

# Pulsar kicks via spin-1 color superconductivity

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We propose a new neutrino propulsion mechanism for neutron stars which can lead to strong velocity kicks, needed to explain the observed bimodal velocity distribution of pulsars. The spatial asymmetry in the neutrino emission is naturally provided by a stellar core containing spin-1 color-superconducting quark matter in the A phase. The neutrino propulsion mechanism switches on when the stellar core temperature drops below the transition temperature of this phase.

*Introduction.*—The first pulsar was discovered more than 37 years ago [1]. Since then, pulsars remain among the most interesting celestial objects in our Galaxy. Their observation allows to test experimentally the existence of the gravitational radiation as predicted by general relativity [2]. The recent discovery of a double-pulsar system [3] is likely to further strengthen the status of pulsars as an astrophysical laboratory for testing general relativity. In this Letter, we suggest that the physics of pulsars may also serve as the ultimate laboratory for testing the theory of strong interactions, quantum chromodynamics (QCD), in the regime of large baryon density.

Matter at large baryon density is expected to be deconfined and color-superconducting (for reviews see, e.g., Ref. [4]). Color-superconducting quark matter could exist inside neutron stars, whose central densities are highest in Nature. Therefore, it is important to study the physical implications of such a possibility in detail. At present, this is not easy because our current knowledge regarding the ground state of neutral,  $\beta$ -equilibrated dense matter is very limited [5, 6]. Many color superconducting phases were proposed [7, 8, 9, 10, 11, 12], but it is not clear which of them can be realized inside stars.

It has been known for a long time that typical spatial velocities of pulsars are an order of magnitude larger than those of their progenitors [13, 14]. Taking into account the violent conditions at the pulsar birth, this may not be so surprising. Even a small asymmetry in the supernova explosion may result in a kick velocity of several hundred  $\text{km s}^{-1}$  [15]. However, the bimodal velocity distribution of pulsars [16, 17], which is unlikely to result from a single physical mechanism, might be more surprising. If one associates the low-velocity component ( $\lesssim 100 \text{ km s}^{-1}$ ) with asymmetric supernova explosions, then what is the origin of the high-velocity component ( $\gtrsim 500 \text{ km s}^{-1}$ )? Several mechanisms were proposed [18, 19, 20], but the issue does not seem to be settled. For short reviews see, for example, Ref. [21].

In this Letter, we propose a new neutrino propulsion mechanism, resulting from a color superconductor in the transverse A phase [22]. In this phase, quarks of the same flavor form Cooper pairs with total spin one. The neu-

trino emission from this phase, dominated by direct Urca-type processes, is not symmetric in space. This emission, as we shall see, provides a natural mechanism to power strong, e.g., of order  $1000 \text{ km s}^{-1}$ , velocity kicks for neutron stars, which could explain the high-velocity component in the pulsar distribution [16, 17]. Unlike most other mechanisms, this one turns on not immediately after the supernova explosion, but after the temperature of the stellar core drops below the critical temperature of the A phase. A distinctive prediction of this mechanism is the alignment of the kick velocity direction with the rotational axis.

*Neutrino emission.*—Let us start by outlining the main steps in the derivation of a general expression for the neutrino emissivity in spin-1 color superconducting phases. We use the Kadanoff-Baym formalism [23, 24] to derive the following differential expression for the emissivity [25]:

$$\begin{aligned} \frac{d\epsilon_\nu}{dp_\nu d\Omega_\nu} &= \frac{G_F^2}{8(2\pi)^6} \int p_e dp_e \int d\Omega_e p_\nu^2 n_B(p_\nu - p_e + \mu_e) \\ &\times n_F(p_e - \mu_e) L_{\lambda\sigma}(P_e, P_\nu) \text{Im}\Pi_R^{\lambda\sigma}(\delta P_e - P_\nu), \end{aligned} \quad (1)$$

where  $G_F$  is the Fermi coupling constant,  $\mu_e$  is the electron chemical potential, and  $\delta P_e^\lambda \equiv P_e^\lambda - \delta_0^\lambda \mu_e$ . Here, particle four-momenta are denoted by capital Latin letters, while the absolute values of the three-momenta are denoted by lowercase letters. The metric tensor is  $g_{\lambda\sigma} = \text{diag}(1, -1, -1, -1)$ . The Bose and the Fermi distribution functions are denoted by  $n_B(\omega) \equiv [\exp(\omega/T) - 1]^{-1}$  and  $n_F(\omega) \equiv [\exp(\omega/T) + 1]^{-1}$ , respectively. The lepton tensor  $L_{\lambda\sigma}(P_e, P_\nu)$  is defined as follows:

$$L_{\lambda\sigma}(P_e, P_\nu) = \text{Tr} [P_e^\kappa \gamma_\kappa \gamma_\sigma (1 - \gamma^5) P_\nu^\rho \gamma_\rho \gamma_\lambda (1 - \gamma^5)]. \quad (2)$$

Finally, the last factor in the integrand on the right hand side of Eq. (1) is the imaginary part of the retarded polarization tensor of the  $W$ -boson,

$$\Pi^{\lambda\sigma}(Q) = \frac{T}{2} \sum_n \int \frac{d^3\mathbf{k}}{(2\pi)^3} \text{Tr} [\Gamma_-^\lambda S(K) \Gamma_+^\sigma S(K+Q)], \quad (3)$$

with the trace running over flavor, color, Dirac and Nambu-Gorkov indices. The quark propagator  $S(K)$  is diagonal in flavor space, and its components have the following Nambu-Gorkov structure:

$$S_f = \begin{pmatrix} G_f^+(K) & \Xi_f^-(K) \\ \Xi_f^+(K) & G_f^-(K) \end{pmatrix}, \quad \text{for } f = u, d. \quad (4)$$

We consider the ultrarelativistic limit. The explicit color and Dirac structure of  $G_f^\pm$  and  $\Xi_f^\pm$  can be found in Ref. [22]. Here, we note only that the poles of the quark propagators appear at  $k_0 = k + \mu_f$  (antiquarks) and at

$$k_0 = \epsilon_{k,r,f} \equiv \sqrt{(k - \mu_f)^2 + \lambda_{k,r}|\phi_f|^2}, \quad r = 1, 2, 3, \quad (5)$$

where  $\phi_f$  is the gap parameter, and the functions  $\lambda_{k,r}$  are specified by the choice of the phase.

The order parameter of the A phase has a special direction in color space: quarks of one color do not pair. Also, it has a special direction in momentum space, say, the  $z$ -direction. If  $\theta_{\mathbf{k}}$  denotes the angle between the three-momentum of a quasiparticle and the  $z$ -axis, the three low-energy quasiparticle modes in the transverse A phase are defined by  $\lambda_{k,1} = (1 + |\cos \theta_{\mathbf{k}}|)^2$ ,  $\lambda_{k,2} = (1 - |\cos \theta_{\mathbf{k}}|)^2$ , and  $\lambda_{k,3} = 0$ . Here, ‘‘transverse’’ refers to the fact that quarks of opposite chirality form Cooper pairs.

The explicit expressions of the vertices in Eq. (3) read

$$\Gamma_\pm^\lambda = \begin{pmatrix} \gamma^\lambda(1 - \gamma^5)\tau_\pm & 0 \\ 0 & -\gamma^\lambda(1 + \gamma^5)\tau_\mp \end{pmatrix}, \quad (6)$$

where the flavor matrix  $\tau_\pm \equiv (\tau_1 \pm i\tau_2)/2$  is constructed from Pauli matrices.

By substituting the quark propagators (4) and the vertices (6) into Eq. (3), we calculate the imaginary part of the retarded polarization tensor. Then, by making use of the result, we derive an expression for the emissivity in the following approximate form [25]:

$$\begin{aligned} \epsilon_{\text{tot}} &\approx 2 \left( \epsilon_\nu^{(11)} + \epsilon_\nu^{(22)} + \epsilon_\nu^{(33)} \right) \\ &\approx \frac{457}{630} \left[ \frac{2}{3} G \left( \frac{\phi_u}{T}, \frac{\phi_d}{T} \right) + \frac{1}{3} \right] \alpha_s G_F^2 \mu_e \mu_u \mu_d T^6, \end{aligned} \quad (7)$$

where  $\epsilon_\nu^{(rr)}$  is the partial contribution involving the  $r$ th type quasiparticle modes, see Eq. (5). The factor 2 comes from taking into account both the neutrino and the anti-neutrino emissivities. In the final result, the contribution of the ungapped modes  $\epsilon_\nu^{(33)}$  is, up to a factor 1/3, the same as in the normal phase of quark matter [26, 27]. The contribution of the other two modes is suppressed by the following function:

$$G(\varphi_u, \varphi_d) \approx \frac{2520}{457\pi^6} \int_0^\infty dv v^3 \int_{-1}^1 d\xi f(v, \xi), \quad (8)$$

where

$$f(v, \xi) = \sum_{e_1, e_2 = \pm} \int_0^\infty \int_0^\infty \frac{(e^{v+e_1\tilde{\epsilon}_u - e_2\tilde{\epsilon}_d} + 1)^{-1} dx_u dx_d}{(e^{-e_1\tilde{\epsilon}_u} + 1)(e^{e_2\tilde{\epsilon}_d} + 1)}, \quad (9)$$

and  $\tilde{\epsilon}_f = \sqrt{x_f^2 + (1 + \xi)^2 \varphi_f^2}$  with  $f = u, d$ . Note that  $0 \leq G(\varphi_u, \varphi_d) \leq 1$  and  $G(0, 0) = 1$ .

In passing, we note that the pair breaking processes [28] do not play any significant role in the case of spin-1 color-superconducting quark matter under consideration. The corresponding contribution to the emissivity [29] is parametrically suppressed by factor  $T/\mu_e \sim 10^{-3}$ .

From Eq. (7), we see that the (anti-)neutrino emissivity in the A phase differs only by a factor of order 1 from the corresponding result in the normal phase of quark matter [26, 27]. Therefore, the A phase should have qualitatively the same effect on cooling of stars as the normal phase.

Nevertheless, the neutrino emission from the A phase is very unusual. It is not symmetric with respect to reversing the direction of the  $z$ -axis. To quantify the asymmetry, we calculate the value of the  $z$ -component of the momentum carried away by neutrinos per unit volume of quark matter, per unit time. This is obtained by replacing one power of  $p_\nu$  on the right hand side of Eq. (1) by  $p_\nu \cos \theta_{\mathbf{p}_\nu}$ . Taking into account that the neutrino and the antineutrino emissions give the same contributions, we arrive at the final result [25]

$$\frac{dP_z^{(\text{tot})}}{dV dt} \approx \frac{2}{3} H \left( \frac{\phi_u}{T}, \frac{\phi_d}{T} \right) \frac{457}{630} \alpha_s G_F^2 \mu_e \mu_u \mu_d T^6, \quad (10)$$

where

$$H(\varphi_u, \varphi_d) \approx -\frac{840}{457\pi^6} \int_0^\infty dv v^3 \int_{-1}^1 d\xi \xi f(v, \xi). \quad (11)$$

Note that  $H(0, 0) = 0$  which is consistent with the fact that the momentum kick is vanishing in the normal phase of quark matter. The numerical result for the function  $H$  at equal values of its two arguments is well approximated by the following expression:

$$H(\varphi, \varphi) \approx \sum_{n=1}^5 \frac{h_n}{[1 + (r_0 \varphi)^2]^{n/2}}, \quad (12)$$

with  $h_1 = 0.3068$ ,  $h_2 = -0.1977$ ,  $h_3 = -0.7838$ ,  $h_4 = 1.0286$ ,  $h_5 = -0.3539$ , and  $r_0 = e^{\gamma + \bar{\zeta}}/\pi \approx 0.8125$  is the ratio of the critical temperature to the value of the gap at  $T = 0$  in the A phase, i.e.,  $r_0 = T_c/\phi_0$  which is expressed in terms of the Euler constant  $\gamma \approx 0.577$  and  $\bar{\zeta} = \ln 2 - 1/3$  [22]. Note that  $\sum_{n=1}^5 h_n = 0$ . The representation in Eq. (12) is particularly convenient when the following simplified temperature dependence for the gap parameter is used:  $\phi(T) = \phi_0 \sqrt{1 - (T/T_c)^2}$ . In this case, the function  $H(T)$  becomes a polynomial:  $H(T) = \sum_{n=1}^5 h_n (T/T_c)^n$  for  $T < T_c$ , and  $H(T) = 0$  for  $T > T_c$ .

The physical reason for the breakdown of the reflection symmetry lies in the pairing pattern of the transverse A phase. Quasiquarks of the first branch,  $r = 1$ , have helicity  $+1$  if the projection of their momentum onto the  $z$ -axis is negative,  $\cos \theta_{\mathbf{k}} < 0$ , and helicity  $-1$  if  $\cos \theta_{\mathbf{k}} > 0$ . Quasiquarks of the second branch,  $r = 2$ , have opposite helicities. Only left-handed quarks (in the ultrarelativistic limit, quarks with negative helicity) participate in the Urca processes. Thus, the quasiquarks of the first (second) branch contribute only if their momenta are in the upper (lower) half-space. Taking this into account, the effective branch relevant for the emission has gap  $\phi_{\text{eff}} \sim 1 + \cos \theta_{\mathbf{k}}$ , which discriminates between  $+z$  and  $-z$  directions, see Fig. 1. Since neutrinos are emitted preferably in the direction opposite to the quark momenta, this asymmetry manifests itself in the neutrino emission.

*Calculation of the velocity kick.*—In order to make a simple estimate of the velocity kick due to the neutrino emission from a color-superconducting quark matter core in the A phase, we use the following model time dependence of the core temperature (see, e.g., Ref. [27]):  $T(t) = T_0(t_0/t)^{1/4}$ . This has the power dependence which is characteristic for bulk matter cooling by neutrino emission with  $\epsilon_\nu \sim T^6$ , see Eq. (7), provided the specific heat of matter is  $c_V \sim T$ . Here, we assume that  $T_c < T_0$ , i.e., the system is too hot for spin-1 pairing initially, and then it cools through the transition point.

The velocity kick for a star of mass  $1.4M_\odot$  with the quark core of radius  $R_c$  is given by the expression:

$$\delta v \equiv \frac{\Delta P_z^{(\text{tot})}}{1.4M_\odot} = \frac{457\alpha_s}{945} G_F^2 \mu_e \mu_u \mu_d \frac{4\pi}{3} \frac{R_c^3}{1.4M_\odot} T_0^4 T_c^2 t_0 \times \theta(t - t_c) \sum_{n=1}^5 \frac{4h_n}{2+n} \left[ 1 - \left( \frac{t_c}{t} \right)^{(2+n)/4} \right], \quad (13)$$

where we used the approximate expression in Eq. (12), as well as a simplified temperature dependence of the gap parameters,  $\phi_u(T) = \phi_d(T) = \phi_0 \sqrt{1 - (T/T_c)^2}$ . The notation  $t_c \equiv t_0(T_0/T_c)^4$  stands for the time when the temperature of the quark core drops below the critical value.

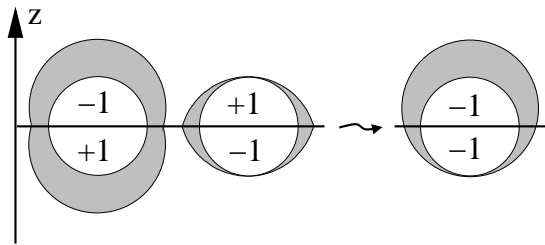


FIG. 1: Gap functions for the first (left) and the second (middle) excitation branch with specified helicities of quasiparticles in the upper and the lower half-spaces. The “effective” gap relevant for the neutrino emission is shown on the right.

From Eq. (13), we can also derive the expression for the maximum velocity kick ( $t = \infty$ ):

$$\delta v_{\text{max}} \approx 0.033 \alpha_s G_F^2 \mu_e \mu_u \mu_d \frac{4\pi}{3} \frac{R_c^3}{1.4M_\odot} T_0^4 T_c^2 t_0. \quad (14)$$

The results for the maximum velocity kicks are presented graphically in Fig. 2. To make the plot, we take  $\alpha_s = 1$ , and use the initial condition:  $T_0 = 100$  keV at  $t_0 = 100$  yr. Also, we use the following values of the chemical potentials:  $\mu_u = 400$  MeV,  $\mu_d = 500$  MeV,  $\mu_e = 100$  MeV.

When the stellar age  $t$  is finite, all curves in Fig. 2 shift to the right. Numerically, the value of the shift is 10 keV at  $t = 10^6$  yr and about 30 keV at  $t = 10^4$  yr. This is easy to understand: at  $t < \infty$ , the step function in Eq. (13) cuts out a range of low values of critical temperatures,  $T_c < T_0(t_0/t)^{1/4}$ , which are not yet accessible at time  $t$ . Other than this shift, the shape of the curves remains similar even at  $t$  as low as  $10^3$  yr.

*Discussion.*—The main observation of this Letter is that the neutrino emission from the spin-1 color-superconducting transverse A phase (unlike from other spin-1 color-superconducting phases [25]) is not symmetric with respect to reversing one spatial direction.

Therefore, we propose a hypothesis according to which the inner cores of some (hybrid) neutron stars are made of the A phase. The neutrino emission from such stars could generate strong velocity kicks, needed to explain the observed bimodal velocity distribution of pulsars [16, 17]. Its bimodal structure results from an overlap of two distributions: one describing “normal” neutron stars, and the other describing hybrid stars with color-superconducting quark cores in the A phase.

The above hypothesis has several specific (and, thus, falsifiable) predictions, which are directly related to the nature of the A phase. Let us start by discussing the direction of the kick velocity. As should be clear from our

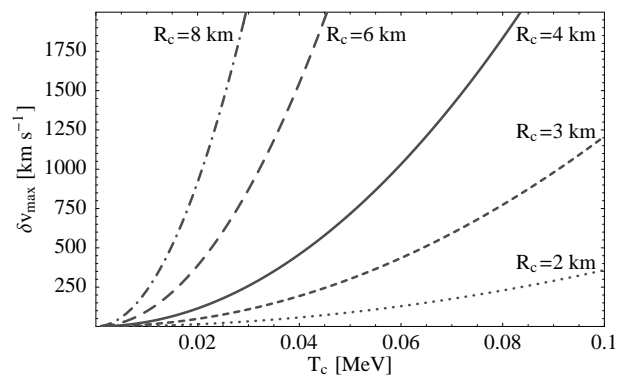


FIG. 2: The dependence of the maximum velocity kick of a neutron star versus the value of the critical temperature of the spin-1 color superconducting phase transition. Results for several values of the quark core radius are shown.

analysis, this is aligned with the direction spontaneously picked by the spin-1 condensate. In the case of an infinite isotropic medium, this direction can be arbitrary. Inside a rotating and magnetized star, however, the degeneracy with respect to the orientation is likely to be removed because of a nonvanishing interaction with the angular momentum and/or with the magnetic field. Therefore, the kick velocity averaged over time should be parallel to the rotational axis. The observational data seems to favor this possibility [30].

The proposed neutrino propulsion mechanism also predicts a correlation between the velocity distribution and the age of young neutron stars. This has not been seen in the observational data yet [16]. Most likely, however, this correlation is hard to detect because it takes typically less than  $10^4$  yr for a star to accumulate almost all of the maximum velocity kick. The future high statistics studies of young neutron stars may resolve the issue.

Along with the advantages of the proposed mechanism, it might be also appropriate to mention several potential difficulties. One of them is a rigorous justification that the spin-1 color-superconducting A phase is the ground state of dense quark matter inside a star. In the case of an infinite isotropic medium, it has been shown that the color-spin locked phase is the favored spin-1 phase at asymptotically large density [11, 22]. We could only speculate that this may change when the density is realistic, and the rotation and the magnetic field are accounted for. The other potential difficulty is related to fast cooling of the A phase by direct Urca processes, which might not be compatible with the observational data [31].

The mechanism, proposed here, may well have other important implications that we did not discuss in this Letter. We hope, however, that they will be addressed in the future studies, providing much stronger tests for the hypothesis of the hybrid stars accelerated by the asymmetric neutrino emission from the A phase. If it passes the tests, the observational data from pulsars could shed light on some details of the QCD phase diagram. Thus, the pulsars should be viewed not only as a unique laboratory for testing the theory of general relativity, but also the ultimate laboratory for testing the theory of QCD.

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# Erratum: Pulsar kicks via spin-1 color superconductivity [Phys. Rev. Lett. 94, 211101 (2005)]

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The model time dependence of the temperature,

$$T(t) = T_0 \left( \frac{t_0}{t} \right)^{1/4}, \quad (1)$$

which was used in our calculation, is unphysical. As a result, the estimate of the kick velocities due to spin-one color superconductivity was wrong.

The correct time dependence is approximately given by

$$T(t) = T_0 \frac{\tau^{1/4}}{(t - t_0 + \tau)^{1/4}}, \quad (2)$$

which is the same as for normal quark matter. This time dependence is obtained by integrating the energy balance equation,  $\epsilon_\nu = -c_V dT/dt$ , in which an approximate analytical expression for the specific heat of quark matter,  $c_V = (\mu_u^2 + \mu_d^2)T$  (see, for example, Ref. [1]) and Iwamoto's analytical expression for the neutrino emissivity  $\epsilon_\nu$  [2] were used. This is justified for the long-term stellar cooling after the thermal relaxation epoch is completed ( $t \gtrsim 10^2$  yr) and before the surface cooling by photons starts to dominate ( $t \lesssim 10^5$  yr), e.g., see Ref. [3]. In the above expression,  $\tau$  is defined by

$$\tau \equiv \frac{315}{914} \frac{\mu_u^2 + \mu_d^2}{\alpha_s G_F^2 \mu_e \mu_u \mu_d} \frac{1}{T_0^4} \approx 10^{-5} \text{ yr}. \quad (3)$$

Taking this into account, Eqs. (13) and (14) in our Letter should be replaced by

$$\delta v \equiv \frac{\Delta P_z^{(\text{tot})}}{1.4M_\odot} = \frac{457\alpha_s}{945} G_F^2 \mu_e \mu_u \mu_d \frac{4\pi}{3} \frac{R_c^3}{1.4M_\odot} T_0^4 T_c^2 \tau \theta(t - t_c) \sum_{n=1}^5 \frac{4h_n}{2+n} \left[ 1 - \left( \frac{t_c - t_0 + \tau}{t - t_0 + \tau} \right)^{(2+n)/4} \right], \quad (4)$$

and

$$\delta v_{\text{max}} \approx 0.033 \alpha_s G_F^2 \mu_e \mu_u \mu_d \frac{4\pi}{3} \frac{R_c^3}{1.4M_\odot} T_0^4 T_c^2 \tau, \quad (5)$$

respectively. Here  $t_c \equiv \tau(T_0^4/T_c^4 - 1) + t_0$  is the time when the temperature is equal to the critical value.

The revised expression for  $\delta v_{\text{max}}$  differs by a factor  $\tau/t_0 \approx 10^{-7}$  from the old one. Consequently, the predicted kick velocities become extremely small. For example, instead of velocities of the order of  $\delta v_{\text{max}} \sim 1000$  km/s, one gets  $\delta v_{\text{max}} \sim 10^{-4}$  km/s.

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