

# Dual string from lattice Yang-Mills theory

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**Abstract.** We review properties of lower-dimension vacuum defects observed in lattice simulations of SU(2) Yang-Mills theories. One- and two-dimensional defects are associated with ultraviolet divergent action. The action is the same divergent as in perturbation theory but the fluctuations extend over submanifolds of the whole 4d space. The action is self tuned to a divergent entropy and the 2d defects can be thought of as dual strings populated with particles. The newly emerging 3d defects are closely related to the confinement mechanism. Namely, there is a kind of holography so that information on the confinement is encoded in a 3d submanifold. We introduce an SU(2) invariant classification scheme which allows for a unified description of  $d = 1, 2, 3$  defects. The scheme fits known data and predicts that 3d defects are related to chiral symmetry breaking. Relation to stochastic vacuum model is briefly discussed as well.

## INTRODUCTION

Studies of the confinement mechanism have become since long a prerogative of the lattice simulations, for a recent review see [1]. The continuum theory provided in fact little guidance for search of the confinement mechanism. Equations which one borrows from the continuum physics refer mostly to U(1) Higgs models or instantons, see, e.g., [2]. However, these hints from the continuum theory could be used at a qualitative level at best.

Painstaking analysis of the lattice simulations did allow to extract vacuum fluctuations which are actually responsible for the confinement. These are so called monopoles and central vortices, for review see, e.g., [3] and [1, 4]. By construction, monopoles are infinitely thin closed trajectories while the central vortices are infinitely thin closed 2d surfaces<sup>1</sup>. Separation of the two types of the defects is actually superficial. Rather, one observes vortices populated with monopoles. Monopoles live on 2d surfaces, not in the whole 4d space and there can be no vortices without monopoles, [6, 7].

Infinitely thin (with size of the lattice spacing  $a$ ), percolating trajectories and surfaces look very different from, say, instantons which are bulky fields, with size of order  $\Lambda_{QCD}^{-1}$ . Thus, one is tempted to say that lattice simulations uncovered existence of objects of lower dimensions in the vacuum state of Yang-Mills theories. However, a prevailing viewpoint, for a recent presentation see, e.g., [2], is that apparent point-likeness of the monopoles and vortices is an artifact of their definition and in fact they only mark some

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<sup>1</sup> both trajectories and surfaces are defined actually on the dual lattice.

bulky field fluctuations.

It is only rather recently that it was understood that the monopoles and vortices might still be physical lower-dimensional defects. The basic observation which brings about such a conclusion is the ultraviolet divergence in the corresponding *non-Abelian* action<sup>2</sup> associated with the monopoles [8] and vortices [6]. The power of the ultraviolet divergence in the action is the same as for pointlike particles and infinitely thin strings, respectively. To explain the survival of the monopoles and vortices on the  $\Lambda_{QCD}$  scale – despite of their ultraviolet divergent action – one is forced to postulate [9] self-tuning of the ultraviolet divergent action and of ultraviolet divergent entropy<sup>3</sup>. Moreover, the d=2 defects appear to be nothing else but dual strings with excitations of scalar field living on them.

The ultraviolet divergences in the action are the earliest evidence in favor of relevance of singular fields to confinement. There exist further observations [11, 12, 13] indicating that lower dimensional defects are of physical significance<sup>4</sup>.

When the lattice studies were undertaken first, there was no theory of extended objects at all. However, more recently the idea that strings are relevant to QCD has become quite common. More specifically, one expects that if a dual formulation of YM theories exists, it would be a string theory [14]<sup>5</sup>.

Thus, there appears a possibility that the languages of lattice and continuum theories would get unified again, this time in terms of theory of extended objects. A possible feedback from lattice studies to the continuum theory is that topological excitations observed within a ‘direct’ formulation might become fundamental variables of the dual formulation of the same theory, see, e.g., [15]. Thus, if the strings are indeed observed as excitations in lattice simulations of YM theories, this is an indication that there exists a dual formulation in terms of fundamental strings [16].

Here we address a problem of reformulating some of the lattice results in terms of the continuum theory. The point is that many results, especially on the lattice strings, are obtained originally in terms of so called projected fields, see, e.g., [1]. We will discuss a classification scheme of the defects in explicitly SU(2) invariant terms. Also we will comment on possible relation to the stochastic picture of the vacuum [17, 18].

## LATTICE STRINGS

We have reviewed recently the properties of the two-dimensional defects, or lattice strings [19] and will be brief here.

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<sup>2</sup> It is worth emphasizing that the non-perturbative ultraviolet divergent fields are no more divergent than perturbation theory, for details see [10].

<sup>3</sup> Visible entropy of the 2d and 1d defects explodes exponentially with the lattice spacing  $a \rightarrow 0$  [7].

<sup>4</sup> Note that for lower-dimension defects to be relevant the corresponding fields are to be singular.

<sup>5</sup> Usually one believes that it is only in the limit of infinite number of colors,  $N_c \rightarrow \infty$ , that one might find a dual formulation.

## Magnetic monopoles

Theoretically, the most difficult point about the monopoles is their definition on the lattice. Monopoles are topological excitations of the compact  $U(1)$  [20]. To define them in non-Abelian case one uses projection of the original YM fields onto the ‘closest’ Abelian configuration. The physical idea behind considering the monopoles is that confinement is mostly due to Abelian degree of freedom [21].

While the definition of the monopoles is not so transparent, many observed properties are beautiful and formulated in perfectly  $SU(2)$  invariant way. Monopoles are observed as clusters of trajectories. Infinite, or percolating cluster corresponds to the classical expectation value,  $\langle \phi_M \rangle$  of a magnetically charged field  $\phi_M$ . Short, or ultraviolet clusters correspond to quantum fluctuations of the field  $\phi_M$ . The total length of the clusters is trivially proportional to the total volume of the lattice,  $V_4$ :

$$L_{tot} = 4\rho_{tot} \cdot V_4 = 4(\rho_{perc} + \rho_{finite}) \cdot V_4 . \quad (1)$$

According to the data [5]:

$$\rho_{tot} \approx 1.6(fm)^{-3} + 1.5(fm)^{-2} \cdot a^{-1} , \quad (2)$$

where  $a$  is the lattice spacing. The  $a^{-1}$  term is entirely due to the finite clusters. For the percolating cluster the density is a constant in the physical units.

One can translate (2) into the standard field theoretic language by observing that [9]:

$$\langle |\phi_M|^2 \rangle = (const)a \cdot \rho_{tot} . \quad (3)$$

Thus, we have

$$\langle |\phi_M|^2 \rangle \sim \Lambda_{QCD}^2 . \quad (4)$$

Theoretically, the estimate (4) can be derived as a constraint implied by the asymptotic freedom of YM theories [10].

## Point-like facet of the monopoles

The monopoles action diverges with  $a \rightarrow 0$  and the power of the divergence is the same as for point-like particles [8]:

$$S_{mon} \equiv M \cdot L_{mon} , \quad M(a) \approx \ln 7 \cdot a^{-1} , \quad (5)$$

where  $M(a)$  corresponds to the radiative mass and is found by measuring extra *non-Abelian* action associated with the monopoles. We quote the data in a way which allows for a straightforward theoretical interpretation. Namely, in field theory (see, e.g., [22]) if one starts with the classical action of a particle,  $S = M \cdot L$  the propagating mass is not  $M$  but:

$$m_{prop}^2 = \frac{(const)}{a} \left( M(a) - \frac{\ln 7}{a} \right) , \quad (6)$$

where the constants  $const, \ln 7$  are of pure geometrical origin and depend on the lattice used. In particular,  $\ln 7$  corresponds to the hypercubic lattice. Note that in Euclidean space a physical mass of a point-like particle can appear only as a result of tuning between divergent action and entropy.

Thus, the data (5) correspond to a small monopole mass. Moreover, data (2) imply that globally monopoles live on a 2d surface. For ordinary point-like particles  $\rho_{tot} \sim a^{-3}$ .

## Closed strings

Closed surfaces are topological defects of the  $Z_2$  gauge theory. In simulations of SU(2) theory these surfaces are defined in terms of the closest  $Z_2$  projection which replaces the original YM fields with  $Z_\mu(x) = \pm 1$ . The central vortices are defined as unification of all the plaquettes on the dual lattice which pierce negative plaquettes in the  $Z_2$  projection, for review see [1, 4].

Two most striking properties of the central vortices is that their total area scales in physical units, for review see [1, 4] while non-Abelian action is ultraviolet divergent [6]:

$$A_{tot} \approx 4 (fm)^{-2} V_4, \quad S_{tot} \approx 0.54 \frac{A_{tot}}{a^2}. \quad (7)$$

Moreover, the excess of the action disappears on the plaquettes next to those belonging to the vortex. In other words, the vortices are infinitely thin, at least on the presently available lattices.

It is worth emphasizing that the properties (7) amount to observing an elementary string. Indeed, the data on the total area imply that the tension is of order  $\Lambda_{QCD}^2$  while the ultraviolet divergence in the action assumes vanishing thickness. The suppression due to the action is to be compensated by enhancement due to the entropy. Fine tuning of the entropy and action is a generic feature of any consistent theory of an elementary string in Euclidean space.

Another striking feature of the lattice strings is that the monopole trajectories, discussed in the preceding subsection, lie in fact on the central vortices [6, 7, 23]. Thus, the two types of defects merge with each other.

## THREE DIMENSIONAL DOMAINS

The 3d defects are more recent than the strings and have been studied in less detail. Moreover, there are a few independent pieces of evidence in favor of existence of 3d defects which are, in fact, not necessarily related to each other.

### ‘Strong’ potentials

The central vortices are defined in terms of negative plaquettes in  $Z_2$  projection. In  $Z_2$  projection links take on values  $\pm 1$ . Generically, the values  $(+1)$  and  $(-1)$  are the same

frequent. One can, however, minimize the number of negative links, using remaining  $Z_2$  invariance. Physicswise, one fixes the gauge by localizing large potentials on as a small number of links as possible. Since link values correspond to potentials and are gauge dependent, one can wonder what is the objective meaning of such minimization. The point is that minimizing, say, potential squared one arrives at a gauge invariant quantity [24]. Minimizing number of negative links is a variation of such a procedure.

And, indeed, one finds [12] that volume of negative links scales as a physical 3d defect:

$$V_3 = c_3 \Lambda_{QCD} \cdot V_4 . \quad (8)$$

Note that by construction the volume is bound by the central vortices. This volume can be called Dirac volume [1]. Eq (8) then states that the minimal Dirac volume scales in physical units or, alternatively, has a zero fractal dimension.

## Holography and confinement

Relation of the volume discussed above to the confinement is revealed through a remarkable observation of the authors of Ref [25]. One replaces the original link matrices  $U_\mu(x)$  by  $\tilde{U}_\mu(x)$  where

$$\tilde{U}_\mu(x) \equiv U_\mu(x) \cdot Z_\mu(x) , \quad (9)$$

where  $Z_\mu(x)$  is the projected value of the same link. Next, one evaluates the Wilson loop and quark condensate  $\langle \bar{q}q \rangle$  in terms of the modified links  $\tilde{U}$ . The result [25] is that both the confining potential and spontaneous breaking of the chiral symmetry disappear.

Originally [25] the change (9) affected approximately half of the total number of links. Now, we see that it is enough to perform the change (9) on a 3d submanifold to kill the confinement and chiral symmetry breaking. In other words, substitution (9) is an ad hoc modification in the ultraviolet of the fields on a 3d volume plus pure gauge transformations. Thus, we observe a kind of holography, with information on the confinement being encoded on a submanifold of the whole space.

In more detail, consider a plane on which we will draw a Wilson line. Consider, furthermore, a particular configuration of the gauge fields generated with the standard SU(2) action. Determine then the 3d volume described in the preceding section. Intersections of this volume with the plane considered are segments of 1d lines. Now, we can draw any Wilson line on the plane. The statement is that the sign of the Wilson line can be determined by counting the number of intersections with segments of 1d defects. It is a highly non-trivial observation, challenge to interpret. Note that there is no logical contradiction, though. Indeed, there are gauges where the confining fields are soft, of order  $A_\mu \sim \Lambda_{QCD}$ . Apparently, one can use gauge invariance to choose a gauge where the confining fields are of order  $A_\mu \sim 1/a$  but occupy a 3d volume <sup>6</sup>.

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<sup>6</sup> Some considerations on possible relation between gauge invariance and holography in the gravitational case can be found in [26]

## Chiral symmetry breaking

There is a series of observations, not directly related to each other that indicate relevance of some 3d defects to the spontaneous breaking of the chiral symmetry <sup>7</sup>:

(a) procedure of Ref [13] described above makes also the quark condensate vanish:

$$\langle \bar{q}q \rangle_{\bar{U}} \approx 0 \quad (10)$$

Now, we know [12] that the change (9) affects not a finite part of the 4d space but only a 3d submanifold.

(b) there is evidence in favor of long range topological structure in QCD vacuum which is related to chiral symmetry breaking [11]. The search process for the topological structure is formulated in terms of eigenfunctions of the Dirac operator and explicitly gauge invariant <sup>8</sup>.

(c) One introduces the so called inverse participation ratio, see in particular [27], defined in terms of eigenfunctions of the Dirac operator:

$$I = N \sum_x \rho_i^2(x) , \quad (11)$$

where  $N$  is the number of lattice sites  $x$ ,  $\rho_i(x) = \psi_i^\dagger \psi_i(x)$ , and  $\psi_i(x)$  is the  $i$ -th normalized ( $\sum_x \rho_i(x) = 1$ ) lowest eigenvector of the Dirac operator.

Dependence of the inverse participation ratio on the lattice spacing  $a$  was studied in Ref [13]. The result is:

$$\langle I \rangle = c_1 + c_2 \cdot a^{-\gamma} , \quad (12)$$

with a non-vanishing exponent  $\gamma$ :

$$1 \leq \gamma \leq 2 .$$

Note that the value  $\gamma = 1$  would correspond, in the limit  $a \rightarrow 0$  to localization of the eigenfunctions on a 3d volume. It is worth emphasizing that the  $a$  dependence observed refers to an explicitly gauge invariant quantity.

To summarize, there are indications that the chiral symmetry breaking is determined by gauge fields living on a subspace. Since the confinement itself also seems to be related to a 3d volume (see above), it is not clear whether we deal with a phenomenon specific for chiral symmetry breaking or with an effect common to confinement.

## CLASSIFICATION SCHEME

### Invariants

There is no theory of the defects in the non-Abelian case. However, even in the absence of such a theory one can try to find a  $SU(2)$  invariant classification scheme.

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<sup>7</sup> We are considering the quenched approximation.

<sup>8</sup> Moreover, measurements [11] refer to  $SU(3)$  color group.

Generically, the first example of such a scheme for monopoles was proposed long time ago [28]. In pure YM theory, there are no classical monopole solutions. However, imagine that there exists a scalar field, vector in the color space  $H^a$ ,  $a = 1, 2, 3$ . Then one could fix the gauge by rotating vector  $H^a$  to the third direction at each point. This fixation of the gauge would fail however at the points where

$$H^a = 0 . \quad (13)$$

Condition (13) can be viewed as three equations defining 1d defects in the 4d space which can be identified with monopole trajectories [28]. It is crucial that (13) is SU(2) invariant.

For various reasons, this idea does not seem to work in the realistic case, for review and references see [29]. Rather, monopoles are associated with singular non-Abelian fields (see above). Let us try to adjust the classification scheme to this set up [19, 29, 30].

Begin with YM theory in three dimensions and assume that monopoles violate the Bianchi identities. If the Bianchi identities

$$D\tilde{G} = 0 , \quad (14)$$

hold, the potential  $A$  can be expressed in terms of the field strength tensor, see, e.g., [31]:

$$A = \frac{1}{g}(\partial\tilde{G})\tilde{G}^{-1} . \quad (15)$$

The inverse matrix exists unless the determinant constructed on the components of  $\tilde{G}$  vanishes. Denoting  $\tilde{G}_{ik}^a \equiv \varepsilon_{ikl}B_l^a$  we have, therefore, the following condition for the Bianchi identities to be violated:

$$\det(B_i^a) \equiv \varepsilon_{ikl}\varepsilon_{abc}B_i^aB_k^bB_l^c = 0 . \quad (16)$$

Note that the condition (16) is perfectly gauge and rotation invariant. Moreover, it singles out a surface (or a line on the dual lattice) while monopoles are usually 0d defects in the 3d case.

Let us now consider the 4d Euclidean case. The rotational group in 4d splits into a product of two  $O(3)$  groups,  $O(4) = O(3) \times O(3)$ . The corresponding representations of the  $O(3)$  groups are chiral gluon fields ( $H_i^a \pm E_i^a$ ). Looking for a generalization of (16) we notice that there are now two possibilities:

$$\det(E_i^a + H_i^a) = 0 , \text{ or } \det(E_i^a - H_i^a) = 0 . \quad (17)$$

Imposing either of them we specify a 3d defect. On this 3d submanifold one can use as independent three fields of a certain chirality but not of the opposite one. Thus, association of 3d defects with chiral symmetry breaking arises as a consequence of the symmetry of the problem.

The boundary of these 3d defects is determined by conditions;

$$\det(E_i^a + H_i^a) = 0 , \text{ and } \det(E_i^a - H_i^a) = 0 . \quad (18)$$

which determine 2d defects. Moreover, if both conditions (18) are satisfied, there is no inversion of the Bianchi identities similar to (15).

Finally, zeros of a second order of the determinant would define 1d defects. They automatically fall onto the 2d defects as well.

### Classification scheme vs data

The classification scheme proposed above is based on symmetry alone and is not unique. But, nevertheless, let us try to identify the 2d and 1d defects arising within this scheme with the central vortices and monopoles. There are a few quite remarkable confirmations of such an identification:

(a) the 2d defects are associated, according to the scheme, with singular fields and, possibly, violations of the Bianchi identities. And, indeed, the central vortices carry a singular action [6]. Moreover, monopoles live on the vortices, on one hand, and may well signify violation of the Bianchi identities, on the other;

(b) non-Abelian fields associated with the 2d defects are aligned with the surface. This is confirmed by the measurements, according to which the excess of the action vanishes already on the plaquettes next to the central vortices [6];

(c) the monopole trajectories are predicted to lie on the central vortices, in agreement with the data [23, 6];

(d) ‘monopoles’ appear to be Abelian fields since zero of second order of the determinant constructed on three independent (within a 3d defect) fields implies that there is only a single independent color vector. Thus, monopoles can well be detected through the U(1) projection.

(e) on the other hand, the non-Abelian field of the monopoles is not spherically symmetrical but rather aligned with the surface. This collimation of the field was observed in measurements, [23].

It is worth emphasizing that all the properties (a) - (e) are gauge invariant. Thus, the data so far do confirm that through projections one detects gauge invariant objects.

Finally, the scheme predicts that breaking of the chiral symmetry is associated with 3d defects. The corresponding lattice data were summarized in the preceding section.

## STOCHASTICITY

In the continuum limit, association of the confining fields with lower-dimension defects implies stochastic-type of correlators<sup>9</sup>. Indeed, the 3d volumes, e.g., are ‘not visible’ in the continuum limit.  $a \rightarrow 0$ . Denote by  $\bar{A}$  the confining potential obtained in the gauge minimizing the number of negative links (see above). Then

$$\langle \bar{A}(x), \bar{A}(y) \rangle = \Lambda_{QCD} \cdot \Lambda_{UV} f_{sing}(x-y) + (regular\ terms), \quad (19)$$

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<sup>9</sup> The material of this section is based on discussions with M.I. Polikarpov.

where

$$f_{sing}(0) = 1, \quad f_{sing}(x \neq 0) = 0 .$$

The singular nature of the confining potential could explain observed dependence of the localization of zero modes on the lattice spacing, see above.

It is worth emphasizing, however, that reduction of the confining potential to the ‘white noise’ would be a great oversimplification<sup>10</sup>. Indeed, the 3d nature of the domains assumes also non-trivial correlators for the derivatives of the potential. The issue deserves further consideration.

Consider now contribution of strings into an explicitly gauge invariant correlator:

$$\langle G^2(x), G^2(y) \rangle_{strings} = (const) \Lambda_{QCD}^4 \Lambda_{UV}^4 f_{sing}(x-y) + (const) \Lambda_{QCD}^8 f_{phys}(x-y) , \quad (20)$$

where  $f_{phys}$  depends on the physical mass scale. Note that appearance of the extra factor  $\Lambda_{QCD}^4$  in front of  $f_{phys}$  is of pure geometrical origin and reflects relative suppression of the 2d volumes compared to a 4d volume. On the other hand, appearance of the ultraviolet cut off in a non-local term would contradict the asymptotic freedom. It is one more example of consistency of the lattice strings with the asymptotic freedom, see also [10].

Finally, for a stochastic model of the confinement (see, e.g., [17, 18]) it is the correlator of two non-Abelian fields connected by a ‘Dirac-string’ operator,

$$\langle G_{\mu\nu}^a(x) \Phi_{ab}(x-y) G_{\mu\nu}^b \rangle = D(x-y) ,$$

which is crucial. The contribution of the string, discussed above, to this correlator is of the form:

$$D_{string}(x-y) = (const) \cdot f_{sing}(x-y) \Lambda_{QCD} \cdot \Lambda_{UV}^2 . \quad (21)$$

Moreover, using standard approximations of the stochastic model<sup>11</sup> one obtains for the string tension  $\sigma$  determining the heavy quark potential at large distances:

$$\sigma \approx \theta_{string} \Delta S_{string} \approx \frac{1}{2} \sigma_{exp} , \quad (22)$$

where  $\theta_{string}$  is the probability of a given plaquette to belong to the lattice string,  $\Delta S_{string}$  is the extra action associated with a plaquette belonging to the string,  $\sigma_{exp}$  is the value obtained in simulations.

It is interesting that the correlator  $D(x-y)$  is singular in any case,

$$\lim_{|x-y| \gg a} D(x-y) \sim \exp(-c|x-y|/a)$$

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<sup>10</sup> Actually, the ‘white noise’ would not confine.

<sup>11</sup> Using the minimal area spanned on the Wilson line is the most sensitive point, difficult to justify theoretically [17].

because the Dirac string,  $\Phi(x - y)$  is a color object and has infinite self energy<sup>12</sup>. Thus, the singular nature of the confining fields, see (7) is the only mechanism which can make the stochastic model relevant.

## CONCLUSIONS

Physics of confinement might undergo quite a dramatic change soon. There have been emerging data indicating relevance to confinement of lower-dimension defects, or singular fields. Two-dimensional defects with divergent action and entropy, which selftuned to each other are naturally interpreted as the dual string, observed as a vacuum excitation. The string possesses many SU(2) invariant properties but is detected through projections. Other emerging phenomena, a kind of holography and localization of modes on a submanifold shrinking to zero with  $a \rightarrow 0$ , are observed in explicitly SU(2) invariant terms. The price is that the structure of the fields responsible for these observational phenomena is less transparent.

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<sup>12</sup> This point was emphasized to me by Ph. de Forcrand. The well known measurements of the correlator  $D(x - y)$  [32] use smoothened, or cooled fields. We discuss here description in terms of the original field configurations. We assume also that pure perturbative fields drop out from the string-tension evaluation.

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