

Top Compositeness and Precision Unification

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The evolution of Standard Model gauge couplings is studied in a non-supersymmetric scenario in which the hierarchy problem is resolved by Higgs compositeness above the weak scale. It is argued that massiveness of the top quark combined with precision tests of the bottom quark imply that the right-handed top must also be composite. If, further, the Standard Model gauge symmetry is embedded into a simple subgroup of the unbroken composite-sector flavor symmetry, then precision coupling unification is shown to occur at $\sim 10^{15}$ GeV, to a degree comparable to supersymmetric unification.

The ambitious ideas of grand unification [1], and variants such as string unification [2] and orbifold unification [3], are founded on the structure and successes of the Standard Model (SM). A central quantitative prediction is the evolution of SM gauge couplings from a single unified coupling, α_U , at a unification scale, M_U . Schematically, in the minimal scenario,

$$\alpha_{i=1,2,3}(\mu) = \alpha_U + \text{SM} + M_U\text{-physics}, \quad (1)$$

where the second term represents SM running, while the third represents model-dependent threshold effects from unification physics at a scale $\sim M_U$. Fortunately, it is natural for these M_U -scale effects to be much smaller than SM running, unless the M_U -sector is very large or has large non-degeneracies. Neglecting these effects allows one to test unification with just SM data.

There are several shortcomings: (i) The couplings do not meet very precisely. This does not falsify unification since M_U -effects may be unexpectedly large, but a more precise meeting without invoking such effects would have been much stronger circumstantial evidence for unification. Nevertheless, the results are intriguing. (ii) With the best fit, $M_U \sim 10^{14}$ GeV, so large that there is certainly no prospect of experimentally verifying any unified symmetry. (iii) The requisite SM extrapolation to such high M_U results in a severe gauge hierarchy problem. (iv) This M_U is still low enough that exchange of massive states can result in excessive proton decay. Such states can however be avoided in string or orbifold unifications [2, 3].

By comparison, unification in the context of weak scale supersymmetry (SUSY) is a striking success. The coupling evolution is given schematically by

$$\alpha_i(\mu) = \alpha_U + \text{SM} + \text{superpartners} + M_U\text{-physics}, \quad (2)$$

and again can be tested neglecting M_U -effects: (i) Adding the superpartner-induced running yields a high precision meeting of couplings. The level of precision can be quantified by the postdiction $\delta_3 \equiv (\alpha_3^{\text{theory}} - \alpha_3^{\text{exp}})/\alpha_3^{\text{exp}} \sim 10\%$ at the scale m_Z .¹ This size of δ_3

can be naturally accounted for by threshold effects from M_U -physics. (ii) While the unification scale $M_U \sim 10^{16}$ GeV is still high, we rely less on directly seeing the unified gauge symmetry given the stronger circumstantial evidence. (iii) SUSY can solve the gauge hierarchy problem, so that two important issues are addressed simultaneously. (iv) M_U is high enough to adequately suppress proton decay. String unification or orbifold unification are still attractive for solving the doublet-triplet splitting problem [2, 3].

In this letter we pursue a very different scenario, namely that the hierarchy problem is solved by having the Higgs doublet be a composite of some new (non-supersymmetric) strong dynamics [4].² While such dynamics is necessarily non-perturbative and theoretically challenging, there has been a recent revival of interest because of two extensions which allow one to understand weak scale symmetry breaking, precision tests and phenomenology, *independent of many of the details of the strong sector*. Little Higgs theory [6] is one such extension, which we will not pursue here. Our work is motivated by (but not strongly reliant on) Refs. [7, 8], a realistic Randall-Sundrum (RS) extra-dimensional scenario with most or all of the SM fields in the bulk. Via the AdS/CFT correspondence [9, 10], such a scenario is *dual* to a purely 4D composite Higgs scenario. The Kaluza-Klein excitations map to some low-lying hadrons at the compositeness scale, Λ_{comp} . The ratio of higher-dimensional curvature to the effective field theory cut-off maps to a new small parameter of the strong sector, with the help of which many weak-scale observables can be calculated independently of microscopic details of the strong dynamics.

An attractive feature of (the 4D dual of) this type of RS set-up is its simple extrapolation (at least for some important inclusive observables) to energies far above the weak scale. This leads to an elegant mechanism for gen-

few percent. This must be multiplied by $\alpha_3(m_Z)/\alpha_3(M_U) \sim 2.5$ in order to compare at m_Z , as we do.

² We will not follow the technicolor approach [5], in which a Higgs scalar is effectively absent.

¹ Often in the SUSY literature δ_3 is evaluated at M_U as being a

erating hierarchical Yukawa couplings [11, 12, 13, 14]. The light SM fermions are taken to be elementary particles, weakly coupled to strong-sector operators. Running down to Λ_{comp} , these operators induce small Yukawa couplings to the Higgs composite. Hierarchies arise naturally from the different scaling dimensions of different strong operators. The weak couplings to the strong sector also naturally suppress modifications of couplings to the W, Z [15] and compositeness and flavor-changing effects in the light SM fermions [13], in accord with modern data.

The top quark is, however, a special case. Its Yukawa coupling to the Higgs composite is so large that either t_R, t_L , or both must effectively also be composite. However, precision data such as tests of $Z \rightarrow b\bar{b}$ strongly suggest that b_L , and hence t_L by electroweak symmetry, can have at most a small admixture of a TeV-scale composite [16][7]. We deduce that t_R must be the composite.

With these broad motivations and expectations, we will show that under quite simple and plausible conditions an attractive scheme for precision unification emerges. We will first derive our central result to leading order (LO) in the couplings of elementary fields to the strong sector, and then consider subdominant corrections. Our discussion will be mostly from the 4D viewpoint of the strong sector, rather than an RS description, for reasons we will explain later. Closely related ideas in a SUSY RS context were presented in Ref. [17].³

Since composites carrying electroweak quantum numbers and color emerge from the new strong sector, the SM vector fields must gauge some $SU(3) \times SU(2) \times U(1)$ subgroup of the global “flavor” group, G . Above Λ_{comp} , the running due to the light composites, H, t_R , must be replaced by the full strong dynamics,

$$\alpha_i(\mu) = \alpha_U + \text{SM} - \{t_R, H\} + \text{strong sector} + M_U\text{-physics.} \quad (3)$$

Fortunately, the non-perturbative strong sector contributions to SM running cancel to one-SM-loop order in computing *differential running*, that is the running of $(\alpha_i - \alpha_1)$ say, if the SM gauge group is embedded in a simple factor of G ⁴ (such as $SU(5)$ for example):

$$\alpha_i(\mu) - \alpha_1(\mu) = \text{SM} - \{t_R, H\} + M_U\text{-physics.} \quad (4)$$

This is all we need to check gauge coupling unification. We assume the simple embedding of the SM into G from now on.

Eq. (4) exhibits a remarkable twist in the unification paradigm. Instead of adding the running from physics beyond the SM, here compositeness instructs us to *subtract*

the running due to some SM particles! Before checking unification we must explain why the light composites, H, t_R , do not fill out complete representations of the global symmetry, G . There are two distinct cases following from the possibility of spontaneous symmetry breaking in the strong sector at Λ_{comp} , $G \rightarrow K$. (Indeed the original, and still attractive, proposal for a composite Higgs is as a (pseudo-)Goldstone boson of this type of symmetry breaking [4], though this is not essential for the present paper.) The two cases are (a) the SM gauge group remains embedded in a simple factor of $K \subset G$, or (b) it does not. In (b) there is no contradiction with H, t_R being the only light composites, and Eq. (4) applies. One finds that the subtractions certainly improve unification, but it is still not very precise. We will not study this case further here.

Here, we focus on (a), where H, t_R *must* be accompanied by other composites, filling out complete K -representations. Having extra (colored) scalar composites does not pose a robust problem, since the perturbations of the SM coupling to the strong sector can easily split the Higgs doublet from its K -partners, allowing the former to condense and be light while the latter do not condense and are massive enough to avoid present bounds. But the chiral fermionic K -partners of the t_R do pose a robust problem (and introduce SM anomalies). The only way to remove these unwanted states is to assume there exist exotic elementary fermions beyond the SM with couplings to the strong sector, which induce Dirac masses with the K -partners of the t_R below Λ_{comp} . That is, the exotics must have SM quantum numbers which are charge-conjugate to the K -partners of the t_R [7, 21].

The elementary exotics also contribute to SM running above Λ_{comp} ,

$$\alpha_i(\mu) = \alpha_U + \text{SM} - \{t_R, H\} + \text{exotics} \\ + \text{strong sector} + M_U\text{-physics.} \quad (5)$$

We assume here that the exotic couplings to the strong sector are weak enough that their contributions to SM running are approximately undressed by strong-sector corrections. At one-SM-loop the *differential* running only depends on the fact that the exotics fill out a complete K -representation *except for a missing t_R^c* ,

$$\alpha_i(\mu) - \alpha_1(\mu) = \text{SM} - \{t_R, t_R^c, H\} + M_U\text{-physics.} \quad (6)$$

Neglecting the M_U -threshold, as usual, yields near perfect unification, with $M_U \sim 10^{15}$ GeV. See Fig. 1.

Eq. (6) and Fig. 1 summarize our central quantitative result to LO in SM gauge couplings and zeroth order in the couplings of elementary fermions to the strong sector. We now discuss the subleading corrections. It is difficult to couple elementary fermions to strong sector operators at $M_U \sim 10^{15}$ GeV without the couplings being highly irrelevant in the IR, resulting in negligible

³ Other RS unification proposals not tied to top/Higgs compositeness appear in Refs. [18, 19, 20].

⁴ This also includes the case in which G contains some discrete generators. For example, $SU(N)_L \times SU(N)_R$ with parity is simple.

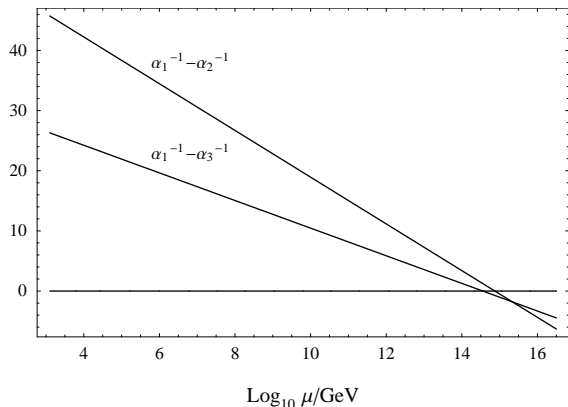


FIG. 1: LO differential running of SM gauge couplings in the top/Higgs compositeness scenario (a).

Yukawa couplings, unless *the strong sector is strongly-coupled throughout the large hierarchy*. This happens naturally when the strong sector is near an IR-attractive fixed point above Λ_{comp} . In this case, working to next-to-leading order (NLO), the gauge coupling running above Λ_{comp} is given by

$$\frac{d}{d \ln \mu} \left(\frac{1}{\alpha_i} \right) = \frac{b_i}{2\pi} + \frac{B_{ij}}{2\pi} \frac{\alpha_j}{4\pi} + \frac{C_{i\alpha}}{2\pi} \frac{\lambda_\alpha^2}{16\pi^2}, \quad (7)$$

where the b, B, C are constants and $\lambda_\alpha, \alpha = exotic, \alpha_L^3 \equiv (t_L, b_L)$, denote the largest couplings of the elementary fermions to the strong sector (resulting in the largest masses with composite fermions). We further decompose

$$\begin{aligned} b_i &\equiv b_i^{SM-} + b_i^{exotic} + b^{strong} \\ B_{ij} &\equiv B_{ij}^{SM-} + B_{ij}^{exotic} + B_{ij}^{strong}, \end{aligned} \quad (8)$$

where “ $SM-$ ” refers to $SM - \{t_R, H\}$. Note that $b_i^{SM-}, b_i^{exotic}, B_{ij}^{SM-}, B_{ij}^{exotic}$ are just representation-theoretic factors. For concreteness we consider the SM gauge group embedded in $SO(10) \subset K$ in the usual way, with the t_R being part of a composite 16 of $SO(10)$, so that the elementary exotics $\equiv \overline{16} - \{t_R\}$.

By contrast, $b^{strong}, B_{ij}^{strong}, C_{i\alpha}$ include unknown $\mathcal{O}(1)$ strong interaction factors. We will treat b^{strong} as an unknown (i -independent by the SM embedding into a simple factor of G), which can usefully be thought of as a crude measure of the $SO(10)$ -charged content of the strong sector. A rough but reasonable expectation is that $b^{strong} \gtrsim b^{comp}$, where b^{comp} is the LO renormalization group coefficient due to the light composites alone in the far IR. For a real scalar 10 and a Weyl fermion 16 of $SO(10)$ ($\ni H, t_R$), $b^{comp} = 1.5$. We will use crude estimates of the NLO coefficients, $B_{ij}^{strong} \sim 3 \cdot 3 \cdot b^{strong}$, $C_{i\alpha} \sim 3 \cdot b^{strong}$, as part of our theoretical error estimates. These estimates follow from the fact that the (non-perturbative) diagrams contributing to the NLO

coefficients arise from diagrams contributing to b^{strong} with insertions of intermediate elementary gauge bosons or fermions via SM gauge couplings or λ_α . Such insertions can result in summation over QCD colors or weak isospins, giving rise to an extra factor of at most 3. Further, experience with perturbative gauge loops shows that they give an extra factor ~ 3 beyond the naive loop-counting parameter. This accounts for the second factor of 3 in B^{strong} .

Following (the AdS/CFT dual of) the scenario of Ref. [7], we assume that above Λ_{comp} the couplings λ_α are slightly relevant, driving the theory away from the original fixed point (of the isolated strong sector) to a nearby fixed point. We can approximate $\lambda_\alpha(\mu)$ in the gauge coupling running by their new-fixed-point values, $\lambda_{*\alpha}$. Integrating Eq. (7) down to $\mu \lesssim \Lambda_{comp}, m_{exotic}$,

$$\begin{aligned} \frac{1}{\alpha_i(\mu)} &= \frac{1}{\alpha_U} - \frac{b_i}{2\pi} \ln \frac{M_U}{\mu} - \frac{C_{i\alpha}}{32\pi^3} \lambda_{*\alpha}^2 \ln \frac{M_U}{\mu} \\ &\quad - \frac{B_{ij}}{4\pi b_j} \ln \left[1 + \alpha_j(\mu) \frac{b_j}{2\pi} \ln \frac{M_U}{\mu} \right] \end{aligned} \quad (9)$$

+ threshold corrections.

Let us first discuss the thresholds at $M_U \sim 10^{15}$ GeV, $\Lambda_{comp} \sim$ few TeV, and $m_{exotic} \sim$ TeV. The detailed physics at M_U is unknown, but the threshold effects associated with the strong sector can be subsumed into α_U, B^{strong}, C . As in standard unification schemes, the minimal natural size of threshold effects associated with elementary fields is $\delta(1/\alpha_i) \sim \mathcal{O}(1)/2\pi$. The expectation $\Lambda_{comp} \sim$ few TeV follows from the requirements of reasonable naturalness of the weak scale as well as passing electroweak precision tests. This was demonstrated in RS modelling using the extra-dimensional calculability [7, 8]. In the present scenario, we do not expect a useful extra-dimensional dual description, as we explain later, but the RS calculations of precision observable serve as plausible estimates, with at most $\mathcal{O}(1)$ unknown correction factors. A central requirement however, is having an approximate “custodial isospin” symmetry of the strong sector to protect the electroweak ρ -parameter. Our choice of $SO(10)$ flavor symmetry ensures this, with custodial $SU(2)_R$ as well as SM subgroups. As a consequence, one exotic has the gauge quantum numbers of ν_R , is stable given a baryon number symmetry of the strong sector, and can serve as a dark matter candidate if its mass is \sim few hundred GeV [22]. However, the exotic $SU(2)_R$ -partner of t_R must have mass $\gtrsim 1.5$ TeV in order to avoid it forming too large a component of the observed bottom quark, in contradiction with precision tests. These considerations motivate $m_{exotic} \sim$ TeV, with mild $SO(10)$ -violating splittings. A simple (but not the only) way for this to happen is for $SU(5)$ to be an exact flavor symmetry of the strong sector, with the remainder of the $SO(10)$ symmetry being only approximate. Exact $SU(5)$ is all we need here.

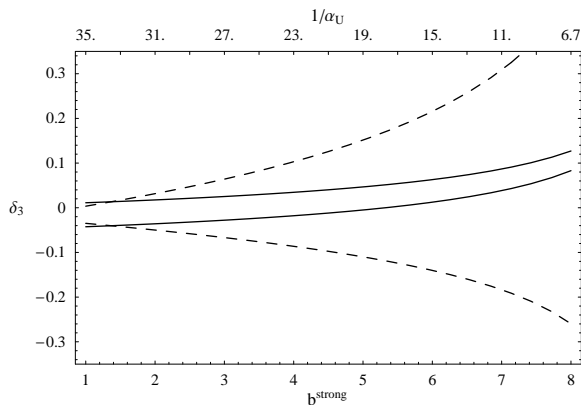


FIG. 2: Postdiction of $\alpha_3(m_Z)$ at NLO as given in terms of $\delta_3 = (\alpha_3^{theory} - \alpha_3^{exp})/\alpha_3^{exp}$. Solid lines denote the error band from threshold effects at M_U , Λ_{comp} , m_{exotic} , whereas dashed lines denote the estimated error from B_{ij}^{strong} , $C_{i\alpha}$.

In SUSY unification the one-loop superpartner threshold effects are generally significant because of large non-gauge-universal splittings in their spectrum induced by running from M_U , and also by the need to avoid existing search constraints and extreme fine-tuning. In the present scenario this does not happen because the Λ_{comp} threshold approximately has the global K -symmetry of the strong sector, while the exotics and their Dirac partners also come in an almost K -symmetric form, with only the t_R^c state missing. We will therefore probe our sensitivity to the associated threshold corrections by simply varying Λ_{comp} from 3 – 5 TeV, and insert a single (for simplicity) exotic threshold from 0.5 – 2 TeV in running SM couplings measured at m_Z up to Λ_{comp} .

Finally, we need to estimate the weakly-perturbed fixed-point couplings, $\lambda_{*\alpha}$. $\lambda_{*Q_L^3}$ is responsible for coupling the elementary t_L to the strong sector, yielding a Yukawa coupling below Λ_{comp} of $\lambda_{*Q_L^3}$ times an $\mathcal{O}(1)$ strong interaction factor. Thus we have $\lambda_{*Q_L^3} \sim 1$. $\lambda_{*exotic}$ is responsible for generating a Dirac mass for the exotics with the excess fermion composites, $m_{exotic} \sim \lambda_{*exotic} \Lambda_{comp} \sqrt{b^{strong}}/4\pi$. For $m_{exotic} \sim (\text{TeV})$ we also need $\lambda_{*exotic} \sim 1$. Thus, in our analysis $\lambda_{*\alpha} \sim 1$.

In Fig. 2 we exhibit a simple and standard test of unification, given the high precision of electroweak data, namely using measured values of $\alpha_{1,2}$ to postdict $\alpha_3(m_Z)$. We use separate bands to denote the variation in postdicted $\alpha_3(m_Z)$ coming from the above threshold ranges and from the theoretical error arising from our B_{ij}^{strong} , $C_{i\alpha}$ bounds. Note that our central predictions are excellent, we do not need large corrections, our largest uncertainties just reflect the conservative bounds put on the B^{strong} , C . The regime of controlled unification involves modest b^{strong} , not much larger than the size of the strong flavor group. Also, requiring that the SM gauge couplings do not have a Landau pole below M_U implies

$b^{strong} \lesssim 9$. This suggests that an AdS dual description, requiring a large ratio of strong colors ($\sim \mathcal{O}(b^{strong})$) to flavors, will not be useful. This is why we have not pursued RS modelling in the present context.

Let us assess our scenario with the criteria used to discuss earlier unification scenarios: (i) The couplings meet very precisely, strong circumstantial evidence for this form of unification. The postdiction of $\alpha_3(m_Z)$ works to better than $\delta_3 \sim 15\%$ over a wide range of b^{strong} . Alternatively, postdiction of $\sin^2\theta_W$ using α_3^{exp} leads to an error $\delta \sin^2\theta_W \simeq 0.03 \delta_3$. This is quite comparable with the level of success of SUSY unification [23]. (ii) With the best fit, $M_U \sim 10^{15}$ GeV, so large that experiments will not directly see the unification physics. However, there will be a striking signature of unification surviving to accessible energies [18], namely the strong sector resonances will fill out approximately degenerate multiplets of a *unified* flavor symmetry, K . Since their masses are expected to be in the few TeV range (based on naturalness), the cross-section for their single production at the LHC could be significant. These resonances will decay mostly into t_R and Higgs (including longitudinal W/Z) due to the strong coupling involved, quite distinctly from other models such as SUSY. (iii) We have arrived at precision unification here by considering one of the simplest non-supersymmetric scenarios for solving the hierarchy problem of the SM. (iv) $M_U \sim 10^{15}$ GeV is still low enough that exchanges of X, Y bosons at this scale can result in excessive proton decay. Even more importantly, composite states with the same quantum numbers can also mediate proton decay. The only known schemes in which both problems are solved are string or orbifold unification [2, 3, 20], so this is a requirement of our scenario. There could also be UV model-dependent states at M_U contributing to proton decay. These effects can be suppressed by imposing a (gauged) baryon-number symmetry, compatible with orbifold unification, as long as it is broken somewhat below M_U . This symmetry should also be an accidental flavor symmetry of the strong dynamics, to extend the usual accidental baryon-number symmetry of the SM. A second reason for preferring string/orbifold unification is that it makes it simpler to understand the appearance of incomplete grand-unified fermion multiplets in the IR, such as our exotics. In orbifold unification the global strong-sector symmetry G (or K) may even be the grand unified gauge group, surviving orbifold projections in this sector, but not in the elementary fermion/gauge-boson sectors. (v) As mentioned earlier, an attractive dark matter candidate emerges as a K -partner of t_R [22].

The scenario in which the SM hierarchy problem is solved non-supersymmetrically with top/Higgs compositeness, or a RS dual depiction, is attractive from several phenomenological points of view. In this letter, we have studied one of the key features that has been taken as

strong evidence in favor of a supersymmetric solution to the hierarchy problem, namely precision gauge-coupling unification. We have found an equally striking (but very different) unification that follows rather minimally from top/Higgs compositeness. We hope to have shown that taking unification as a serious consideration, one must still keep an open mind as to how the hierarchy problem is resolved in Nature, supersymmetrically or non-supersymmetrically. This is not a passive state, extracting new physics from upcoming colliders is challenging and requires planning ahead.

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