figure 1 is easily calculated using the REDUCE. To do the phase space integrations in (12), first we consider the integral,

$$\begin{aligned} I = & \int \frac{\delta^3(\overline{\mathbf{p}} + \mathbf{k} + \mathbf{k}' - \mathbf{p}')\mathrm{d}^3\overline{p}}{p'_\circ(\overline{p}_\circ + k_\circ + k'_\circ - p'_\circ)^2} \\ = & \frac{p'_\circ}{f(z)^2}, \end{aligned} \tag{13}$$

where

$$f(z) = -\frac{{p'_T}^2}{3m^2} + \frac{3}{z} + \frac{4}{3}\Big(1 + \frac{{p'_T}^2}{4m^2}\Big)\frac{1}{1-z}. \tag{14}$$

Also we note that

$$\int f(z,\mathbf{k}_T)d^3\mathbf{k} = \int f(z,\mathbf{k}_T)\mathrm{d}k_L\mathrm{d}^2k_T$$
$$= m^2k_\circ f(z,\langle {k_T}^2\rangle) = m^2k_\circ f(z,\frac{1}{2}\langle {p'_T}^2\rangle), \tag{15}$$

and

$$\int f(z,\mathbf{k'}_T)d^3\mathbf{k'} = \int f(z,\mathbf{k'}_T)\mathrm{d}k'_L\mathrm{d}^2k'_T$$
$$= m^2k'_\circ f(z,\langle {k'_T}^2\rangle) = m^2k'_\circ f(z,\frac{1}{2}\langle {p'_T}^2\rangle). \tag{16}$$

Here, instead of performing transverse momentum integrations, for simplicity we have replaced them by their average values. Putting all this back in (12), we obtain the fragmentation function as,

$$\begin{aligned} D_{Q\to QQQ}(z,\mu_\circ) = & \frac{\pi^4\alpha_s^4 f_B^2 C_F^2}{108m^2z^4(1-z)^4f(z)^2g(z)^6} \\ & \times\Big[\xi^8z^8 + 4\xi^6z^6(83-130z+51z^2) \\ & +6\xi^4z^4(1413-3084z+3022z^2-2156z^3+821z^4) \\ & +4\xi^2z^2(18711-51678z+69417z^2-70308z^3 \\ & +53529z^4-25950z^5+6343z^6)+222345-740664z \\ & +1179036z^2-1253448z^3+90126z^4-388872z^5 \\ & +109916z^6-49912z^7+20649z^8\Big]. \end{aligned} \tag{17}$$

Here we have defined $\xi = \langle p'^2_T\rangle/m^2$. $g(z)$ comes from the propagators and have the following form

$$g(z) = 1+\frac{3}{z}+\frac{4}{3}\Big(1+\frac{\mathbf{p}'^2_T}{4m^2}\Big)\frac{z}{1-z}. \tag{18}$$

and $f(z)$ is due to the energy denominator given by (14). Replacement of $f(z)$ by $g(z)$ which we have done in the original manuscript, changes the fragmentation function only slightly.

The fragmentation function $D_{c\to\Omega_{ccc}}(z,\mu_\circ)$ and $D_{b\to\Omega_{bbb}}(z,\mu_\circ)$ are easily obtained from the above by letting $m = m_c, m_b$ and using appropriate f_B, α_s and $\mu_\circ$ values.

4 Results and Discussion

We were able to calculate the process of direct c and b quark fragmentation into Ω_{ccc} and Ω_{bbb} baryons. In doing so we had to follow certain assumptions. Firstly we have considered only the dominant contributing Feynman diagram in leading order. This assumption reduced the complexity and the length of the calculation and enabled us to obtain analytic forms of the fragmentation functions. Our second assumption concerns kinematics. We believe that the high momentum of the process has to be taken away in the forward direction and let the two antiquarks carry the transverse momentum of the initial heavy quark. Furthermore since they are identical, we have considered equal contribution from them both in magnitude an in direction. Therefore, we have established equations (5) and (6) and used them in our calculation. To see how our later assumption works, we have set the kinematics by allowing k and k' to share the jet energy-momentum. We have let $k = x(1-z)p'$ and $k' = (1-x)(1-z)p'$ where x is a variable which is between zero and one. We have repeated our calculations and studied the behaviour of Ω_{ccc} fragmentation function with the same parameters as before. It is revealed that as x increases, the function grows rapidly and gives the highest peak at $x = 1/2$. As x increases further, the peak falls rapidly. Since there is not much information about the wave functions of the triply heavy baryons at hand, we have reduced the non-perturbative smearing of the bound state to a delta function times a factor which is much like the meson decay constant. We have denoted this constant by f_B and assumed to take 0.25 GeV both for Ω_{ccc} and Ω_{bbb} baryons.

In obtaining (17) we have not performed the transverse momentum integrations. Instead we have replaced the variables by their average values. However the numerical integration converges well for sufficiently large transverse momentum. Let us now sketch the behaviour of our frag-

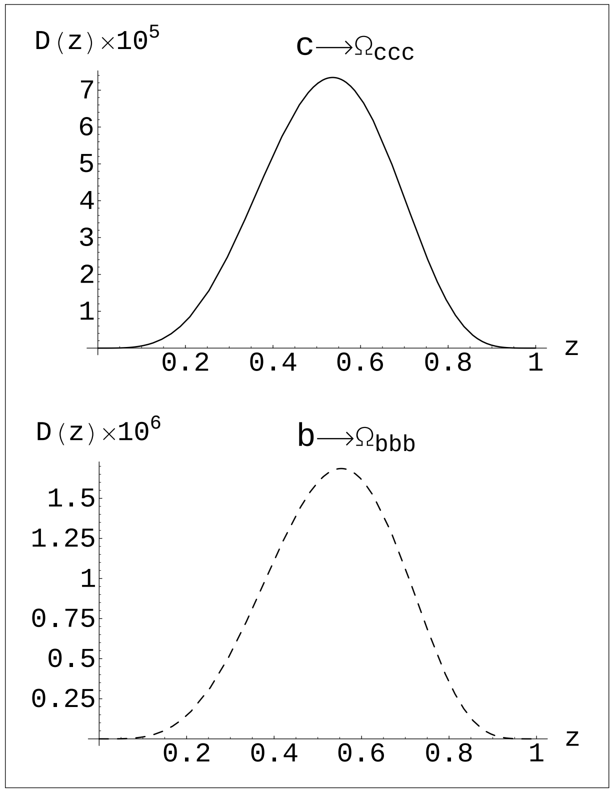

Fig. 2. The behavior of Ω_{ccc} (solid) and Ω_{bbb} (dashed) fragmentation function at the respective fragmentation scale.

mentation functions. Figure 2 shows the behaviour of $D_{c\to\Omega_{ccc}}(z,\mu_\circ)$ and $D_{b\to\Omega_{bbb}}(z,\mu_\circ)$ in the fragmentation scale $\mu_\circ$. In drawing them we have assumed that $m_c = 1.25$ GeV and $m_b = 4.25$ GeV. The scales are $\mu_\circ$=6.25 GeV and $\mu_\circ$=21.25 GeV respectively. We have set $\langle {p'_T}^2\rangle$= 1 GeV and included the colour factor of C_F=7/6 obtained using color line counting rule. Consistent with the study of B_c and B_c^* states, we have taken $\alpha_s = 0.26$ for Ω_{ccc} and $\alpha_s = 0.18$ for Ω_{bbb} [3].

At leading order in α_s one has $\int_0^1 P_{Q\to Q}(z,\mu)\mathrm{d}z = 0$ [12], and the evolution equation implies that the fragmentation probability $\int_0^1 D_{Q\to B}(z,\mu)\mathrm{d}z$ does not evolve with the scale μ. Therefore, the fragmentation probability is a universal characteristic of the production rates. The evolution only moves the z-distribution to small values of z. We have obtained this quantity for Ω_{ccc} and Ω_{bbb} using our fragmentation functions. The other relevant kinematical parameter is the average fragmentation parameter. Our results for the fragmentation probabilities and $\langle z\rangle$ appear in Table 1. It is seen that our analysis give very close $\langle z\rangle$ values for Ω_{ccc} and

Table 1
Fragmentation probability and $\langle z \rangle$ for different states.

	Ω_{ccc}	Ω_{bbb}
Frag. Prob.	2.789×10^5	6.459×10^7
$\langle z \rangle$	0.522	0.535

Ω_{bbb}. The fragmentation probabilities in Table 1 suggest that considerable event rate is expected both at the Tevatron and the LHC .

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