

# EXPLICIT EQUATIONS FOR RENORMALIZATION PRESCRIPTIONS IN THE CASE OF PION-NUCLEON SCATTERING \*

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This talk is a natural continuation of that given by V. Vereshagin [1]. We discuss some details not covered in that talk and review the calculational technique using the  $\pi N$  elastic scattering as an example. Finally, we briefly mention some results of numerical comparison with experimental data. More technical details can be found in the talk by K. Semenov-Tian-Shansky [2] devoted to the analysis of elastic  $KN$  scattering.

## 1 General notes

As explained in the talk [1], our approach does not assume any “nuclear democracy”. In contrast, it discriminates between stable particles and resonances. Only stable particles survive as asymptotic states, and it is the stable sector where the  $S$ -matrix is unitary (see, e.g. [3]). If we restrict ourselves by a consideration of the strong non-strange sector, then the only stable particles are pions and nucleons. Hence, to illustrate the application of our technique by the relatively simple process, we can choose among  $\pi\pi$ ,  $NN$ , and  $\pi N$ -elastic scattering (along with the cross-symmetric processes). Our choice of ( $\pi N$ ) is mainly dictated by the absence of extra phenomenological symmetries appearing in the former two reactions and, at the same time, by the relatively rich set of experimental data.

When working in the framework of effective theory one has to take account of all possible vertices and resonances which can contribute to the amplitude of the reaction under consideration. Since the perturbation theory which we rely upon is of Dyson’s type, we need to construct the perturbation series order by order, starting from the tree level. However, at this very first step we immediately meet the difficulty because to obtain the tree level amplitude we need to sum an infinite number of contact vertices and exchange graphs (Fig. 1).

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$$\sum_{\text{vertices}}^{\infty} \text{diagram}_1, \quad \sum_{\text{vertices, resonances}}^{\infty} \text{diagram}_2, \quad \sum_{\text{vertices, resonances}}^{\infty} \text{diagram}_3, \quad \sum_{\text{vertices, resonances}}^{\infty} \text{diagram}_4.$$

Figure 1: Tree graphs:  $R_s$ ,  $R_t$  and  $R_u$  stand for all admissible resonances in the  $s$ -,  $t$ -, and  $u$ -channels, respectively; summation over all possible kinds of vertices is implied, though the summation order is still unspecified.

The resulting sum is nothing but functional series, thus the problem of summation order is essential one. As it is demonstrated in [1, 4, 5, 6], our approach gives a way to overcome the obstacle. Simply speaking, the recipe we suggest reads:

1. Classifying all possible graphs and switching to the *minimal parametrization* [6] single out the set of *resultant* parameters of the given level (here — tree level). The latter are assigned the *physical* values with the help of relevant renormalization prescriptions (RP's).
2. Being guided by the *uniformity* and *summability* [1] principles use the Cauchy formula for given order (tree level) amplitude in certain domain of the space of kinematical variables.
3. Equating different expressions for the amplitude (the latter results from the Cauchy formula application) in the domains of their mutual validity, obtain the system of *bootstrap* equations. The latter allow one to specify the exact expressions of the amplitude under consideration and give restrictions for the values of *physical* parameters of the theory.

In this talk we shall take a closer look at the first and the last steps.

## 2 Minimal (resultant) vertices and renormalization conditions

As it is seen from Fig 1, there are Hamiltonian<sup>1</sup> three- and four-leg couplings and masses which parametrize the tree level amplitude in our case.

Minimal parametrization is a first step toward the constructing of so-called *essential* parameters [7, 6] — the *independent* parameters needed to describe the (on-shell) S-matrix. In case of general process amplitude of arbitrary loop order the minimal couplings

<sup>1</sup>In [6] it is explained why it is preferably to use the effective Hamiltonian, rather than Lagrangian when constructing a theory with unlimited number of field derivatives.

are the natural building blocks for the resultant parameters of which, in turn, the essential parameters can be constructed. However, in case of triple vertices at tree level, this structure gets simplified, and all the contributing three-leg minimal couplings appear also to be “resultant”.

The minimal vertices are, roughly speaking, the on-shell vertices. One just needs to take the *effective vertex* of a given order (at tree level this is a matrix element of the sum of all Hamiltonian vertices constructed of a given set of fields with all possible derivatives and matrix structures), put it on the mass shell, present the result in a Lorentz-covariant form and cross the wave functions out. The structure surviving after this is done, being considered as a function of independent components of *off-shell* momenta<sup>2</sup> is called the minimal vertex. The coefficients in the formal series for the corresponding formfactors are called the minimal couplings<sup>3</sup>. One easily observes that the tree-level triple minimal couplings are constants, because on the mass shell any triple vertex does not depend upon external momenta. For example, all the minimal vertices with resonances of isospin  $\frac{1}{2}$  and half-integer spin  $l + \frac{1}{2}$  contributing to our process at tree level can be listed as the following “Hamiltonian monomials”<sup>4</sup>:

$$g_{\widehat{R}} \overline{N} \sigma \widehat{R}_{\mu_1 \dots \mu_l} \partial^{\mu_1} \dots \partial^{\mu_l} \pi + H.c. \text{ for the resonance parity } P = (-1)^{l+1}, \text{ and}$$

$$ig_R \overline{N} \sigma \gamma_5 R_{\mu_1 \dots \mu_l} \partial^{\mu_1} \dots \partial^{\mu_l} \pi + H.c. \text{ for the resonance parity } P = (-1)^l,$$

where  $\sigma_a$  stands for Pauli matrix,  $\pi$ ,  $N$ , and  $R$  denote pion, nucleon and resonance fields, respectively, while  $g$ 's are the minimal coupling constants which, of course, depend on the resonance spin and mass. The essence of the reduction theorem proved in [6] is that any vertex that differs from the listed above by the number (or/and position) of derivatives, when added to the Feynman rules will only result in certain *rescaling* of  $g$ 's as long as one computes the  $S$ -matrix.

In the same way we can specify all the 4-leg minimal couplings contributing at tree level, but in our case it appears to be unnecessary. The reason is not simple, so let us not discuss their structure at this stage and suppose that transition to the minimal

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<sup>2</sup>Energy-momentum conservation is, of course, implied. For the precise definition of minimal vertex and the related classification see [6]

<sup>3</sup>They are, of course, functions of initial Hamiltonian couplings. However the latter functional dependence is not of interest anymore: we are not going to fix any of couplings in the initial Hamiltonian, rather, we will prefer to operate with minimal (resultant) parameters directly.

<sup>4</sup>Lacking space here, we do not list the remaining vertices with half-integer spin resonances and those with integer spin contributing in  $t$ -channel.

parametrization has been done. The main thing one should keep in mind is that the  $S$ -matrix is completely specified when the values of all the minimal couplings are given.

The way one assigns certain values to the  $S$ -matrix parameters in perturbation theory is the renormalization prescriptions (RP's). To obtain our tree level amplitude, we need to specify 3- and 4-leg couplings and masses. Forgetting for a while about 4-leg couplings we concern ourselves with the remaining parameters. As pointed out in [6], the resultant parameters are the natural candidates to impose the RP's under the condition that the renormalization point is taken on shell and *renormalized perturbation theory* is used. In this scheme the action is written in terms of *physical* parameters plus counter terms, the latter are tuned in a way that the values of those parameters remains unchanged after renormalization. So, we imply that the Feynman rules are written in the form of physical part plus counter terms at every loop order and it is the *real parts of physical masses* that appear in bare propagators. Simply speaking, we impose the following set of RP's:

$$\mathbf{Re} V(p_1, p_2, p_3) = G_{phys} \text{ at } p_i^2 = M_{i_{phys}}^2,$$

and

$$\mathbf{Re} \Sigma(p) = 0 \text{ at } p^2 = M_{phys}^2,$$

for every self-energy  $\Sigma$  and every three-point vertex  $V$ . Now we are at tree level, thus there are no counter terms relevant, therefore the couplings  $g$  are also physical (experimentally measurable).

There is no phenomenological evidence that the mass spectrum and spin values of resonances are bounded from above. Therefore we need to reserve the possibility to work with infinite set of resonances of arbitrary high spin value. In other words, there is still infinite number of minimal couplings coming even from three-leg vertices. One of the main points of our work is that these couplings are *not independent*: there are *self-consistency conditions* that restrict their values. We call this conditions as the *bootstrap* equations.

### 3 Bootstrap and experimental data

Because of lack of space we do not discuss here the method of constructing the well defined expressions for the amplitude at tree level or at any given order of perturbation theory. It is enough to say that the main tool allowing to do this is just the celebrated Cauchy integral formula with the *summability* and *asymptotic uniformity* conditions discussed in [1]. The final expression turns out to be completely parametrized by the minimal

couplings. Moreover, in the case of tree level  $\pi N$  elastic scattering amplitude only triple resultant vertices enter this expression. The joint contribution of four-leg vertices turns out to be uniquely determined by masses and triple couplings<sup>5</sup>.

The bootstrap equations mirror the crossing symmetry of a given order amplitude within our perturbation scheme. They can be rewritten in a form of infinite set of numerical equations for the amplitude parameters [4, 5]. What is essential to stress here is that the parameters that enter those equations are all minimal, and hence, as explained in the previous section, they are physical or (at least, in principle) *measurable*.

Using the renormalized perturbation theory with on-shell RP's at each loop level one obtains certain set of bootstrap equations which should be satisfied to ensure self-consistency (usually crossing symmetry). The form of these equations may vary from level to level, but *all* of them are the equations for physical parameters, and the full set of RP's should be compatible with all of them. To put it another way, the set of renormalization prescriptions for couplings and masses must be a solution to the full set of the bootstrap constraints.

We do not know how the solution of this latter set looks like. Even at tree level their form is highly non-linear. However, if our perturbation scheme can describe nature, then the experimentally fitted values of coupling constants and masses must fulfil the system of bootstrap conditions. That is why we have performed various calculations to check the consistency of our approach with the experimental data. Namely, we checked the tree level bootstrap equations for  $\pi\pi$  and  $\pi K$  elastic scattering amplitudes (see [4] and references therein), and recently analogous calculations were performed for the cases of  $\pi N$  [8] and  $KN$  elastic scattering (the latter case is discussed in the talk by K. Semenov-Tian-Shansky [2]). There were no contradiction found so far, and in most cases examined the experimental data seem to support our approach nicely.

Apart from the question of formal compatibility with experiment, there is a question of efficiency. One can ask how many loops should be taken into account and how many parameters fixed to obtain the amplitude that could fit well the data at least at some kinematical region. To check this point we performed a calculation of low energy coefficients<sup>6</sup> for the  $\pi N$  amplitude. This coefficients measured and fitted in [9] are reproduced in our approach with very good accuracy already at tree level<sup>7</sup>, and to gain reasonable

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<sup>5</sup>This statement is by no means trivial and requires separate consideration. The main reason for it is the known values of Regge intercepts which, by uniformity principle, define the asymptotic behavior of the tree level amplitude. This analysis will be published elsewhere.

<sup>6</sup>Taylor expansion coefficients around the crossing symmetry point.

<sup>7</sup>Of course, it is partly because this region is relatively far from the branch cut points. In case if the

precision it is enough to specify the parameters of just few lightest resonances. The results of this analysis were summarized in [8]; the details will be published elsewhere.

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latter appears close to the investigated region one should necessary include loops.