

How a Non-hierarchical Neutrino Mass Matrix Can Arise

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Abstract

One puzzle of neutrino masses and mixings is that they do not exhibit the kind of strong “hierarchy” that is found for the quarks and charged leptons. Neutrino mass ratios and mixing angles are not small. A possible reason for this is proposed here. It is based on the fact that typical realistic grand unified models contain particles with unification-scale masses which, when integrated out, can yield a neutrino mass matrix that is not of the standard seesaw form.

In the standard (or type I) see-saw mechanism [1], the observed left-handed neutrinos derive their masses from their Dirac couplings to right-handed neutrinos. These right-handed neutrinos have Majorana masses that in grand unified theories (GUTs) are expected to be of order the unified scale M_{GUT} . When the right-handed neutrinos are integrated out, an effective $d = 5$ operator is induced that produces a mass matrix for the left-handed neutrinos given by the well-known “see-saw formula”:

$$M_\nu = -M_N M_R^{-1} M_N^T. \quad (1)$$

Here M_N is the Dirac mass matrix of the neutrinos, whose elements are less than or of order the Weak scale. M_R is the Majorana mass matrix of the right-handed neutrinos. And M_ν is the effective mass matrix of the light left-handed neutrinos.

In GUTs based on $SO(10)$ or related groups, there is typically a close connection between the Dirac mass matrix of the neutrinos M_N and the (Dirac) mass matrices of the up quarks, down quarks, and charged leptons (M_U , M_D , and M_L respectively). For instance, in minimal $SO(10)$ these matrices are all proportional. The small interfamily mass ratios and small CKM angles suggest that these other mass matrices are “hierarchical” in structure (i.e. their entries decrease as one goes up and to the left in the matrix, in the usual convention). This would lead us to expect a hierarchical structure also for M_N . In that case, the see-saw formula would make M_ν hierarchical unless there were a “Dirac-Majorana conspiracy” [2], i.e. unless there were hierarchies in both M_N and M_R that practically cancelled each other out in M_ν . Such a conspiracy is not easy to achieve in a natural way.

On the other hand, the experimental evidence suggests that the light neutrinos have at most a very weak hierarchy. The solar mixing angle and atmospheric mixing angle are large, and the ratio of neutrino masses m_2/m_3 is not very small either, about $\frac{1}{6}$ judging from the fact that $\delta m_{atm}^2 \sim 3 \times 10^{-3}$ eV [3] and $\delta m_{atm}^2 \sim 7.3 \times 10^{-5}$ eV [4]. Thus we have a problem: if M_ν is given by the standard see-saw formula, why doesn’t it exhibit a strong interfamily hierarchy? In this letter we suggest a possible solution.

In realistic grand unified models, it is typical that there are many fermions with masses of order M_{GUT} besides the three right-handed neutrinos. When integrated out, these other superheavy fermions can give effective higher-dimension Yukawa operators that contribute to the masses of the observed quarks and leptons. In fact, in many models, this is precisely the way in which the quarks and leptons of the lighter families get their small masses, i.e. the way the interfamily mass hierarchies arise. (See [5, 6] for examples). The crucial point for the present discussion is that these superheavy fermion multiplets typically contain neutrino-like states; and these, when integrated out, give contributions to the masses of the observed light neutrinos that are not necessarily of the standard (or “type I”) see-saw form shown in Eq. (1). These non-type-I contributions to M_ν do not involve the effective Dirac neutrino mass matrix M_N and consequently there is no reason why they have to be hierarchical.

The possibility of non-type-I contributions to M_ν arising from integrating out superheavy fermion multiplets in grand unified models was analyzed in a general way in Ref. [7]. Here we present a concrete model that illustrates in a transparent way how a non-hierarchical M_ν can arise from these contributions

even when family symmetry causes M_N , M_U , M_D , and M_L to be hierarchical.

Consider an $SO(10)$ model in which, in addition to the three families (contained in three spinor multiplets, denoted $\mathbf{16}_i$, $i = 1, 2, 3$), there are three family-antifamily pairs, denoted $\mathbf{16}'_i$, $\overline{\mathbf{16}}'_i$, $i = 1, 2, 3$. There is a vector multiplet of Higgs fields, denoted $\mathbf{10}_H$, which contains the MSSM doublets of Higgs, H_u and H_d . There are three Higgs multiplets denoted Ω_i , $i = 1, 2, 3$, which contain components that are neutral under the Standard Model gauge group, have $B = L = 0$, and obtain $O(M_{GUT})$ vacuum expectation values (VEVs). These will break a $U(1)$ family symmetry and play the role of “familons” or “Froggatt-Nielson fields” [5].

Under the family symmetry, called $U(1)_F$, the multiplets just itemized have the following charges. The $\mathbf{16}_i$ and $\overline{\mathbf{16}}'_i$ have charge q_i , and the $\mathbf{16}'_i$ has charge $-q_i$, where $q_1 = y$, $q_2 = z$, and $q_3 = 0$. The Higgs multiplet $\mathbf{10}_H$ is neutral under $U(1)_F$, and the Froggatt-Nielson fields Ω_1 , Ω_2 , and Ω_3 have charges $(-y - z)$, $-z$, and 0, respectively.

These assignments allow the following renormalizable terms to appear in the Yukawa part of the superpotential.

$$\begin{aligned}
W_{Yukawa} &= \sum_i M_i (\overline{\mathbf{16}}'_i \mathbf{16}'_i) + (\overline{\mathbf{16}}'_3 \mathbf{16}'_3) \Omega_3 \\
&+ \sum_i (\mathbf{16}_i \mathbf{16}'_i) \mathbf{10}_H + (\mathbf{16}_3 \mathbf{16}_3) \mathbf{10}_H + (\mathbf{16}'_3 \mathbf{16}'_3) \mathbf{10}_H \\
&+ (\overline{\mathbf{16}}'_3 \mathbf{16}_2 + \overline{\mathbf{16}}'_2 \mathbf{16}_3) \Omega_2 + (\overline{\mathbf{16}}'_2 \mathbf{16}_1 + \overline{\mathbf{16}}'_1 \mathbf{16}_2) \Omega_1 + (\overline{\mathbf{16}}'_3 \mathbf{16}_3) \Omega_3.
\end{aligned} \tag{2}$$

We do not show the dimensionless Yukawa couplings, which we assume all to be $O(1)$. Note that the explicit mass terms and the couplings of the $\mathbf{10}_H$ are both flavor-diagonal. The non-trivial flavor structure in the model comes from the couplings of Ω_i

In addition to all of these multiplets, we require there to be Higgs that break $B - L$ at the GUT scale. We take these to be in several $\overline{\mathbf{126}}_H$ and $\mathbf{126}_H$ multiplets. (There are reasons, including not having the unified coupling blow up as it runs from M_{GUT} to M_{Pl} , that it might be more realistic to assume that $B - L$ is broken by spinors of Higgs fields. However, for simplicity of exposition, we assume that it is broken by $\overline{\mathbf{126}}_H$ and $\mathbf{126}_H$. It would be quite straightforward to modify the model to replace the $\overline{\mathbf{126}}_H$ and $\mathbf{126}_H$ by spinor Higgs.) The mass matrix M_R of the right-handed neutrinos comes from terms of the form $(\mathbf{16}_i \mathbf{16}_j) \overline{\mathbf{126}}_H$. We will also assume that a mass matrix \overline{M}'_R arises

from terms of the form $(\overline{\mathbf{16}}'_i \overline{\mathbf{16}}'_j) \mathbf{126}_H$. The $\overline{\mathbf{126}}_H$ and $\mathbf{126}_H$ multiplets will have whatever $U(1)_F$ charges are required to allow these couplings.

The family hierarchy is assumed to arise from a hierarchy in the VEVs of the familons, with

$$\Omega_1 \ll \Omega_2 \ll \Omega_3. \quad (3)$$

Before considering neutrino masses, let us see how the hierarchies in M_U , M_D , and M_L arise. For specificity, let us consider M_L . The multiplets $\mathbf{16}_i$, $\mathbf{16}'_i$, and $\overline{\mathbf{16}}'_i$ contain charged leptons that we will denote (ℓ_i^+, ℓ_i^-) , $(\ell_i^{+'}, \ell_i^{-'})$, and $(\overline{\ell}_i^{+'}, \overline{\ell}_i^{-'})$, respectively. If the mixing of the $\mathbf{16}_i$ and $\mathbf{16}'_i$ is small, then it is approximately the case that the light leptons are contained in the ‘‘unprimed multiplets’’ $\mathbf{16}_i$ while the superheavy leptons are contained in the ‘‘primed multiplets’’ $\overline{\mathbf{16}}'_i$ and $\mathbf{16}'_i$. There is a $d = 4$ operator that contributes directly to M_L , namely $(\ell_3^- \ell_3^+) H_d$, coming from the term $(\mathbf{16}_3 \mathbf{16}_3) \mathbf{10}_H$ in Eq. (2). In addition, M_L receives contributions from integrating out the superheavy (primed) leptons, as follows.

The terms in Eq. (2) include leptons mass terms of the form

$$\begin{aligned} W_{lepton} \supset & \sum_{ij} M_{ij} (\overline{\ell}_i^{-'} \ell_j^{-'} + \overline{\ell}_i^{+'} \ell_j^{+'}) \\ & + \sum_{ij} (M_\Omega)_{ij} (\overline{\ell}_i^{-'} \ell_j^- + \overline{\ell}_i^{+'} \ell_j^+) \\ & + \sum_{ij} (m_H)_{ij} (\ell_i^- \ell_j^{+'} + \ell_i^+ \ell_j^{-'}). \end{aligned} \quad (4)$$

where these matrices have the forms

$$M = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \quad M_\Omega = \begin{pmatrix} 0 & \omega_1 & 0 \\ \omega_1 & 0 & \omega_2 \\ 0 & \omega_2 & \omega_3 \end{pmatrix}, \quad m_H = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \quad (5)$$

The matrices M and M_Ω have elements of $O(M_{GUT})$, while the elements of m_H are of order the weak scale. And $\omega_1 \ll \omega_2 \ll \omega_3$, $M_1 \sim M_2 \sim M_3$, $m_1 \sim m_2 \sim m_3$. When the superheavy, primed leptons are integrated out, the diagrams of Fig. 1 lead to a contribution to M_L of the form

$$\begin{aligned} \delta M_L & = m_H M^{-1} M_\Omega + \text{Transpose} \\ & = \begin{pmatrix} 0 & \overline{\omega}_1 & 0 \\ \overline{\omega}_1 & 0 & \overline{\omega}_2 \\ 0 & \overline{\omega}_2 & \overline{\omega}_3 \end{pmatrix}, \end{aligned} \quad (6)$$

where $\bar{\omega}_1 = \omega_1(\frac{m_1}{M_1} + \frac{m_2}{M_2})$, $\bar{\omega}_2 = \omega_2(\frac{m_2}{M_2} + \frac{m_3}{M_3})$, and $\bar{\omega}_3 = \omega_3(2\frac{m_3}{M_3})$. Note the hierarchical form of δM_L . The $d = 4$ operator $\ell_3^- \ell_3^+ H_d$ just gives a contribution to the 33 element that is of the same order as the 33 element of δM_L .

Since the same types of diagrams give the mass matrices M_U , M_D , and M_N , these matrices are all related. In fact, if the fields Ω_i were $SO(10)$ singlets, these matrices would not “know” that $SO(10)$ is broken, and one would have the minimal $SO(10)$ relations $M_L = M_D \propto M_U = M_N$. If, on the other hand, some of the Ω_i are adjoints or other non-singlets, then Clebsch coefficients can appear that distinguish the matrices of fermions of different types (i.e. of different Standard Model quantum numbers).

The Dirac mass matrix M_N thus arises both from the direct $d = 4$ term $(\nu_3 N_3^c) H_u$ (from $(\mathbf{16}_3 \mathbf{16}_3) \mathbf{10}_H$) and from the diagrams of Fig. 2. Like M_L , it is hierarchical in form. The right-handed neutrinos N_i^c get superlarge Majorana mass matrix $(M_R)_{ij} (N_i^c N_j^c)$ from terms of the form $(\mathbf{16}_i \mathbf{16}_j) \overline{\mathbf{126}}_H$. Consequently, one has the type I see-saw contribution $M_\nu^{type-I} = -M_N M_R^{-1} M_N^T$, which arises from graphs of the kind shown in Fig. 3. However, these are not the only kind of graphs that can contribute to M_ν . If there are terms of the form $(\overline{\mathbf{16}}'_i \overline{\mathbf{16}}'_j) \mathbf{126}_H$, then a simpler graph, shown in Fig. 4, contributes. Basically, this graph “short circuits” the usual type-I diagrams by eliminating the standard right-handed neutrinos N_i^c as intermediate particles. To put it another way, it “eliminates the middle-man”. This graph gives

$$M_\nu^{non-type-I} = m_H M^{-1T} \overline{M}'_R M^{-1} m_H^T. \quad (7)$$

Note that the graph in Fig. 4 does *not* contain insertions of the VEVs of the Froggatt-Nielson fields Ω_i , and therefore does not “know” about the family hierarchy. Consequently, this non-type-I contribution to M_ν has no reason to be hierarchical and will not in general be so. Moreover, since these non-type-I contributions do not depend on the small parameters Ω_1/Ω_3 and Ω_2/Ω_3 , they will dominate over the type-I contributions, except perhaps for the 33 element where they should be comparable. One expects, therefore, neutrino mixing angles and neutrino mass ratios that are of order one, as is indeed observed.

It should be noted that if the term $(\mathbf{16}_i \mathbf{16}_j) \overline{\mathbf{126}}_H$ does not exist, i.e. the matrix $M_R = 0$, then the form of M_ν is not well approximated by Eq. (7). The diagrams can be misleading, and one must really do the matrix

diagonalization to find out the form of M_ν . However, if both M_R and \overline{M}'_R are non-zero and M_R is non-singular, then M_ν is given approximately by the sum of Eq. (1) and Eq.(7).

One might ask whether the even simpler kind of diagram shown in Fig. 5 might give a non-type-I, hierarchical contribution to M_ν if there were terms of the form $(\mathbf{16}'\mathbf{16}')\overline{\mathbf{126}}_H$. It would appear so from the structure of the diagram; but it turns out that no non-type-I contribution of this kind arises unless there is also a contribution of the kind given in Fig. 4 and Eq. (7). This is not obvious from inspection of the diagrams, but can be shown by looking in detail at the full neutrino mass matrix diagonalization problem, as was done in the general analysis of Ref. [7].

In conclusion, we see that in realistic GUT models there can be additional contributions to M_ν that are not of the type I form, and that these contributions can be non-hierarchical even if a family symmetry enforces a hierarchical pattern for all the Dirac mass matrices, that of the neutrinos (M_N) and those of the up quarks, down quarks, and charged leptons (M_U , M_D , M_L). This is due to the fact that there typically exist in realistic GUT models superheavy neutrino-like fermions besides the three right-handed neutrinos N^c usually taken into account. Integrating out these other states can induce contributions to M_ν that do not involve the N^c at all, and thus do not depend on the form of the Dirac neutrino mass matrix M_N . Finally, it should be mentioned that a recent paper of Nir and Shadmi [8] has interesting points of contact with the present work. For a discussion of the similarities and differences of their approach and the ideas on which the present work is based, see the discussion at the end of [7].

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Figure Captions

Fig. 1: A contribution to the charged lepton mass matrix M_L coming from integrating out the $O(M_{GUT})$ charged leptons in the “primed” multiplets (denoted by the double lines). The coupling to the familons Ω gives the family hierarchy.

Fig. 2: A contribution to the neutrino Dirac mass M_N coming from integrating out the $O(M_{GUT})$ neutrinos in the “primed” multiplets (denoted by double lines). These diagrams are related by $SO(10)$ symmetry to those in Fig. 1.

Fig. 3: A typical standard (or type I) seesaw contribution to M_ν . The double lines represent the $O(M_{GUT})$ fermions that are integrated out to generate M_N . (See Fig. 2.)

Fig. 4: A contribution to M_ν that is not of the standard type I form. It does not involve the standard right-handed neutrinos N_i^c of the three families. It also does not involve the familon field Ω and is not therefore hierarchical in structure.

Fig. 5: A diagram that seems to give a non-type-I contribution to M_ν but generally does not.

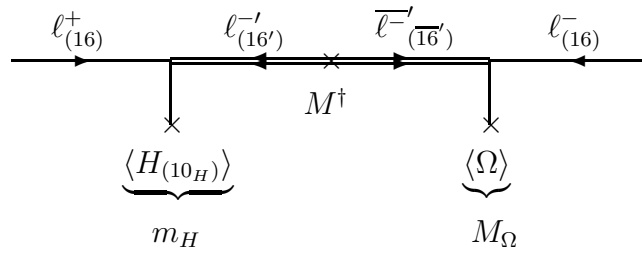


Fig. 1(a)

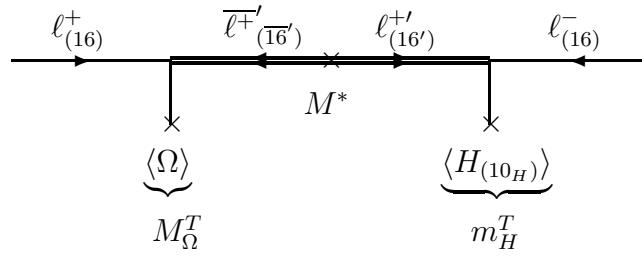


Fig. 1(b)

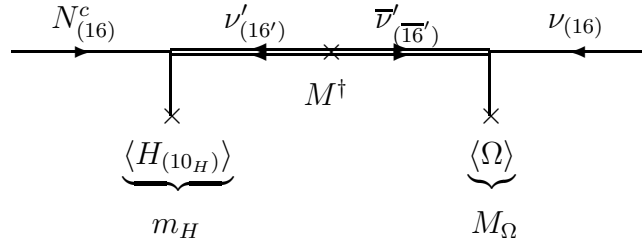


Fig. 2(a)

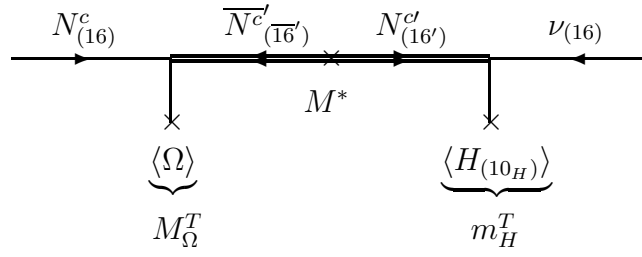


Fig. 2(b)

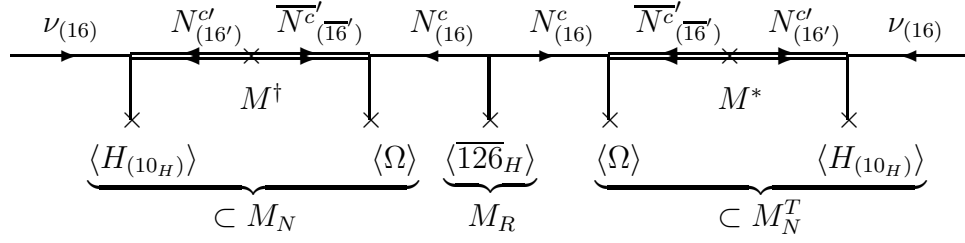


Fig. 3

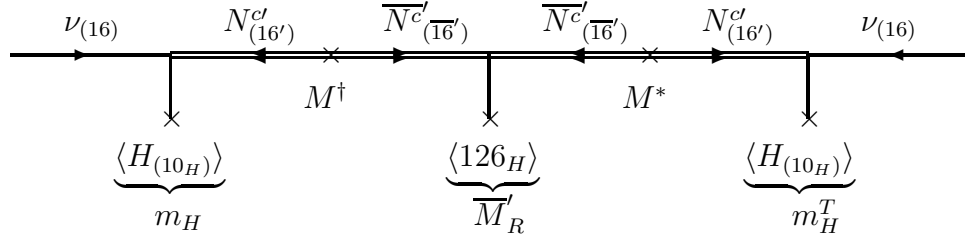


Fig. 4

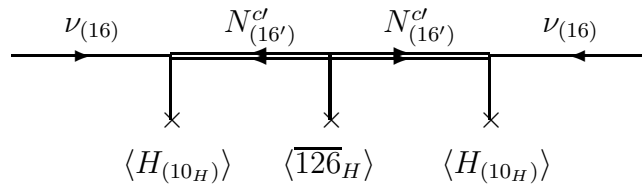


Fig. 5