

**RENORMALIZATION IN EFFECTIVE THEORIES:
PRESCRIPTIONS FOR KAON-NUCLEON RESONANCE
PARAMETERS ***

K. M. SEMENOV-TIAN-SHANSKY
St. Petersburg State University

E-mail: semenov@pdmi.ras.ru

In this talk we show how it is possible to apply the general scheme of effective scattering theory to the description of hadronic processes. We perform the numerical tests of the tree level bootstrap constraints for renormalization prescriptions in the case of elastic kaon-nucleon scattering process.

1 Introduction

In papers [2]-[4] an attempt is made to develop an effective field theory formalism suitable for description of hadronic scattering processes (see also [1]). It was shown that the requirements of consistency of perturbation series for scattering amplitude lead to certain restrictions for the effective Hamiltonian parameters that are called *bootstrap equations*. Actually, we are unable to solve the bootstrap system explicitly. So, roughly speaking, the only way to check the consistency in our effective theory approach is the numerical testing.

An important property of the bootstrap system is its renormalization invariance. This property allows one to compare with experiment the results that follow already from the tree level bootstrap system. In many cases this data fitting leads to reasonable consequences. This can be considered as a strong evidence in favor of consistency of our approach. The similar verification was successful in the cases of πK [2] and πN elastic scattering processes (see references in [1]).

In this talk we discuss the application of our formalism to the case of KN elastic scattering. The resonance spectrum of KN reaction is measured with much less precision

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than that of πN reaction. However, it is possible to single out the set of sum rules that are well saturated by the known experimental data.

On the other hand, those sum rules that are not so well saturated with available data permit us to speculate about the possible scenarios that allow to amend the situation. So here we also aim to show that our approach is a powerful tool to study the resonance spectrum.

2 Bootstrap for KN scattering

The amplitude of KN elastic scattering $M_{\alpha i}^{\beta j} = \langle N_{\beta}(k')K_j(p') | (S-1) | N_{\alpha}(k)K_i(p) \rangle$ can be presented in the following form:

$$M_{\alpha i}^{\beta j} = \delta_{\alpha}^{\beta} \delta_i^j M^+(\lambda, \lambda') + \delta_{\alpha}^j \delta_i^{\beta} M^-(\lambda, \lambda'),$$

where $M^{\pm}(\lambda, \lambda', s, t, u) = \bar{u}^+(\lambda', k') \left\{ A^{\pm}(s, t, u) + \frac{\hat{p} \pm \hat{p}'}{2} B^{\pm}(s, t, u) \right\} u^-(\lambda, k)$. Here k, k' (p, p') stand for the nucleon (kaon) momenta, $\hat{p} \equiv p_{\mu} \gamma^{\mu}$; $\alpha, i, \beta, j = 1, 2$ are the isospin indices; λ, λ' stand for nucleon spin variables; $\bar{u}(k', \lambda')$, $u(k, \lambda)$ — for Dirac spinors. Invariant amplitudes A^{\pm} and B^{\pm} are the functions of an arbitrary pair of Mandelstam kinematical variables s, t, u .

The detailed theoretical background of our calculations is discussed in [1]. Here we shall only briefly recall the main steps needed to construct the set of tree level bootstrap constraints for renormalization prescriptions (RP's) in KN reaction.

We work in the framework of the general formalism of effective theories. This means that the corresponding interaction Hamiltonian contains all local terms consistent with given algebraic symmetry requirements. We consider a very narrow class of so-called *localizable* effective theories. In this case to construct a consistent tree level approximation it is necessary to turn to the *extended perturbation scheme* which, along with the fields of stable particles, also contains an infinite number of fields corresponding to auxiliary unstable ones (resonances) of arbitrary high spin and mass. The tree level amplitude of a scattering process $2 \rightarrow 2$ calculated in this formalism takes a form of an infinite sum of resonance exchange graphs plus another (also infinite) sum of all possible contact terms. Thus one needs to establish certain guiding principle that would allow to fix the order of summation of this formal series for tree level amplitude. This problem can be solved by passing to the *minimal parametrization* (see [4]) and by using the method of Cauchy forms. Minimal parametrization allows one to get rid of those combination of Hamiltonian parameters which do not contribute to the renormalized S -matrix. It can be

shown that the tree level amplitude is completely determined by the values of three-leg minimal vertices (in some cases, one also needs to impose one additional RP fixing the value of the amplitude at certain kinematical point).

The method of Cauchy forms allows one to present the tree-level $2 \rightarrow 2$ scattering amplitude as a uniformly convergent series of pole contributions in three mutually intersecting (near the corners of Mandelstam triangle) layers $B_s\{s \sim 0\}$, $B_t\{t \sim 0\}$, $B_u\{u \sim 0\}$. Bootstrap system naturally arises as a requirement that the Cauchy forms (different in different layers) should coincide in the domains of intersection of layers. For example, let us consider the system of those tree level bootstrap constraints for A^- invariant amplitude which appear from the domain where the layers B_s and B_u intersect. Namely, the difference of Cauchy forms in two layers $\tilde{A}^-(s, u)\Big|_{B_s} - \tilde{A}^-(u, s)\Big|_{B_u} \equiv \Phi_A^-(u, s)$ should be identically zero in the vicinity of the point $s = 0, u = 0$:

$$\frac{\partial^{p+k}}{\partial u^k \partial s^p} \Phi_A^-(u, s) \Big|_{\substack{u=0 \\ s=0}} = 0, \quad p, k = 0, 1, 2, \dots \quad (1)$$

The explicit form of the generating function $\Phi_A^-(u, s)$ is given in the Appendix.

The point of major importance is that, if the calculations are carried out in the scheme of *renormalized perturbation theory* with on-shell normalization conditions, the bootstrap equations are nothing but a system of restrictions for the admissible values of RP's (real parts of pole positions and triple couplings). In that way bootstrap system results in a set of constrains for observable physical spectrum of the theory. Thus once established on the tree-level, these relations must also hold at higher loop orders, just because at each loop order one should impose the same RPs. This explains our direct use of the experimental values of resonance masses and coupling constants (e.g. given in [5]) to perform the numerical comparison with data. If our scheme is somehow suitable for the description of physical world the bootstrap constrains must hold.

3 Numerical tests

Now we pass to our numerical tests. As a first example we show how it is possible to obtain the estimate for the $G_{\Sigma(1385)\bar{K}N}$ coupling with the help of sum rules that follow from the bootstrap system. Our first goal is to find the sum rules that can be saturated with a small number of well established resonances. The up-to-date information on the KN resonance spectrum is incomplete in the region of high mass and spin. Much is unclear with $M > 1 \text{ GeV}$ meson resonances in the t -channel of elastic reaction. One also

needs to keep in mind the possible existence of s -channel $S = +1$ exotic resonances. Let us consider the sum rule that follows from the bootstrap condition (1) for the invariant amplitude A^- and corresponds to $k = p = 1$. It turns out that in this sum rule the contributions of certain not well established resonances is wiped out. This sum rule can be considered as purely baryonic one (only baryons with $J = \frac{3}{2}, \frac{5}{2}, \dots$ can contribute), because in the meson sector only isospin 1 resonances of odd spin $J \geq 3$ (e.g., $\rho_3(1690)$) can in principle contribute to it. An assumption is made that heavy meson contributions are suppressed by small $\sim \frac{1}{M}$ factors. In our present analysis we also will not take account of possible contributions of exotic resonances with strangeness $S = +1$. However, in what follows we show that several sum rules provide an indirect evidences in favor of existence of exotic resonances.

Thus we try to saturate our sum rule by the contributions of baryons with masses $M < 2 \text{ GeV}$ and spins $J \leq \frac{5}{2}$ (see [5]). Imputing the deficit to the unknown contribution of $\Sigma(1385)$ we can estimate the value of $\Sigma(1385)\bar{K}N$ coupling constant. This gives: $G_{\Sigma(1385)\bar{K}N} = 1.3 \pm 0.4$. The experimental value of this constant (see, e.g., [6] p.61) is: $G_{\Sigma(1385)\bar{K}N} = 1.06 \pm 0.13$. The agreement looks impressive. However, there are several sufficiently well established resonances with $M > 2 \text{ GeV}$. The large contribution of $\Lambda(2100)$ seems to slightly disturb the sum rule. This gives: $G_{\Sigma(1385)\bar{K}N} = 1.5 \pm 0.7$. This shift can be compensated by the contributions of $\Sigma(2100)$ and of the other heavy Σ resonances in this region.

It is very instructive to consider also the sum rules which follow from the bootstrap constrains for A^- (1) with many derivatives. These sum rules can be well saturated with the reliable experimental data on $S = -1$ baryon spectrum with $J = \frac{3}{2}, \frac{5}{2}$ (mesons with $J = 0, 1, 2$ do not contribute). This gives a strong evidence in favor of consistency of our approach, because the shape of these sum rules crucially depends on our assumptions (in particular, on the concrete formulation of the *summability principle* [1]). The results are presented in the Table 1. The fact that the balance becomes worse with the growth of k shows that the contribution of baryons with spin $J > \frac{5}{2}$ becomes relatively more important in these sum rules.

However, not all sum rules are well saturated with known data. For example the sum rules for A^+ look very nasty. At first glance, nothing could compensate the huge positive contribution of ($I = 1, J = \frac{3}{2}$) resonances nearest to the KN threshold. There are certain possibilities to overcome this difficulty. First of all, it is interesting to notice that a similar situation was encountered in the “toy bootstrap model” [3] based on Veneziano string amplitude. In certain sum rules for the resonance parameters of the

p	k	Sum Rule	p	k	Sum Rule
0	1	$13.0 \div 19.8 \simeq 19.8 \div 24.7$	1	1	$15.3 \div 24.2 \simeq 13.7 \div 21.8$
0	2	$20.7 \div 25.7 \simeq 23.4 \div 28.4$	1	2	$16.2 \div 22.7 \simeq 14.9 \div 21.2$
0	3	$48.0 \div 55.1 \simeq 43.8 \div 50.9$	1	3	$23.6 \div 31.2 \simeq 23.8 \div 32.4$
0	4	$151.0 \div 167.3 \simeq 111.4 \div 125.1$	1	4	$44.0 \div 55.7 \simeq 50.2 \div 66.5$
1	0	$23.8 \div 48.5 \simeq 24.3 \div 43.2$	1	5	$99.8 \div 123 \simeq 131.4 \div 171.8$

Table 1: Saturation of sum rules (1) for different values of p , k .

string amplitude it is sufficient to take into account the contribution of a relatively small number of first poles to saturate it with high precision. At the same time, in some another sum rules it is necessary to sum over the contributions of considerable number of poles to compensate the “accidentally large” contribution coming from several first poles. It is possible that heavy resonances with $J^P = \frac{3}{2}^+$ and $J^P = \frac{5}{2}^+$ could in principle gradually compensate the large contribution of $\Sigma(1385)$. The same mechanism could work for other sum rules from this group with $k > 1$. Another interesting possibility is to interpret the deficit in these sum rules as an indirect evidence for the existence of exotic baryons with strangeness $S = +1$ (so-called Z or θ baryons). One can easily check that the contribution of a baryon with $S = +1$ and $J^P = \frac{3}{2}^+$ below the KN threshold, or of a $J^P = \frac{3}{2}^-$ baryon above it, can significantly compensate the deficit. However, one is forced to assume the existence of at least two exotic baryons with isospin 0 and 1, respectively. Otherwise, it is impossible to attain the mutual cancellation of the contributions from exotic sector in those sum rules which are satisfactorily saturated with the $S = -1$ baryons.

4 Conclusions

The numerical tests (that were carried out for πN , KN , πK and $\pi\pi$ reactions) make it possible to conclude that our approach, at least, does not roughly contradict to presently known phenomenology. However, at the moment we are unable to give an answer to the main question: “How many independent RP’s are needed to fix the physical content of effective scattering theory?” To answer it, we need to somehow solve the bootstrap system. A possible way to solution is provided by the application of general theory of analytic continuation along with the tool of infinite-dimensional matrices. We also need to study if the higher order bootstrap constrains (1-loop, ...) impose additional

restrictions on the set of RP's or just follow from the tree-level bootstrap.

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Appendix

Here we give the explicit expressions for the baryon part of the generating functions of bootstrap system for the amplitude A^- : $\Phi_A^-(u, s) = \sum_{S=-1} \frac{c_I^- G_{R_s KN} F_A^l(-\mathcal{N}M, -(\Sigma+u))}{s-M^2} - \sum_{S=-1} \frac{b_I^- G_{R_u \bar{K} N} F_A^l(-\mathcal{N}M, -(\Sigma+s))}{u-M^2}$. The residue of the amplitude in the pole corresponding to a baryon resonance of strangeness $S = \pm 1$, isospin I , spin $j = l + \frac{1}{2}$, normality \mathcal{N} and mass M is given by $F_A^l(M, \chi) = (M+m)P'_{l+1}(1 + \frac{\chi}{2\phi}) + (M-m)\frac{(M+m)^2 - \mu^2}{(M-m)^2 - \mu^2} P'_l(1 + \frac{\chi}{2\phi})$. Here P'_l stands for derivatives of ordinary Legendre polynomials; $m(\mu)$ is the nucleon (kaon) mass; $\phi = \vec{k}_{C.M.F}^2$; b_I^- , c_I^- are the isotopic coefficients; $\Sigma = M^2 - 2(m^2 + \mu^2)$; G_{RKN} is the dimensionless coupling constant.

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