# Diffraction, the Color Glass Condensate and String Theory

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We explain the main ideas of the color glass condensate in high energy collisions. Different approaches to the problem are outlined with emphasis on the resummation approach. We present evidence that the color glass condensate can be described by an effective conformal field theory or even by a string theory.

# 1. DIFFRACTION AND THE COLOR GLASS CONDENSATE

Hadronic scattering processes at high energies are called diffractive if there is a wide angular region in the final state without hadronic activity. Such a rapidity gap can only emerge if a colorless object, the Pomeron, is exchanged between the colliding particles. Clearly, elastic scattering at high energy is a diffractive process. Hence the very same object determines via the optical theorem also the total cross section at high energies. Due to Regge or high energy factorization the elastic amplitude is a convolution of the amplitude for Pomeron exchange with impact factors coupling it to the external particles, and the actual energy dependence is encoded only in the Pomeron.

Although there is plenty of data for different scattering processes, we still have no conclusive picture of the Pomeron in terms of quarks and gluons, and to understand the high energy limit of QCD remains one of the most challenging open problems in the physics of the strong interaction. There are two key points which make the problem complicated. Many scattering processes at high energies involve low momentum scales, in general prohibiting a perturbative approach. Fortunately though, there are some scattering processes involving momentum scales sufficiently large for making perturbation theory applicable, like for example the scattering of two highly virtual photons which split into quark-antiquark pairs and then scatter off each other. But even if one concentrates on these latter processes there is still a second problem, namely the occurrence of high parton densities. At high energies there is a large phase space for the emission of soft partons off the incoming hadrons. Eventually, the usual picture breaks down in which different parton cascades in the hadron are viewed as being independent of each other. At this point the partons in the hadron start to overlap and recombination effects have to be

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taken into account. As a consequence, the evolution of the system with energy becomes nonlinear, asking for new theoretical methods to describe it. An advantage of high parton densities, on the other hand, is that multiple scattering in a dense medium can induce hard scales, possibly extending the applicability of perturbation theory to a larger number of scattering processes.

The dense system of partons in colliding hadrons at high energies consists mainly of gluons, because these are the particles of highest spin and largest color charge. Since gluons carry a color charge and are bosons, one calls this dense parton system the color glass condensate (CGC). The 'glass' refers to the fact that the quantum evolution of the system, that is the emissions and recombinations of gluons, takes place on much longer time scales than the interaction in the collision process. In other words, the hadron evolves for a long time before the actual collision takes place.

## 2. THEORETICAL DESCRIPTION OF THE COLORED GLASS

The color glass condensate has been studied in a number of different approaches in perturbative QCD. The approach on which we will concentrate in the present talk is based on the concept of resummation of large logarithms of the energy  $\sqrt{s}$  which can compensate the smallness of the coupling constant. In the leading logarithmic approximation (LLA) [1,2] one collects all terms of the form  $(\alpha_s \log s)^n$  in the perturbative series. The resulting linear evolution equation, the BFKL equation, describes the Pomeron as an exchange of two interacting reggeized gluons in the t-channel. At higher energies it becomes necessary to go beyond the LLA by taking into account nonlinearities, as we will describe below. Among the other theoretical approaches to the color glass condensate are the dipole picture [3], the operator expansion of Wilson lines [4], and the 'color glass condensate approach'<sup>3</sup>, for a review see [5]. In all these approaches one finds identical or at least similar results for the main properties of the color glass condensate. One of them is the fact that the main dynamics takes place in the two-dimensional transverse space of the scattering process. In the approximation of low parton densities, in which nonlinear terms can be neglected, all the approaches reproduce the BFKL Pomeron. We are convinced that a more detailed understanding of the relations between the different approaches will be crucial for future developments in the theory of the CGC.

In the resummation approach one can include high density effects and go beyond the LLA by taking into account exchanges with more than two gluons in the *t*-channel. Here one collects the maximally possible number of logarithms for a given number of exchanged gluons, giving rise to the generalized leading logarithmic approximation (GLLA). This can be done either by keeping the number of gluons fixed during the *t*-channel evolution [6,7,8], or, more interestingly, in the extended form of that approximation (EGLLA) in which the number of *t*-channel gluons is allowed to fluctuate [9,10]. In the EGLLA, amplitudes for the production of *n* gluons in the *t*-channel are described by a hierarchy of coupled integral equations. It is important to note that in the EGLLA it is not necessary to take the large- $N_c$  limit at any stage.

The analysis [11,12,10] of the amplitudes in the EGLLA has shown that they can be cast into the form of an effective field theory of reggeized gluons. In this effective

 $<sup>^{3}</sup>$ Somewhat confusingly, that approach has been given the same name as the object it describes.

theory only states with (fixed) even numbers of gluons occur which are coupled to each other by effective vertices  $V_{2\to 2l}$ . So far the two-to-four [12] and the two-to-six gluon vertex [10] have been calculated explicitly. The emerging structure is that of a twodimensional field theory in transverse space, with rapidity  $Y \sim \log s$  as an additional time-like parameter. Due to lack of space we mention only in passing here that the field theory structure emerges because of the reggeization of the gluon at high energies [13,14]. In short, reggeization reflects the fact that at high energies the correct degrees of freedom are collective excitations of the Yang-Mills field rather than elementary gluons.

A very important property of the effective transition vertices is their conformal invariance in two-dimensional impact parameter space [15,16], a property which they share with the kernel of the BFKL equation [17]. This observation makes it appear likely that the effective field theory of the color glass condensate in the EGLLA is in fact a conformal field theory (CFT). There is some hope that the known effective vertices are already sufficient to identify the underlying CFT. It would clearly be a significant step towards understanding the color glass condensate if one could apply the powerful methods of CFT to this problem. It could also help to establish a relation of high energy QCD to string theory via the AdS/CFT correspondence.

A slightly less ambitious but still very interesting goal would be to look for an effective field theory of interacting Pomerons for the color glass condensate. Clearly, it is an additional approximation to restrict oneself to color singlet states of two gluons, and depending on the process under investigation it is not even necessarily a good one. For hard scattering processes on large nuclei it should be a valid approach though, since the coupling of the Pomerons to different nucleons makes these exchanges the leading ones in an expansion in  $1/N_c$ . In a first approximation one takes into account only the splitting of a Pomeron into two Pomerons, that is the three-Pomeron vertex, as the first nonlinear term in the evolution. In the limit of large  $N_c$  this gives rise to the BK equation [4,18,19,20] which is obtained in all the approaches to the CGC mentioned above in suitable approximations, and which is now widely used in the phenomenological study of high energy scattering processes. One clearly expects that with increasing parton densities also higher nonlinear terms and in general also  $N_c$ -suppressed terms become relevant. Obviously, it is very important for understanding the high energy limit of QCD to know whether higher Pomeron vertices exist, how they look like, and whether an effective field theory of interacting Pomerons can be formulated. In the following sections we will present new results [21] which bring us closer to an answer to these questions. Again, the conformal invariance plays a key role here.

## 3. CONFORMAL BOOTSTRAP FOR POMERON VERTICES

Due to the conformal invariance of the BFKL equation in impact parameter space the resulting Pomeron states can be classified according to their behavior under  $SL(2, \mathbb{C})$ transformations. The representations of this symmetry group are characterized by the conformal weight  $h = (1 + n)/2 + i\nu$  with  $n \in \mathbb{Z}$  and  $\nu \in \mathbb{R}$ . Consequently, the Pomeron vertices depend on the quantum numbers  $h_i$  of the Pomerons and on their coordinates  $\rho_i$  in impact parameter space (understood as a complex plane). For simplicity we will restrict ourselves to Pomeron states with vanishing conformal spin  $n_i = 0$  in the following. In the resummation approach the three-Pomeron vertex  $V_{3\mathbb{P}}$  is obtained from the effective two-to-four gluon vertex  $V_{2\to4}$  by projecting the four outgoing gluons onto a pair of BFKL Pomerons [22,23]. The four-Pomeron vertex  $V_{4\mathbb{P}}$  has been calculated from the two-to-six gluon vertex  $V_{2\to6}$  in a similar way in [24]. Note that a four-Pomeron vertex has also been obtained in the approach based on the expansion of Wilson lines in [25], but its relation to the vertex  $V_{4\mathbb{P}}$  has not yet been studied.

The three-Pomeron vertex computed from the EGLLA consists of two terms which come with different powers of  $N_c$ ,

$$V_{3\mathbb{P}}(\rho_i, h_i) = g^4 C_1 \left[ V^{(0)}(\rho_i, h_i) + \frac{C_2}{N_c^2} V^{(1)}(\rho_i, h_i) \right]$$
(1)

where  $C_{1,2}$  do not depend on the positions  $\rho_i$ . The corresponding four-Pomeron vertex has been obtained in [24] as a sum of different terms which are all of the same order in  $N_c$ . These terms can be expressed in terms of three different functions  $\Phi(\rho_j, h_j)$ ,  $\Theta(\rho_j, h_j)$ , and  $\Pi(\rho_j, h_j)$  for which explicit integral representations exist. Interestingly, the vertex functions in  $V_{3\mathbb{P}}$  and  $V_{4\mathbb{P}}$  are all completely symmetric in the Pomerons, despite the fact that they have been calculated from effective vertices which are not symmetric under the exchange of the two incoming and the four (resp. six) outgoing gluons. So far, the two vertices  $V_{3\mathbb{P}}$  and  $V_{4\mathbb{P}}$  are the only Pomeron vertices which have been derived in the EGLLA, and a calculation of higher Pomeron vertices appears prohibitively complicated if the same techniques were to be used. As we will see, the properties of the two known vertices make it already possible to predict many properties of all higher vertices.

A first important observation is that these two Pomeron vertices depend on the Pomeron coordinates in the particular form required by conformal symmetry, that is they are conformal three- and four-point functions, respectively. We emphasize that this is a nontrivial outcome of the EGLLA and has not been put in. Further, an interpretation of these vertices in the framework of an effective CFT of interacting Pomerons would require that the four-point function is related to the three-point function in the form of a conformal bootstrap relation. Remarkably, relations of exactly this type can be found [21]. They express the functions  $\Phi$ ,  $\Theta$  and  $\Pi$  appearing in the four-Pomeron vertex as products of those appearing in the three-Pomeron vertex,  $V^{(0)}$  and  $V^{(1)}$ . For example, the function  $\Phi(\rho_j, h_j)$  (j = a, b, c, d) appearing in the four-Pomeron vertex  $V_{4\mathbb{P}}$  can be expressed as

$$\Phi = \sum_{n_k=-\infty}^{+\infty} \int_{-\infty}^{+\infty} d\nu_k f(h_k) \int d^2 \rho_k V^{(1)}(\rho_a^*, \rho_b^*, \rho_k, h_a^*, h_b^*, h_k) V^{(1)}(\rho_k^*, \rho_c^*, \rho_d, h_k^*, h_c^*, h_d)$$
(2)

with a weight factor  $f(h_k) = |h_k - 1/2|^2/\pi^4$ . This result is derived with the help of a completeness relation and it is therefore crucial to sum over a full set of intermediate Pomeron states with label k. Similarly, one can express the other two functions  $\Theta$  and  $\Pi$  in the four-Pomeron vertex as products of the two functions  $V^{(0)}$  and  $V^{(1)}$ , where for  $\Pi$  this product has to be taken with crossed arguments. The product of  $V^{(0)}$  with itself does not occur. A closer inspection shows that it cannot be obtained in the four-Pomeron vertex  $V_{4\mathbb{P}}$  derived in the EGLLA because the Feynman diagrams required for it are not part of that approximation. The latter observation might help in identifying possible important contributions beyond the EGLLA, and to compare them for example with those of [26].

These bootstrap relations between the three-Pomeron vertex and the four-Pomeron vertex strongly suggest that there exists in fact an effective CFT of interacting Pomerons. Assuming that they are not simply accidental, one can use them to formulate conjectures about higher *n*-Pomeron vertices for arbitrary *n*. Our analysis of the two known Pomeron vertices makes it hence possible to predict – up to normalization factors – all higher Pomeron vertices as they would be obtained in the EGLLA. Explicit formulae will be given in [21].

## 4. STRING AMPLITUDES FOR POMERON VERTICES

It has long been conjectured that QCD at high energies is a string theory or can at least be related to one. In fact high energy hadron scattering is even the origin of string theory. It is therefore interesting to see whether the known interactions of Pomerons resemble string amplitudes. A relation of this kind was discussed in the context of a conjecture for higher Pomeron vertices in the dipole picture in [26]. We have found that the Pomeron vertices obtained in the EGLLA do not coincide with those conjectured there. Nevertheless, it turns out that the Pomeron vertices in the EGLLA can be related to string amplitudes in a way similar to that proposed in [26].

Specifically, we find that both  $V_{3\mathbb{P}}$  and  $V_{4\mathbb{P}}$  can be expressed as integrands of Virasoro-Shapiro amplitudes of a closed bosonic string theory [21]. Writing for example  $V^{(1)}$  as a conformal three-point function (again assuming vanishing conformal spins  $n_i$ ),

$$V^{(1)} = \Lambda(\nu_a, \nu_b, \nu_c) \prod_{\substack{i < j \\ i, j \in \{a, b, c\}}} |\rho_{ij}|^{-2\Delta_{ij}}$$
(3)

with  $\Delta_{ab} = h_a + h_b - h_c$  and  $\rho_{ij} = \rho_i - \rho_j$ , it can be shown that

$$A_6(p_a, p_b, p_c, p_\delta, p_1, p_2) = \int d^2 \rho_\delta \left| \frac{\rho_{ca}}{\rho_{c\delta} \rho_{\delta a}} \right|^2 \Lambda(\nu_a, \nu_b, \nu_c)$$
(4)

is a Virasoro-Shapiro amplitude for the scattering of six closed string tachyon states after suitable identification of the string momenta with the scaling dimensions  $\nu_i$  of the Pomerons. More precisely, we have to identify the scalar products of the string momenta in the target space with combinations of the scaling dimensions. Similar relations have been found for the four-Pomeron vertex, and one can also find them for the higher Pomeron vertices which follow from our conjecture in the previous section. Intriguingly, that correspondence to string amplitudes holds also for vertices of Pomerons with nonvanishing conformal spins which can be related to amplitudes of excited string states.

At present, there are still many open questions concerning the relation of Pomeron vertices to string amplitudes. It turns out, for example, that the string amplitude for a given n-Pomeron vertex is not unique. Instead, there is only a minimal number of strings required for each given n. Due to that it is possible that the number of strings matches the number of gluons, but an identification of closed strings with Pomerons seems to be excluded. Other key problems are to interpret the Pomeron quantum numbers via string momenta in a suitable Minkowskian target space, and to find the meaning of the critical dimension of closed string theory in the context of Pomeron vertices. Finding an effective string theory for interacting Pomerons would open many interesting possibilities, including the computation of phenomenologically relevant Pomeron loop amplitudes.

### 5. SUMMARY

We have studied the color glass condensate in the approach based on the perturbative resummation of logarithms of the energy. The Pomeron vertices obtained in this approach exhibit bootstrap relations and hint at an underlying effective conformal field theory of interacting Pomerons. We find some evidence for the exciting possibility that the color glass condensate can even be described by an effective string theory.

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