

Parton distributions from the lattice

W. Schroers ^a

^aNIC/DESY Zeuthen, Platanenallee 6, D-15738 Zeuthen, Germany

I review the current status of lattice calculations for two selected observables related to nucleon structure: the second moment of the unpolarized parton distribution, $\langle x \rangle_{u-d}$, and the first moment of the polarized parton distributions, the non-singlet axial coupling g_A . The major challenge is the requirement to extract them sufficiently close to the chiral limit. In the former case, there still remains a puzzling disagreement between lattice data and experiment. For the latter quantity, however, we may be close to obtaining its value from the lattice in the immediate future.

Preprint: DESY 05-010

1. INTRODUCTION

Deep inelastic scattering of electrons off nuclei has historically provided an indispensable way to resolve the substructure of hadrons and thus to learn how the strong interaction is responsible for their properties. Using the short distance and the light cone operator product expansions, one can relate nucleon matrix elements of twist-two quark bilinear operators to moments of parton distributions,

$$\langle n | \bar{\psi} \Gamma^{\{\mu_1} D^{\mu_2} \dots D^{\mu_n} \psi | n \rangle = \mathcal{F}^{\{\mu_1 \dots \mu_n\}} \int dx x^{n-1} f(x). \quad (1)$$

For $\Gamma = \gamma$ the parton distribution function (PDF) $f(x) = \Theta(x)q(x) - \Theta(-x)\bar{q}(-x)$, $-1 \leq x \leq 1$, represents the probability (minus probability) to find a quark (antiquark) inside the nucleon with longitudinal momentum fraction x ($-x$). Choosing $\Gamma = \gamma_5 \gamma$ one finds the associated spin-dependent PDF $\tilde{f}(x)$. $\mathcal{F}^{\{\mu_1 \dots \mu_n\}}$ contains kinematical prefactors and the curly braces indicate symmetrization and subtraction of traces. The matrix elements on the l.h.s. of eq. (1) are accessible in lattice simulations. For technical details see [1,2,3] and also [4] for a recent review.

Unfortunately, lattice simulations are limited by computer resources and the most expensive aspect of such calculations is to make the quark mass small. Today even the most sophisticated calculations are still far from the chiral limit. One may suspect that the properties of hadrons in a world with very heavy quarks — and thus very heavy pions — are different from the world nature has provided us with. This in fact appears to be the case. The qualitative and quantitative behavior of quantities as functions of mass and energy scales can in principle be assessed by small scale expansion techniques like chiral perturbation theory. For a recent review on chiral extrapolation techniques for nucleon structure, see [5]. For a general review on lattice chiral perturbation techniques, see [6].

This presentation will concentrate on two representative quantities: the second moment of the unpolarized parton distribution, $\langle x \rangle_{\text{u-d}}$, and the first moment of the polarized distribution, the non-singlet axial coupling g_A . All numbers are given in the $\overline{\text{MS}}$ -scheme with a scale of $\mu = 2\text{GeV}$. The purpose of this presentation is to illustrate the current status of the field by discussing if and how lattice calculations can provide an understanding of the values of these quantities in the experiment.

Table 1

Compilation of different lattice investigations of $\langle x \rangle_{\text{u-d}}$ and g_A .

Group & Ref.	m_π	Technique	$\langle x \rangle_{\text{u-d}}$	g_A
Kentucky [7]	?	Wilson (quenched)	-	1.20(11)
KEK [8]	> 530 MeV	Wilson (quenched)	-	0.985(25)
QCDSF [1]	> 600 MeV	Wilson (quenched)	0.263(17)	1.074(90)
LHPC [9]	> 650 MeV	Wilson (full)	0.269(23)	1.031(81)
RBCK [10]	> 390 MeV	DWF (quenched)	-	1.212(27)
LHPC [11]	> 360 MeV	Hybrid+Wilson (full)	-	⁻¹
QCDSF [2]	> 550 MeV	CI-Wilson (quenched)	0.245(19)	-
QCDSF [12]	> 550 MeV	CI-Wilson (full)	-	⁻¹
QCDSF [13]	> 300 MeV	Overlap (quenched)	0.20(2)	1.13(5)
RBCK [14]	> 390 MeV	DWF (quenched/full)	⁻¹	⁻¹
Experimental values: $\langle x \rangle_{\text{u-d}} = 0.154(3)$ [15], $g_A = 1.248(2)$ [16], see also [17,18]				

¹ Work in progress and/or no prediction quoted

A selection of different lattice studies is compiled in table 1. These studies use a variety of different techniques and thus cover a wide range of parameters. However, none of them approaches the chiral limit closer than 300 MeV. The results for $\langle x \rangle_{\text{u-d}}$ will be discussed in more detail in section 2, and the results for g_A in section 3. Finally, section 4 contains the conclusions and outlook.

2. THE MOMENT $\langle x \rangle_{\text{u-d}}$

As it is evident from table 1 almost all results for $\langle x \rangle_{\text{u-d}}$ systematically exceed the experimental result by about 50%. Given the variety of techniques and parameters used, neither finite size, unquenching, nor lattice artifacts can account for this discrepancy.

Hence, this discrepancy can only be attributed to the large quark masses used. Only reference [13] finds a systematically smaller value than all other studies. Since the domain-wall calculation in reference [14] is quite similar, but does not show the same behavior, the discrepancy could be explained by a systematic effect at the matching between lattice and $\overline{\text{MS}}$ -schemes. A final assessment will be possible once a non-perturbative matching for this quantity has been performed [19].

The proposal to resolve this discrepancy in [20] introduces a cut-off by hand into the leading order chiral perturbation theory expansion of $\langle x \rangle$ which effectively limits the size of the pion cloud. For this proposal to have predictive power it is required that this cut-off parameter is independent of the observable under consideration. Even before the

advent of modern calculations in the chiral regime the approach has been criticized for just employing the leading order chiral expansion. This may be inadequate for pion masses beyond the physical one [21]. The calculations that have appeared since then at quark masses down to 300 MeV have found no evidence of such a “bending down”.

3. THE AXIAL COUPLING g_A

The situation for the axial coupling g_A is quite different. First, it is well established that g_A is very sensitive to finite-size effects. This has initially been realized in [10] and later been confirmed by other groups in [12] and [11]. On the other hand, there is an improving understanding of how to treat this quantity within chiral perturbation theory [12,22].

Hence, combining results from different lattice sizes and taking an “enveloping” curve of all numbers not influenced by strong finite-size effects may lead to an accurate prediction compatible with experiment. This procedure has been hinted at in [11]. Alternatively, one can try to fit data from different box sizes and pion masses directly in a combined fit similar to what has been done in [12]. Although these two reports describe work still in progress, it is likely that both approaches will soon lead to a consistent picture and a quantitative result for g_A at the physical value of the pion mass.

4. CONCLUSION AND OUTLOOK

The field of nucleon structure lattice calculations is at a turning point — on one hand it has turned out that nucleon matrix elements of quark bilinears exhibit strong quark mass dependence and — in some cases — substantial finite-size effects. On the other hand recent progress in algorithms and hardware has provided theoreticians with the means to perform computations at substantially smaller quark masses and larger volumes than has previously been achieved.

Despite this progress, the puzzling mismatch between experiment and theory for the second moment of the unpolarized parton distribution, $\langle x \rangle_{u-d}$ has not been resolved. At this time, it is not clear how the apparent disagreement can be reconciled. If there should indeed be a drastic change of 50% at masses of $m_\pi \ll 300$ MeV, this pattern would indeed be unique and the physical mechanism behind such a behavior yet requires understanding.

The situation for the axial coupling, however, is different. With improved chiral expansion techniques becoming available, see in particular ref. [22], it is now understood how this quantity is sensitive to finite box sizes. Taking this observation into account, the availability of numerical results at sufficiently small pion masses, allows for a consistent picture to be drawn how g_A can be obtained from current lattice data. Several groups are close to obtaining sufficiently accurate data to conclusively postdict g_A from lattice calculations.

While the field of parton distributions so far has only allowed lattice calculations to make postdictions, the associated field of GPDs [23] is in the unique position to make quantitative predictions [24] which are otherwise very hard to extract experimentally. Therefore, the solution of the puzzles forward parton distributions pose to us would be one of the milestones lattice QCD faces today.

REFERENCES

1. M. Göckeler *et al.* [QCDSF], Phys. Rev. D **53** (1996) 2317.
2. M. Göckeler *et al.* [QCDSF], arXiv:hep-ph/0410187.
3. P. Hägler and J. Zanotti, these proceedings.
4. R. Horsley, arXiv:hep-lat/0412007.
5. M. Göckeler, arXiv:hep-lat/0412013.
6. O. Bär, arXiv:hep-lat/0409123.
7. K. F. Liu *et al.*, Phys. Rev. D **49**, 4755 (1994).
8. M. Fukugita, Y. Kuramashi, M. Okawa and A. Ukawa, Phys. Rev. Lett. **75** (1995) 2092.
9. D. Dolgov *et al.* [LHPC], Phys. Rev. D **66** (2002) 034506.
10. S. Sasaki, K. Orginos, S. Ohta and T. Blum [RBCK], Phys. Rev. D **68**, 054509 (2003).
11. D. B. Renner *et al.* [LHPC], arXiv:hep-lat/0409130.
12. A. A. Khan *et al.*, arXiv:hep-lat/0409161.
13. M. Gürtler *et al.*, arXiv:hep-lat/0409164.
D. Galletly *et al.* [QCDSF-UKQCD], Nucl. Phys. Proc. Suppl. **129**, 453 (2004).
14. S. Ohta and K. Orginos [RBCK], arXiv:hep-lat/0411008.
S. Ohta and K. Orginos [RBCK], Nucl. Phys. Proc. Suppl. **129** (2004) 296.
15. H. L. Lai *et al.*, Phys. Rev. D **55**, 1280 (1997).
M. Glück, E. Reya and A. Vogt, Eur. Phys. J. C **5**, 461 (1998).
A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. C **23**, 73 (2002).
16. J. Blümlein and H. Böttcher, Nucl. Phys. B **636** (2002) 225.
M. Glück, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D **63** (2001) 094005.
Y. Goto *et al.* [AAC], Phys. Rev. D **62**, 034017 (2000).
M. Hirai, AIP Conf. Proc. **675**, 365 (2003).
17. J. Blümlein, H. Böttcher and A. Guffanti, Nucl. Phys. Proc. Suppl. **135** (2004) 152.
18. J. Blümlein, these proceedings.
19. M. Gürtler, private communication.
20. W. Detmold *et al.*, Phys. Rev. Lett. **87**, 172001 (2001).
21. Th. Hemmert, private communication.
22. M. Procura and Th. Hemmert, these proceedings.
23. M. Diehl, Phys. Rept. **388** (2003) 41.
24. M. Göckeler *et al.* [QCDSF], Phys. Rev. Lett. **92**, 042002 (2004).
P. Hägler *et al.* [LHPC], Phys. Rev. D **68**, 034505 (2003).
P. Hägler *et al.* [LHPC], Phys. Rev. Lett. **93**, 112001 (2004).
J. W. Negele *et al.* [LHPC], Nucl. Phys. Proc. Suppl. **129**, 910 (2004).
W. Schroers *et al.* [LHPC], Nucl. Phys. Proc. Suppl. **129**, 907 (2004).
M. Göckeler *et al.* [QCDSF], Nucl. Phys. Proc. Suppl. **128**, 203 (2004).
J. W. Negele *et al.* [LHPC], Nucl. Phys. Proc. Suppl. **128**, 170 (2004).
M. Göckeler *et al.* [QCDSF], Nucl. Phys. Proc. Suppl. **135**, 156 (2004).
M. Göckeler *et al.* [QCDSF], arXiv:hep-lat/0409162.
P. Hägler *et al.* [LHPC], arXiv:hep-ph/0410017.
M. Göckeler *et al.* [QCDSF], arXiv:hep-lat/0410023.