

# Some relations between hadron masses

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## **Abstract**

The mass of some hadrons are reproduced in terms of the mass of the nucleon. A possible reason for emission of soft gammas is proposed.

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During the last 4-5 decades, many new mesons and baryons have been observed [1]. In this note we show that some of their masses probably are related, and that some phenomenological formulas seem to reproduce some hadron masses fairly well.

According to the quark model, mesons are quark-antiquark systems  $q\bar{q}$ . For each possible  $q\bar{q}$  combination, the minimum mass meson [1] is listed in Table I. In the following, masses are written in  $\text{MeV}/c^2$ .

**Table I**

	$\bar{u}$	$\bar{d}$	$\bar{s}$	$\bar{c}$	$\bar{b}$	$\bar{t}$
$u$	$\pi^0(135)$	$\pi^+(140)$	$K^+(494)$	$\bar{D}^0(1865)$	$B^+(5279)$	
$d$	$\pi^-(140)$	$\pi^0(135)$	$K^0(498)$	$D^-(1869)$	$B^0(5279)$	
$s$	$K^-(494)$	$\bar{K}^0(498)$	$\phi(1020)$	$D_s^-(1968)$	$B_s^0(5370)$	
$c$	$D^0(1865)$	$D^+(1869)$	$D_s^+(1968)$	$\eta_c(2980)$	$B_c(6400)$	
$b$	$B^-(5279)$	$\bar{B}^0(5279)$	$\bar{B}_s^0(5370)$	$B_c(6400)$	$\Upsilon(9460)$	
$t$						

It is seen from Table I that

$$\begin{aligned}
 m(\phi) + m(D_s) &= 1020 + 1968 \approx m(\eta_c), \\
 m(\phi) + m(B_s) &= 1020 + 5370 \approx m(B_c), \\
 m(\eta_c) + m(B_c) &= 2979 + 6400 \approx m(\Upsilon),
 \end{aligned}$$

which suggest that the masses of some mesons are correlated.

The relation

$$m(N)/2\pi \approx 150 \text{ MeV}/c^2 \approx m(\pi)$$

[2], which reproduces the mass  $m(\pi)$  of the pion in terms of the mass  $m(N)$  of the nucleon fairly well, corresponds to

$$\lambda_c(\pi) \approx 2\pi\lambda_c(N)$$

where  $\lambda_c(\pi)$  and  $\lambda_c(N)$  are the the Compton wave lengths of the pion and the nucleon, respectively, which suggests a geometric relation between the two Compton wave lengths.

By means of an extra factor  $(4\pi)^{n/2}$  on the left hand side of the mass formula above, [2], the trend of increase of mass along the quasi-diagonal

$$u\bar{d}, d\bar{s}, s\bar{c}, c\bar{b}$$

in Table I is fairly well reproduced for  $n = 0, 1, 2, 3$ , i.e.

$$\begin{aligned}(4\pi)^{0/2} m(N)/2\pi &\approx 150 \approx m(\pi), \\(4\pi)^{1/2} m(N)/2\pi &\approx 530 \approx m(K), \\(4\pi)^{2/2} m(N)/2\pi &\approx 1880 \approx m(D), \\(4\pi)^{3/2} m(N)/2\pi &\approx 6660 \approx m(B_c).\end{aligned}$$

If also

$$(4\pi)^{4/2} m(N)/2\pi \approx 23610 \approx m(b\bar{t})$$

and by means of an extrapolation of the sums above, one might expect

$$m(\Upsilon) + m(b\bar{t}) \approx 9460 + 23610 \approx 33070 \approx m(t\bar{t}).$$

By means of the ratio

$$r = m/m(\pi)$$

we measure the mass of some mesons in  $m(\pi)$ -units.  $r$  and the value of

$$2^k \pi^{n/2}$$

which for integer  $k$  and  $n$  gives the best fit to  $r$  are listed in Table II.

**Table II**

Meson	$I$	$J^P$	$r = m/m(\pi)$	$2^k \pi^{n/2}$
$\eta$	0	$0^-$	$547/140 = 3.91$	$2^2 \pi^{0/2} = 4.00$
$K$	1/2	$0^-$	$497/140 = 3.55$	$2^1 \pi^{1/2} = 3.54$
$\omega$	0	$1^-$	$782/140 = 5.59$	$2^0 \pi^{3/2} = 5.56$
$K^*$	1/2	$1^-$	$892/140 = 6.37$	$2^1 \pi^{2/2} = 6.28$
$J/\psi$	0	$1^-$	$3097/140 = 22.12$	$2^2 \pi^{3/2} = 22.26$

According to this table, these mesonic masses are fairly well reproduced by the formula

$$m \approx 2^k \pi^{n/2} m(\pi)$$

in favour of some mutual relationship between these mesons.

While the mass differences between the non-strange, strange, double and triple strange baryons in the well known baryon decouplet is about  $140 \text{ MeV}/c^2$ , which is the mass of the pion, the baryons in the octet do not show the same differences. While the difference between the mass of the  $\Delta(1232)$  and the mass of the nucleon is  $290 \text{ MeV}/c^2$ , the difference between the mass of the  $\Lambda(1115)$  and the mass of the nucleon is  $175 \text{ MeV}/c^2$ .

If every baryon heavier than the nucleon is taken to be an excited state of a nucleon, then the relative excitation energies would be important. In Tables III-IV we therefore compare the successive mass differences  $d$  with the smallest one for baryons with  $J^P = 1/2^+$ , and  $J^P = 3/2^+$ , [1], respectively.

**Table III**

Baryon	$I$	$J^P$	$m$	$d \simeq m - m(N)$	$d/175 \simeq 2^{n/k}$
$\Lambda$	0	$1/2^+$	1116	175	$1.0 = 2^{0/2}$
$\Sigma$	1	$1/2^+$	1197	255	$1.4 \approx 2^{1/2}$
$\Xi$	1/2	$1/2^+$	1315	375	$2.1 \approx 2^{2/2}$
$N$	1/2	$1/2^+$	1440	500	$2.9 \approx 2^{3/2}$
$N$	1/2	$1/2^+$	1640	700	$4.0 = 2^{4/2}$

For these five  $J^P = 1/2^+$  baryons, the mass difference  $d$  is for  $k = 2$  and  $n = 0, 1, 2, 3$ , or  $4$ , fairly well described by

$$d \approx 2^{n/k} 175 \text{ MeV}/c^2.$$

**Table IV**

Baryon	$I$	$J^P$	$m$	$d \simeq m - m(N)$	$d/290 \simeq 2^{n/k}$
$\Delta$	3/2	$3/2^+$	1232	290	$1.0 = 2^{0/3}$
$\Sigma^*$	1	$3/2^+$	1385	445	$1.5 \approx 2^{2/3}$
$\Xi^*$	1/2	$3/2^+$	1530	590	$2.0 \approx 2^{3/3}$
$\Omega^-$	0	$3/2^+$	1672	730	$2.5 \approx 2^{4/3}$

For these four  $J^P = 3/2^+$  baryons, the mass difference  $d$  is for  $k = 3$  and  $n = 0, 2, 3$ , or  $4$ , fairly well described by

$$d \approx 2^{n/k} 290 \text{ MeV}/c^2.$$

A  $J^P = 3/2^+$  baryon with mass about  $1308 \text{ MeV}/c^2$  corresponding to  $2^{1/3}$  seems however to be missing according to this table. If such a baryon exists, it is possibly overlooked in the background below the  $J^P = 3/2^+$   $\Xi(1315)$  peak.

Table V shows the ratio

$$R = m/m(N)$$

for some baryons with mass  $m$  and  $F$ -values defines by the formula

$$F = (1 + 1/n)^2$$

for integer  $0 < n < 12$ .

Table V

Baryon	$I$	$J^P$	$R = m/m(N)$	$F = (1 + 1/n)^2$
$\Lambda$	0	$1/2^+$	$1116/940 = 1.19$	$(12/11)^2 = 1.19$
no baryon seen at 1137 corresponding to				$(11/10)^2 = 1.21$
no baryon seen at 1160 corresponding to				$(10/9)^2 = 1.23$
$\Sigma$	1	$1/2^+$	$1197/940 = 1.27$	$(9/8)^2 = 1.27$
$\Delta$	$3/2$	$3/2^+$	$1232/940 = 1.31$	$(8/7)^2 = 1.31$
$\Xi$	$1/2$	$1/2^+$	$1315/940 = 1.40$	$(7/6)^2 = 1.36$
$\Sigma^*$	1	$3/2^+$	$1385/940 = 1.47$	$(6/5)^2 = 1.44$
N	$1/2$	$1/2^+$	$1440/940 = 1.53$	$(5/4)^2 = 1.56$
$\Omega^-$	0	$3/2^+$	$1672/940 = 1.78$	$(4/3)^2 = 1.78$
$\Lambda$	0	$5/2^+$	$2110/940 = 2.24$	$(3/2)^2 = 2.25$
uncertain			$3800/940 = 4.04$	$(2/1)^2 = 4.00$

In the region between  $\Lambda(1115)$  and  $\Sigma(1190)$ , two non-observed baryons are expected to exist according to this formula, as seen from Table V. If their widths are large enough, they might have been taken to belong to the background in this region. While the geometric constant  $\pi$  is needed for the reproduction of mesonic masses,  $\pi$  is not needed for reproduction of the baryonic masses. The formula

$$m \approx F m(N),$$

thus reproduces some baryonic masses as if these baryons belong to a common chain. It is interesting to note that for

$$F \rightarrow 1,$$

i.e. for decreasing mass  $m$ , the density of states increases and tends to appear as a continuum, contrary to e.g. hydrogen atoms where continuum appears for high excitations. For large  $n$ -values corresponding to a  $Q$ -value

$$Q < m(\pi)c^2,$$

i.e. too small for emissions of a pion, only  $\gamma$  emission would be energetically possible. Since final state non-Bremsstrahlung soft  $\gamma$ 's have been observed in high energy  $pp$  reactions [3], such emission is a candidate for  $\gamma$  decay of large  $n$  baryons. Emission of non-Bremsstrahlung soft  $\gamma$ 's has also been observed in high energy  $K^+p$ -reactions [4, 5],  $\pi^+p$ -reactions [5], and in  $\pi^-p$ -reactions [6, 7, 8]. This suggests that not only nucleons but also mesons may be excited to energy levels too low for mesonic decays.

In summary, based only on the mass of the nucleon, some relations are found which reproduce some hadronic mass values fairly well. A possible reason for emission of soft gammas in hadron-proton collisions is proposed.

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