

Comments on Diquarks, Strong Binding and a Large Hidden QCD Scale

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Abstract

We present arguments regarding the possible role of diquarks in low-energy hadron phenomenology that has so far escaped theorists' attention. Good diquarks, i.e. the 0^+ states of two quarks, are argued to have a two-component structure with one of the components peaking at distances several times shorter than a typical hadron size (a short-range core). This can play a role in solving two old puzzles of the 't Hooft $1/N$ expansion: strong quark-mass dependence of vacuum energy density and strong violations of the Okubo-Zweig-Iizuka (OZI) rule in quark-antiquark 0^\pm channels. In both cases empiric data defy 't Hooft's $1/N$ suppression. If good diquarks play a role at an intermediate energy scale they ruin 't Hooft's planarity because of their mixed-flavor composition. This new scale associated with good diquarks may be related to a numerically large scale discovered in [V. Novikov, M. Shifman, A. Vainshtein and V. Zakharov, Nucl. Phys. B **191**, 301 (1981)] in a number of phenomena mostly related to vacuum quantum numbers and 0^\pm glueball channels.

If $SU(3)_{\text{color}}$ of bona fide QCD is replaced by $SU(2)_{\text{color}}$, diquarks become well-defined gauge-invariant objects. Moreover, there is an exact symmetry relating them to pions. In this limit predictions regarding good diquarks are iron-clad. If passage from $SU(2)_{\text{color}}$ to $SU(3)_{\text{color}}$ does not lead to dramatic disturbances, these predictions remain qualitatively valid in bona fide QCD.

1 Introduction

The constituent quark model enjoyed a remarkable success in qualitative and semi-quantitative descriptions of a huge body of data on traditional hadron spectroscopy, static parameters and other regularities (for reviews, see e.g. [1, 2]). At the same time, since the advent of QCD it has been known that some important hadronic phenomena cannot be easily understood in this model. Probably, the most clear-cut example of this type is the masslessness of pion (in the chiral limit), which can only occur due to a “superstrong” attraction in the 0^- quark-antiquark channel. In the 1980’s, a similar strong attraction was argued to exist in the 0^+ quark-quark channel. This conjecture was supported by occasional observations of a special role played by diquarks in low-energy hadronic phenomenology.

Indications of diquark relevance, mentioned above, came (i) from the instanton side [3] (see also [4]). It was argued [3] that an instanton-induced interaction is sufficient to form a bound antitriplet scalar diquark. Then, (ii) there were suggestions [5] (see also [4]) that diquark correlations are important in the enhancement of the operator O_1 , which contributes to the $\Delta I = 1/2$ strangeness-changing decays. In fact, an enhancement of matrix elements of O_1 for the hyperon decays was first advocated in Ref. [6] where this operator, together with penguin ones, was used to calculate all $\Delta I = 1/2$ amplitudes. The penguin operators are enhanced by a strong correlation in the quark-antiquark channel. The diquark representation of the operator O_1 introduced in Ref. [6], (see Eqs. (50) through (55)) was used to fix sign of the O_1 contribution in the hyperon decay amplitudes and roughly estimate its absolute value. The sign was in agreement with experimental data for all S waves.

Recently, the issue was revitalized by experimental evidence for existence of exotic pentaquark baryon (see [7] for recent reviews), the state which was predicted to be narrow in Ref. [8] based on the chiral-quark-soliton model. Diquarks were suggested as an alternative explanation for pentaquark in a number of papers [9–14] which also discussed other phenomenological evidences. The implications are that diquark correlations seem to be instrumental in excited and exotic hadron spectroscopy and in other aspects of hadronic physics, and that “good” diquarks in color-flavor locked antisymmetric combination, due to a strong attraction, are probably bound to the extent that the “mass” of a good diquark roughly coincides with that of a constituent quark.

It is natural to suggest that the characteristic size of the good diquark as well as that of the constituent quark is considerably smaller than the nucleon size. In the case of the constituent quark a relevant momentum scale in the $\bar{q}q$ channel is of order of a few GeV. We will argue that a similar scale appears in the diquark qq channel. A

basic tool here is the observation that if $SU(3)_{\text{color}}$ is replaced by $SU(2)_{\text{color}}$, diquarks become well-defined gauge-invariant objects, related by symmetry to conventional Goldstone bosons (pions).

The existence of hierarchical scales in QCD was discussed long ago in [15]. The largest of these scales shows up in 0^\pm glueball channels. Recent striking (and unexpected) experimental evidence of a nonperturbative momentum scale much higher than Λ_{QCD} comes from heavy ion collisions at Relativistic Heavy Ion Collider (RHIC). There, instead of weakly coupled quark-gluon plasma at short distances, the pattern of hadron production shows hydrodynamical features and signals production of strongly interacting objects.

The existence of a hidden high scale could be helpful in understanding constituent quarks as small objects. In the diquark case a higher momentum scale implies a core-like structure similar to that of a pion. This opens up the possibility of understanding the small width of a pentaquark (if its experimental status is confirmed). In this paper we use the idea of hierarchical scales to analyze diquark features and related phenomenological signals. We also present arguments for special role of diquarks based on an expansion complementary to 't Hooft's large- N_c limit — the so-called “orientifold” large- N expansion [16].

It should be stressed that the introduction of diquarks in low-energy hadron phenomenology is not meant to replace the quark model [1]. The old idea that baryons are built of three (relatively weakly bound) constituent quarks while mesons are similar composites built of quark-antiquark pairs gives a reasonable overall picture of the hadronic world. It serves as a motivation for the $SU(6)$ -based description which produces a large number of results compatible with experiment. For instance, the ratio of proton-to-neutron magnetic moments, -1.47 , is very close to $-3/2$ as predicted in the quark model. The $SU(6)$ -based prediction of the ratio π -to- ρ charge radii is also consistent with what is known from experiment. The reader can easily continue this list.

The diquark short-range correlations we discuss below are meant to supplement the quark model by explaining a few subtle aspects of hadron phenomenology which do not come out right in the quark model. If we had a fully developed dynamical scheme of the quark-gluon low-energy interactions we would first confirm that this scheme is compatible with the quark model in all cases (the majority!) where the latter produces successful predictions, while diquarks show up only in certain aspects (they were mentioned above) where the quark model fails. Alas ... no fully developed dynamical model of this type is available at present. Consideration of diquarks in the present paper is carried out largely at the qualitative rather than quantitative level. In this sense our results are by no means “carved in stone.” Our main intention

is to provoke further discussion of the issue and related activities.

2 Diquarks' progenitors

The usefulness of diquark notion in hadronic physics is based on the assumption of an “abnormally strong” attraction in the 0^+ channel. Moreover, it is meaningful only if there are two scales separated by a rather large numerical factor: the size of a good diquark R_{dq} and that of a typical hadron R_h , which may be larger, say, by a factor of ~ 3 . (The smallness of the ratio R_{dq}/R_h is not parametric, but, rather, of a numerical nature.) Strongly bound diquarks must have a smaller typical size than that of typical hadrons. Otherwise the picture advocated in Refs. [9–14] could hardly be consistent.

Let us ask ourselves if we are familiar with other objects with similar dynamics. The answer is yes. In the 0^- $\bar{q}q$ channel a strong attraction between the quark and antiquark leads, in the chiral limit, to formation of massless pions. One can give a strong argument for the existence of a short-range core in the pion.

Indeed, let us compare two matrix elements,

$$\begin{aligned} \langle 0 | \bar{d} \gamma_\mu \gamma_5 u | \pi^+ \rangle &= i f_\pi q_\mu, \\ \langle 0 | \bar{d} \gamma_5 u | \pi^+ \rangle &= i f_\pi \frac{m_\pi^2}{m_u + m_d}. \end{aligned} \tag{1}$$

The second matrix element is enhanced by the ratio

$$\frac{m_\pi^2}{m_u + m_d} \approx 1.8 \text{ GeV}, \tag{2}$$

which has a smooth chiral limit $m_q \rightarrow 0$ but is rather large numerically, because of $m_\pi/(m_u + m_d) \approx 13$.

The 1^+ and 0^- currents, $\bar{d} \gamma_\mu \gamma_5 u$ and $\bar{d} \gamma_5 u$, look very different in the framework of QCD sum rules [17]. The first one works well and allows one to calculate f_π , while the second essentially does not work with perturbative Operator Product Expansion (OPE) coefficients. In other words, the intervals of duality are quite different,

$$\begin{aligned} \bar{d} \gamma_\mu \gamma_5 u &: & 0.6 \text{ GeV}^2, \\ \bar{d} \gamma_5 u &: & 1.9 \text{ GeV}^2. \end{aligned} \tag{3}$$

How can one interpret this fact? In the quantum-mechanical approach the matrix elements (1) can be viewed as a value of wave function at zero separation, $\psi(0)$. We

have two of them: the leading twist, $t = 2$ for $\bar{d}\gamma_\mu\gamma_5 u$ and nonleading twist, $t = 3$, for $\bar{d}\gamma_5 u$,

$$\psi = \alpha_2\psi_{t=2} + \alpha_3\psi_{t=3}. \quad (4)$$

The large value of $\psi_{t=3}(0)$ for the $\bar{d}\gamma_5 u$ component implies the existence of a smaller size. We try to illustrate this in Fig. 1. The factor $(1/3)^2$ for core size will be interpreted below in the framework of the instanton liquid model. Of course, the

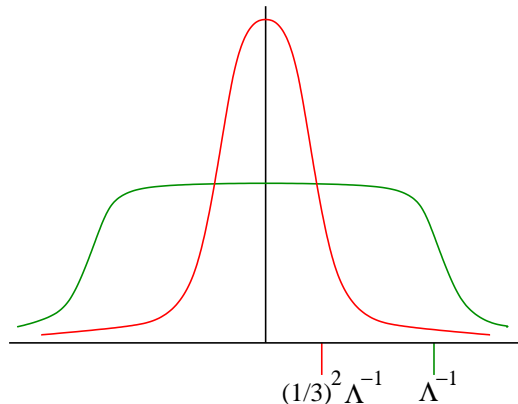


Figure 1: Double-component structure with a core.

picture is rather symbolic, not only characteristic sizes but also relative probability amplitudes (coefficients $\alpha_{2,3}$) influence results for different probes. Moreover, the profile of wave function of the leading twist should also involve a shorter scale. This follows from a smallness of $f_\pi \simeq 135$ MeV in comparison to the characteristic momentum scale $m_\rho \simeq 776$ MeV; note that $f_\pi \propto \sqrt{N_c}$ and grows with the number of colors while m_ρ is stable in this limit.

The instanton liquid model [18] provides a particular way of interpreting a hierarchy of scales. The model operates with two parameters, the average instanton size $\rho = 0.48 \Lambda_{\text{QCD}}^{-1}$ which is a factor of ~ 3 smaller than the average instanton separation $R = 1.35 \Lambda_{\text{QCD}}^{-1}$. Instantons are Euclidean objects and to relate their parameters with our Minkowski world, note that $R^{-1} \sim \Lambda_{\text{QCD}}$ is a typical hadronic scale while ρ^{-1} is the geometrical mean between Λ_{QCD} and the higher glueball scale Λ_{gl} . One can readily verify this at weak coupling in gauge theory with the adjoint Higgs field, where the inverse instanton size is the geometrical mean between W boson mass and that of monopoles (or spalerons for the Higgs field in the fundamental representation).

Hence, we conclude that in the glueball world $\Lambda_{\text{gl}} \sim 3^2 \Lambda_{\text{QCD}}$, which explains $(1/3)^2$ in Fig. 1. This is consistent with the estimate in [15] based on the low-energy theorem for the correlator of $G_{\mu\nu}G^{\mu\nu}$. Let us add that in the instanton liquid model

the smallness of f_π appears as $f_\pi \sim \rho/R^2 \sim \Lambda_{\text{QCD}}(\Lambda_{\text{QCD}}/\Lambda_{\text{gl}})^{1/2}$, while the large scale (2) is $m_\pi^2/(m_u + m_d) \sim R^2/\rho^3 \sim \Lambda_{\text{QCD}}(\Lambda_{\text{gl}}/\Lambda_{\text{QCD}})^{3/2}$.

If pions serve as a textbook dynamical example of a strongly bound quark-antiquark system, what can be said of diquarks? Of course, in discussing diquarks, it is rather difficult to stay fully quantitative since they are not defined as gauge-invariant objects in QCD. Is there a limit in which pions and diquarks are related by a rigorous symmetry?

Such a limit does exist with the assumption that replacing the SU(3) color group of QCD by SU(2) we do not impose a dramatic change on hadron dynamics (for a more precise statement see below). In two-flavor SU(2)_{color} QCD with massless quarks the flavor group is enhanced to SU(4), and the pattern of its spontaneous symmetry breaking is different from bona fide QCD [19, 20]. The reason is that in SU(2) the fundamental representation is (quasi)real, i.e. antiquarks form the same representations as quarks and interact with gluons the same way, too. Two flavors are composed of four two-component Weyl spinors (left-handed) and four complex conjugated ones (right-handed). In [20] it was shown, by 't Hooft's matching [21] of triangles, that the pattern of spontaneous breaking of flavor symmetry is as follows:

$$\text{SU}(4) \rightarrow \text{SO}(5). \quad (5)$$

This pattern was confirmed by other methods (for a recent review see [22]), and used to analyze different phases of the theory depending on chemical potential and temperature [23, 24]. The partnership of pions and diquarks in SU(2)_{color} was also discussed in Ref. [25].

To elucidate (5) let us note that the flavor doublets of the left-handed quarks ($u_{L\alpha}, d_{L\alpha}$) and antiquarks ($\bar{d}_{R\alpha}, -\bar{u}_{R\alpha}$) ($\alpha = 1, 2$) form the fundamental representation of SU(4)

$$\chi^i = \begin{pmatrix} u_L \\ d_L \\ \bar{d}_R \\ -\bar{u}_R \end{pmatrix}, \quad (6)$$

containing entries with different baryon numbers. The six bilinears $\chi^i \chi^j$ (convolution in space and color indices is implied) antisymmetric in i, j can be viewed as an O(6) vector. A nonvanishing vacuum average of this vector which can be aligned as

$$\langle \bar{u}u + \bar{d}d \rangle = \langle \bar{u}_R u_L + \bar{u}_L u_R + \bar{d}_R d_L + \bar{d}_L d_R \rangle \neq 0 \quad (7)$$

implies the spontaneous symmetry breaking which preserves SO(5). Acting on $\bar{u}u + \bar{d}d$

by SU(4) generators we determine five Goldstone bosons corresponding to operators

$$\begin{aligned} \bar{u}_R u_L - \bar{d}_R d_L - (R \leftrightarrow L) &= -\bar{u} \gamma_5 u + \bar{d} \gamma_5 d, \\ \bar{u}_R d_L - (R \leftrightarrow L) &= -\bar{u} \gamma_5 d, \quad \bar{d}_R u_L - (R \leftrightarrow L) = -\bar{d} \gamma_5 u, \end{aligned} \quad (8a)$$

$$u_L d_L + (R \leftrightarrow L) = -u C \gamma_5 d, \quad \bar{d}_L \bar{u}_L + (R \leftrightarrow L) = -\bar{d} C \gamma_5 \bar{u}. \quad (8b)$$

Here C is the charge conjugation matrix. In addition to the triplet of massless 0^- pions we get two “baryon” Goldstone bosons which are 0^+ diquark states.

In terms of symmetry currents, fifteen currents generating SU(4) are classified as ten currents which are generators of the linearly realized SO(5), plus five spontaneously broken currents, which have the form

$$\bar{u} \gamma_\mu \gamma_5 d, \quad \bar{d} \gamma_\mu \gamma_5 u, \quad \bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d, \quad (9a)$$

$$u C \gamma_\mu \gamma_5 d, \quad \bar{u} \gamma_\mu \gamma_5 C \bar{d}, \quad (9b)$$

and are coupled to the Goldstones.

Thus, we have more than the conventional triplet of pions; there emerge two extra diquark states related to pions by exact symmetry. Consequently, these diquark states also have the two-component structure reflecting the existence of a higher scale.

The question is, what happens to these states as we elevate the gauge group from $SU(2)_{\text{color}}$ to $SU(3)_{\text{color}}$? It is not difficult to see that the extra states become diquarks. This passage from $SU(2)_{\text{color}}$ to $SU(3)_{\text{color}}$ can be formalized if we introduce into $SU(3)_{\text{color}}$ theory an additional complex scalar field (“Higgs”) in the triplet representation, and analyze the theory as a function of the vacuum expectation value (VEV) of this field. It is well-known [26] that there is no phase transition in VEV – physics smoothly flow from the $SU(2)_{\text{color}}$ phase at large VEV to $SU(3)_{\text{color}}$ phase at small (vanishing) VEV of fundamental Higgs.

If $N_f > 2$ (and the color group is $SU(2)_{\text{color}}$) the flavor symmetry of the theory is $SU(2N_f)$. The pattern of spontaneous chiral symmetry breaking is [20, 22]

$$SU(2N_f) \rightarrow Sp(2N_f)$$

(note that $Sp(4)$ is the same as $SO(5)$). The linearly realized part of symmetry, $Sp(2N_f)$, has $2N_f^2 + N_f$ generators. There are $2N_f^2 - N_f - 1$ Goldstone states. Out of those, $N_f^2 - 1$ are conventional pions. The remaining $N_f^2 - N_f$ Goldstone states are “baryonic.” Upon lifting $SU(2)_{\text{color}}$ to $SU(3)_{\text{color}}$ these $N_f^2 - N_f$ baryonic Goldstones become diquarks.

Since in $SU(2)_{\text{color}}$ theory baryonic Goldstones and pions are related by an exact symmetry, their spatial structure is the same. In particular, the two-component structure depicted in Fig. 1 is shared by both. As we pass to $SU(3)_{\text{color}}$, the exact symmetry no longer holds, but an approximate similarity in the spatial structure of pions and diquarks is expected to hold.

The symmetry between pions and diquarks in $SU(2)_{\text{color}}$ theory gives us an idea of the momentum interval in which composite diquarks might play a role. If pion loops make any sense, this can only happen at $130 \text{ MeV} \lesssim p \lesssim 700 \text{ MeV}$. The same must be applicable to diquark loops.

In *bona fide* QCD, with $SU(3)_{\text{color}}$, in the energy interval between $\sim R_h^{-1}$ and $\sim R_{dq}^{-1}$ good diquarks act as pointlike color-antitriplet objects whose interaction with gluons is determined only by the color representation to which they belong, in much in the same way as color-triplet quarks.

3 Approaching from the large- N side

Everybody knows that 't Hooft's $1/N$ expansion [27] presents a powerful tool in qualitative and semiquantitative analysis of QCD, and as an organizational principle. A complementary “orientifold” large- N expansion was suggested recently [16] (see also [28]). This expansion is similar in spirit — but not technically — to expansion which goes under the name of *topological expansion* and was suggested long ago [29]. (Topological expansion assumes that the number of flavors N_f scales as N in the large- N limit, so that the ratio N_f/N is kept fixed. A quark-diquark representation of nucleon is natural in topological expansion.)

The orientifold large- N extrapolation treats quarks as Dirac fields in the two-index antisymmetric representation of the $SU(N)$ color group. At $N = 3$ the two-index antisymmetric representation is equivalent to fundamental; hence, the starting point for both extrapolations — orientifold and 't Hooft's — is *bona fide* $N = 3$ QCD, and both are equally suitable for $1/N$ expansion.

In most instances orientifold and 't Hooft large- N expansions lead to overlapping predictions. There are notable exceptions, however, of which the most important are:

- (i) quark-mass dependence of vacuum energy density, and
- (ii) the OZI rule.

In the former case, according to 't Hooft, quark-mass dependence should be suppressed by a factor $1/N$ as all quark loops scale as $1/N$ at large N . On the other

hand, in the orientifold theories quark loops are *not* suppressed, and one should expect quark-mass dependence of a “natural” order of magnitude (see below). The very same property, $1/N$ for each “additional” quark loop in the ’t Hooft limit, is supposed to explain inhibition of transitions between quarks of distinct flavors (the OZI rule). In the orientifold large- N limit quark loops do not carry $1/N$, and, correspondingly, transitions between quarks of distinct flavors are not suppressed by $1/N$.

At large N ’t Hooft and orientifold extrapolations present distinct theories, and, therefore, it is not surprising that some predictions do not coincide. There is one point, $N = 3$, where these theories do coincide, and, hence, must lead to the same results. Logically there are two options that would lead to a reconciliation at $N = 3$: a numerical suppression of appropriate amplitudes in orientifold theory or a numerical enhancement in ’t Hooft’s theory. We will argue that it is the latter option that is realized in certain important cases (see below).

On the phenomenological side, both points (i) and (ii) above, rather than being successful, present a serious challenge to the ’t Hooft approach. Indeed, two fundamental relations

$$\frac{d}{dm_q} \mathcal{E}_{\text{vac}} = \langle \bar{q} q \rangle \quad (10)$$

and

$$\mathcal{E}_{\text{vac}} \Big|_{m_q=0} = \frac{\beta(\alpha_s)}{16\alpha_s} \langle G_{\mu\nu}^a G^{\mu\nu a} \rangle \quad (11)$$

being combined and evaluated numerically [15] imply that in QCD with light quarks the vacuum energy density \mathcal{E}_{vac} changes by a factor of ~ 2 as the strange quark mass increases from zero to its actual value ~ 150 MeV. Moreover, while the extra quark-loop suppression inherent in the ’t Hooft N counting seemingly does explain the OZI rule in the 1^- channel, where the ϕ meson is an almost pure $\bar{s}s$ state, it badly fails in 0^\pm channels where flavor mixing is fully operative: say, the composition of η' meson only slightly deviates from $\bar{u}u + \bar{d}d + \bar{s}s$.

It was suggested [15] that the ’t Hooft-defying enhancement in these two cases is due to “direct” instantons. Instantons are theoretical objects existing in Euclidean space-time. It is natural to ask what actual physics lies behind this phenomenon. What is the relevant Minkowski-space picture? As was mentioned in Sect. 2, simultaneously, the existence of a new numerically large scale in hadronic physics was discovered in [15]. In phenomena where vacuum quantum numbers (e.g. the 0^\pm channels) were involved, a hidden energy scale, larger than the conventional ~ 300 MeV by a factor of ~ 10 or so, was revealed.

The space-time structure of good diquarks is (in Weyl notation): $\chi_\alpha^{[if} \chi^{jg]\alpha}$ where

square brackets denote antisymmetrization with respect to both color indices i, j and flavor indices f, g ; and α is the spinor index. Thus, they are bosons. Good diquarks are scalars. The flavor mixing is maximal in good diquarks. They appear, right from the start, as two-index antisymmetric objects in color representation. This naturally matches them with orientifold theory.

If N is treated as a free parameter, then diquarks propagating in loops with virtual momenta in the interval ($\sim R_h^{-1}$, $\sim R_{dq}^{-1}$) have no $1/N$ suppression. This simple observation suggests that they play a crucial role in strong quark-mass dependence of vacuum energy as well as in the failure of the OZI rule in 0^\pm channels, acting as Minkowski-space “representatives” of the Euclidean instantons much in the same way as sphalerons and monopoles are Minkowski signatures of instantons at weak coupling. Note that, being composite in flavor, diquark loops (with diquarks treated as pointlike objects) also negate the notion of planarity.

At $N = 3$ the failure of the OZI rule in 0^\pm channels should look like a numerical enhancement with regards to the 't Hooft counting. For instance, the VEV $\langle \bar{s}s \rangle$ is contributed by loops of (su) and (sd) diquarks; both can be scalar and pseudoscalar (see Fig. 2). This provides a numerical enhancement factor ~ 4 . Moreover, if this

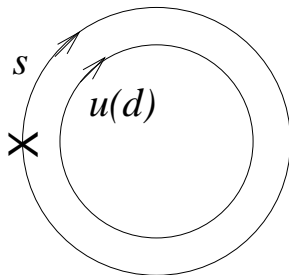


Figure 2: The diquark contribution to $\langle \bar{s}s \rangle$.

picture is correct, it is absolutely obvious that in 0^+ channels at low energies flavor mixing has no suppression whatsoever (the same is true for 0^-). An immediate consequence is that the correlation functions

$$\langle \bar{q}(x)q(x), \bar{q}(y)q(y) \rangle, \quad \langle \bar{q}(x)\gamma^5 q(x), \bar{q}(y)\gamma^5 q(y) \rangle \quad (12)$$

in addition to two-quark mesons in the intermediate state, must also exhibit four-quark exotic mesons.

Why then does flavor transition inhibition work so well in 1^- channels (φ meson decays)? The answer may be purely kinematical. In the $J = 1$ case the diquark pair must be in the P wave which increases effective energies to the extent that

diquarks no longer can be viewed as local; they are essentially destroyed and do not contribute in loops. The destruction of diquarks resurrects planarity. In orientifold large- N expansion the OZI rule in 1^- channels is explained by relatively large effective energies and numerical smallness of corresponding loop factors which become quite suppressing (two loops!).

4 Diquarks and weak nonleptonic decays of heavy baryons

As was mentioned in Sect. 1, the short-distance diquark component is presumably responsible for (a part of) the enhancement of nonleptonic decays of strange hyperons.¹ It is natural to ask whether a similar enhancement occurs for baryons containing heavy c and b quarks.

Let us start from b -containing baryons. The phenomenological issue here is that diquarks could play a role in reconciling experimental trends with theoretical expectations in the problem of lifetimes of b -containing hadrons, in particular, Λ_b . This issue has a long and dramatic history. Asymptotically (at $m_b \rightarrow \infty$) lifetimes of all b -containing hadrons must be equal. At finite but large m_b deviations of the ratio $\tau(\Lambda_b)/\tau(B_d)$ from unity can be calculated as an expansion in powers of $1/m_b$. A (formally) leading power correction m_b^{-2} is due to the dimension-5 chromomagnetic operator parameterized by μ_G^2 . It contributes at the level ~ -0.02 , see e.g. [30–32] and is rather unimportant. Moreover, this correction is flavor blind. Flavor-dependent and more important numerically are dimension-6 four-quark operators whose contribution is suppressed by m_b^{-3} . The original estimates [33] of four-quark operators in heavy baryon lifetimes were at the level ~ -0.02 .

At the same time, experimental measurements conducted in the 1990’s seemingly indicated that the ratio $\tau(\Lambda_b)/\tau(B_d)$ could be as small as ~ 0.8 , i.e. preasymptotic corrections as large as ~ -0.2 . This apparent discrepancy ignited a renewed theoretical effort and extensive debate in the literature [34–37] (the last paper reported lattice results). It is worth emphasizing that there are two distinct mechanisms responsible for m_b^{-3} corrections in $\tau(\Lambda_b)/\tau(B_d)$, namely, *Pauli interference* and *weak scattering*. Pauli interference works in the “unfavorable” direction, increasing the ratio $\tau(\Lambda_b)/\tau(B_d)$, while weak scattering tends to decrease it. Stretching estimates of the matrix elements of relevant four-quark operators in the “favorable” direction

¹ In addition to diquark correlations, the enhancement is also due to the quark-antiquark correlations, i.e. the penguin mechanism which, simultaneously, explains the enhancement in K meson decays [6].

one typically gets ~ 0.03 and ~ -0.07 for Pauli interference and weak scattering, respectively, so that there is significant compensation [34].

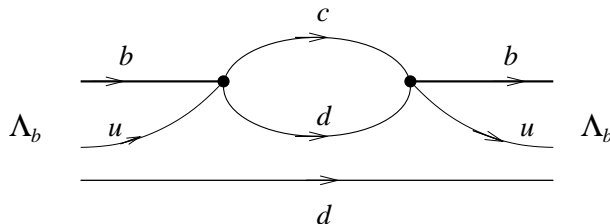


Figure 3: Preasymptotic m_b^{-3} corrections: the weak scattering mechanism giving rise to the four-quark operator (13) in the OPE for $\tau(\Lambda_b)$.

For our purposes it is important to note that the four-quark operators emerging in Pauli interference and weak scattering have spatial structures which do not coincide for these two mechanisms. In weak scattering the only relevant operator occurring in OPE (Fig. 3) has a structure “good diquark density times good diquark density,” of the type mentioned for O_1 in Ref. [6], namely,²

$$O_- = 2 (j_k)^\dagger (j_k), \quad j_k = \varepsilon_{kji} b^j C \frac{1 - \gamma_5}{2} u^i, \quad (13)$$

where i, j, k are color indices. The assumption that a strong (positive) diquark correlation persists in the 0^+ system of one heavy and one light quark, would result in a further enhancement of the weak scattering contribution in Λ_b , effectively destroying the cancellation between Pauli interference and weak scattering. In this case theoretical prediction $\tau(\Lambda_b)/\tau(B_d) \sim 0.9$ would become natural.

Note that recent experimental measurements of the ratio $\tau(\Lambda_b)/\tau(B_d)$ tend to shift the central value of the ratio from ~ 0.8 up to ~ 0.9 . Note also that the argumentation above is applicable to Ξ_b baryons as well.

One can give another argument in favor of a certain enhancement of the Λ_b matrix element of the operator (13), compared to the naive estimates of the 1980’s. This argument goes along the lines discussed in Sect. 2. There, reducing the color group from $SU(3)$ to $SU(2)$, we were able to relate 0^+ light diquarks to pions. If we follow the same strategy here, we will relate the (bu) good diquark to B mesons. Extensive analyses in the 1990’s firmly established the fact that f_B is quite large, $f_B \sim 200$ MeV, a factor of ~ 1.5 larger than was previously believed. In $SU(2)_{\text{color}}$

²Strictly speaking j_k represents a combination of the 0^+ and 0^- diquarks.

the factor of 2 enhancement of f_B^2 immediately translates into a similar enhancement of $\langle \Lambda_b | O_- | \Lambda_b \rangle$.

Now let us briefly comment on the situation with c -containing baryons. This case is marginal: on the one hand the charmed quark can be viewed as heavy, and heavy quark-mass expansion could be applied. As seen from explicit calculations [33,38], this expansion is rather poorly convergent and can be used only for qualitative estimates. This fact indicates that the c quark is not heavy enough. The hidden scale quoted in Eq. (2) is actually of the same order as m_c .

In this marginal situation one can try the opposite description, treating the c quark as “light.” The cd diquark plays a similar role in c -baryon nonleptonic decays as the ud diquark in hyperon decays. What is different is that the penguin mechanism plays no significant role in total widths of c -containing hadrons. That is why one can qualitatively expect a relative enhancement of all c -baryon decays over meson ones. This expectation is indeed confirmed by experimental data.

5 Hidden scale and deep inelastic processes

Deep inelastic scattering (DIS) provides a good probe for diquark correlations in nucleons. At first sight an immediate consequence of these correlations is an enhancement of the higher-twist correction which carries an extra power of $1/Q^2$, namely $Q^{(-t+2)}$. For local operators twist t is the difference between its dimension and spin. The nucleon average of higher-twist operators containing the diquark combinations of fermion fields becomes large if the corelike diquark structure is present.

It is simple, however, to demonstrate [39] that the effect does not show up at the twist-4 (next-to-leading) level. The leading level is twist $t = 2$. Indeed, to generate a twist-4 effect the corresponding operator should contain diquark combinations of fermion fields in a form similar to j_k in Eq. (13),

$$\epsilon^{\alpha\beta} \epsilon_{ijk} \epsilon_{fgh} q_{L\alpha}^{if} q_{L\beta}^{jg}, \quad \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon_{ijk} \epsilon_{fgh} q_{R\dot{\alpha}}^{if} q_{R\dot{\beta}}^{jg}, \quad (14)$$

where $\alpha, \beta, \dot{\alpha}, \dot{\beta} = 1, 2$ are Weyl spinor indices, $i, j, k = 1, 2, 3$ and $f, g, h = 1, 2, 3$ refer to quark colors and flavors. The twist of these combinations equals 3 (it remains intact if some number of covariant derivatives D_μ are inserted). For DIS we consider operators with vanishing baryon charge; therefore, diquark combinations (14) must be multiplied by operators $\bar{q}D \dots D\bar{q}$. The minimal twist of these operators is 2, so the total twist is not less than 5. Actually the minimal total twist is 6. Twist 5 is excluded because of the chiral features of DIS operators under $SU(3)_L \times SU(3)_R$ transformations.

Thus, diquark correlations do not show up at the level of $1/Q^2$ corrections. This is good because no enhancement in these corrections was observed experimentally. What could remain is an enhancement for twist 6, i.e. in the $1/Q^4$ corrections.

On the other hand, once we consider the range of moderate momentum transfers Q , smaller than the hidden scale, we can view corelike diquarks as pointlike spin-zero “quarks.” This implies a strong impact on the longitudinal structure functions which vanish for spin 1/2 partons but not for spin 0. What do experimental data show for longitudinal cross sections σ_L in the range of relatively low $Q^2 < 3 \text{ GeV}^2$?

A glance at the data for σ_L [40–42] shows that σ_L maximizes at $Q^2 \sim 2 \text{ GeV}^2$ (let us recall that σ_L vanishes at $Q^2 = 0$ and at infinity). The amplitude of the effect was first interpreted [40] as excess over predictions based on perturbative QCD (with target mass corrections accounted), i.e. as an indication of nonperturbative higher-twist effects. However, later it was shown in Ref. [41] that inclusion of next-to-next-to leading (NNLO) terms in the perturbative fit allows one to get rid of higher twists.

We think that in the considered range of Q^2 untangling of NNLO corrections from nonperturbative effects is ambiguous. In perturbation theory the observed enhancement implies rather large higher-order corrections, of the order of the effect itself. Such a situation could leave space for a significant role for nonperturbative effects. Thus, the possibility of a diquark enhancement in σ_L is not completely excluded. A dedicated analysis of the moments of longitudinal structure functions at moderate Q^2 could shed light on the issue.

6 Conclusions

This paper could have been called “Connecting diquarks to pions,” or “Diquarks and a Large Scale,” or “Diquarks vs. $1/N$ Expansions.” We discussed various approaches allowing one to attempt to quantify the role of diquarks in hadronic physics. The most solid consideration, albeit somewhat remote from bona fide QCD, is that based on $SU(2)_{\text{color}}$. Reducing the gauge group from $SU(3)$ to $SU(2)$ allows one to relate diquarks and pions through a global symmetry which exists only for $SU(2)_{\text{color}}$. Diquarks become well-defined gauge-invariant objects, which share with pions a two-component structure with a relatively short-range core. Then one can speculate, qualitatively or, with luck, semiquantitatively on what remains of this symmetry upon lifting $SU(2)_{\text{color}}$ to $SU(3)_{\text{color}}$. It is worth noting that all instanton-based calculations carry a strong imprint of the above symmetry since basic instantons are, in essence, $SU(2)_{\text{color}}$ objects.

In the case of pions a hidden large scale is known to exist based on various arguments. Our task was to investigate consequences of the existence of this scale

in diquarks. We argue that short-range diquark correlations play an important role in QCD vacuum structure: they help resolve the apparent contradiction with large- N expectations for vacuum structure. We also discuss consequences of diquarks in nonleptonic baryon decays. In all cases — hyperons, c - and b -containing baryons — a short-range diquark core seems to be instrumental in understanding existing phenomenology.

Finally, we addressed the issue of diquarks in application to deep inelastic scattering. In particular, they could show up in longitudinal structure functions at moderate Q^2 . The current experimental situation seems to be inconclusive and calls for further analysis.

The issue of diquarks was raised and discussed mostly in application to hadron spectroscopy in Refs. [9–14]. We extended the discussion by including for consideration a hidden large scale implying a short-range core in diquarks. We tested the idea in a range of applications other than hadronic spectroscopy. The idea survived these tests. However, our considerations are essentially qualitative; more quantitative analyses are most welcome.

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