

Quantum Theory of Neutrino Spin-Light in Matter

A.Grigoriev*, A.Studenikin †

Department of Theoretical Physics,

Moscow State University,

119992 Moscow, Russia

A.Ternov ‡

Department of Theoretical Physics,

Moscow Institute for Physics and Technology,

141700 Dolgoprudny, Russia

Abstract

The quantum theory of the spin light of neutrino ($SL\nu$) exactly accounting for the effect of the background matter is developed. The $SL\nu$ rate and power, and also the emitted photon's energy are obtained for the different values of the neutrino momentum and density of matter. The spatial distribution of the radiation power and the dependence of the emitted photon's energy on the direction of radiation are studied. It is also shown that, in a wide range of the neutrino momentum and densities of matter, the $SL\nu$ radiation is characterized by nearly total circular polarization. Conditions for the effective neutrino spin-light radiation in the electron plasma are discussed.

1 Introduction

It is well known that a neutrino with non-zero mass may have non-trivial electromagnetic properties. In particular, the Dirac massive neutrino can possess non-vanishing magnetic and electric dipole moments. It is believed that non-zero neutrino magnetic moment could have an important impact on astrophysics and cosmology.

In the minimally extended Standard Model with $SU(2)$ -singlet right-handed neutrino the one-loop radiative correction generates neutrino magnetic moment which is proportional to the neutrino mass [1]

$$\mu_\nu = \frac{3}{8\sqrt{2}\pi^2} e G_F m_\nu = 3 \times 10^{-19} \mu_0 \left(\frac{m_\nu}{1\text{eV}} \right), \quad (1)$$

*ax.grigoriev@mail.ru

†studenik@srd.sinp.msu.ru

‡a_ternov@mail.ru

where $\mu_0 = e/2m$ is the Bohr magneton, m_ν and m are the neutrino and electron masses, respectively. There are also models [2] in which much large values for magnetic moments of neutrinos are predicted.

So far, the most stringent laboratory constraints on the electron, muon, and tau neutrino magnetic moments come from elastic neutrino-electron scattering experiments. The now days constraints for the magnetic moments of the three flavour neutrinos are as follows [3]: $\mu_{\nu_e} \leq 1.5 \times 10^{-10} \mu_0$, $\mu_{\nu_\mu} \leq 6.8 \times 10^{-10} \mu_0$, $\mu_{\nu_\tau} \leq 3.9 \times 10^{-10} \mu_0$.

The recent studies of a massive neutrino electromagnetic properties within the one-loop level, including the neutrino magnetic moment, can be found in [4], the detailed discussion on the neutrino charge radius is also presented in [5, 6]. Electromagnetic properties of a massive neutrino in external electromagnetic fields were studied in [7] and in the background matter were discussed in [8] (see also [9] for a review).

In a series of papers [10,11] we have predicted that a massive neutrino moving in matter and external electromagnetic and/or gravitational fields can emit electromagnetic radiation due the neutrino non-zero magnetic moment. We have named this radiation the "spin light of neutrino" ($SL\nu$) in matter and external fields. The quasi-classical theory of the $SL\nu$ in matter was developed in [10, 11] on the basis of Bargmann-Michel-Telegdi equation generalized [12–15] for the case of a neutrino moving in external fields. In our recent papers [16] we have developed the quantum approach to the $SL\nu$ in matter. However, at the final stage of the performed calculations of the the $SL\nu$ rate and total power we have considered the limit of low matter density. Therefore, the evaluation of the consistent quantum theory of the $SL\nu$ in matter has still remained an open issue. Note that the neutrino radiation due to the magnetic moment interaction with a constant magnetic field was considered in [17].

Here below we present the quantum theory of the neutrino spin-light exactly accounting for the effect of the background matter on the basis of the obtained [16] exact solutions of the modified Dirac equation for the neutrino wave function in matter.

2 Neutrino wave function in matter

For developing of the quantum theory of the $SL\nu$ in matter one has to find the neutrino wave functions with the effect of matter accounted for. To derive the quantum equation for the neutrino wave function in the background matter we start with the effective Lagrangian that describes the neutrino interaction with particles of the background matter. For definiteness, we consider the case of the electron neutrino ν propagating through moving and polarized matter composed of only electrons (the electron gas). Assume that the neutrino interactions are described by the extended standard model supplied with $SU(2)$ -singlet right-handed neutrino ν_R . We also suppose that there is a macroscopic amount of electrons in the scale of a neutrino de Broglie wave length. Therefore, the interaction of a neutrino with the matter (electrons) is coherent. In this case the averaged over the matter electrons addition to the vacuum neutrino Lagrangian, accounting for the charged

and neutral interactions, can be written in the form

$$\Delta L_{eff} = -f^\mu \left(\bar{\nu} \gamma_\mu \frac{1+\gamma_5}{2} \nu \right), \quad (2)$$

$$f^\mu = \frac{G_F}{\sqrt{2}} \left((1 + 4 \sin^2 \theta_W) j^\mu - \lambda^\mu \right),$$

where the electrons current j^μ and electrons polarization λ^μ are given by

$$j^\mu = (n, n\mathbf{v}), \quad (3)$$

and

$$\lambda^\mu = \left(n(\boldsymbol{\zeta} \cdot \mathbf{v}), n\boldsymbol{\zeta} \sqrt{1-v^2} + \frac{n\mathbf{v}(\boldsymbol{\zeta} \cdot \mathbf{v})}{1 + \sqrt{1-v^2}} \right), \quad (4)$$

θ_W is the Weinberg angle.

The Lagrangian (2) accounts for the possible effect of the matter motion and polarization. Here n , \mathbf{v} , and $\boldsymbol{\zeta}$ ($0 \leq |\boldsymbol{\zeta}|^2 \leq 1$) denote, respectively, the number density of the background electrons, the speed of the reference frame in which the mean momenta of the electrons is zero, and the mean value of the polarization vector of the background electrons in the above mentioned reference frame. The detailed discussion on the determination of the electrons polarization can be found in [13–15].

From the standard model Lagrangian with the extra term ΔL_{eff} being added, we derive the following modified Dirac equation for the neutrino moving in the background matter,

$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2}\gamma_\mu (1 + \gamma_5) f^\mu - m \right\} \Psi(x) = 0. \quad (5)$$

This is the most general equation of motion for a neutrino in which the effective potential $V_\mu = \frac{1}{2}(1 + \gamma_5)f_\mu$ accounts for both the charged and neutral-current interactions with the background matter and also for the possible effects of the matter motion and polarization.

If we neglect the contribution of the neutral-current interaction and possible effects of motion and polarization of the matter then from (5) we can get corresponding equations for the left-handed and right-handed chiral components of the neutrino field derived in [18].

The generalizations of the modified Dirac equation (5) for more complicated matter compositions (and the other flavour neutrinos) are just straightforward. For instance, if one considers the case of realistic matter composed of electrons, protons and neutrons, then the matter term in (5) is (see, for instance, [19], [14] and the second paper of [16])

$$f^\mu = \frac{G_F}{\sqrt{2}} \sum_{f=e,p,n} j_f^\mu q_f^{(1)} + \lambda_f^\mu q_f^{(2)} \quad (6)$$

where

$$q_f^{(1)} = (I_{3L}^{(f)} - 2Q^{(f)} \sin^2 \theta_W + \delta_{ef}),$$

$$q_f^{(2)} = - (I_{3L}^{(f)} + \delta_{ef}),$$

$$\delta_{ef} = \begin{cases} 1 & \text{for } f = e, \\ 0 & \text{for } f = n, p. \end{cases}$$

and $I_{3L}^{(f)}$ is the value of the isospin third component of a fermion f , $Q^{(f)}$ is the value of its electric charge and θ_W is the Weinberg angle. The fermion's currents j_f^μ and polarizations λ_f^μ are given by eqs.(3) and (4) with the appropriate substitutions: $n, \boldsymbol{\zeta}, \mathbf{v} \rightarrow n_f, \boldsymbol{\zeta}_f, \mathbf{v}_f$.

In the further discussion below we consider the case when no electromagnetic field is present in the background. We also suppose that the matter is unpolarized, $\lambda^\mu = 0$. Therefore, the term describing the neutrino interaction with the matter is given by

$$f^\mu = \frac{\tilde{G}_F}{\sqrt{2}}(n, n\mathbf{v}), \quad (7)$$

where we use the notation $\tilde{G}_F = G_F(1 + 4 \sin^2 \theta_W)$.

In the rest frame of the matter the equation (5) can be written in the Hamiltonian form,

$$i \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \hat{H}_{matt} \Psi(\mathbf{r}, t), \quad (8)$$

where

$$\hat{H}_{matt} = \hat{\boldsymbol{\alpha}} \mathbf{p} + \hat{\beta} m + \hat{V}_{matt}, \quad (9)$$

and

$$\hat{V}_{matt} = \frac{1}{2\sqrt{2}}(1 + \gamma_5)\tilde{G}_F n, \quad (10)$$

here \mathbf{p} is the neutrino momentum. We use the Pauli-Dirac representation for the Dirac matrices $\hat{\boldsymbol{\alpha}}$ and $\hat{\beta}$, in which

$$\hat{\boldsymbol{\alpha}} = \begin{pmatrix} 0 & \hat{\boldsymbol{\sigma}} \\ \hat{\boldsymbol{\sigma}} & 0 \end{pmatrix} = \gamma_0 \boldsymbol{\gamma}, \quad \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \gamma_0, \quad (11)$$

where $\hat{\boldsymbol{\sigma}} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrixes.

The form of the Hamiltonian (9) implies that the operators of the momentum, $\hat{\mathbf{p}}$, and longitudinal polarization, $\hat{\boldsymbol{\Sigma}} \mathbf{p}/p$, are the integrals of motion. So that, in particular, we have

$$\frac{\hat{\boldsymbol{\Sigma}} \mathbf{p}}{p} \Psi(\mathbf{r}, t) = s \Psi(\mathbf{r}, t), \quad \hat{\boldsymbol{\Sigma}} = \begin{pmatrix} \hat{\boldsymbol{\sigma}} & 0 \\ 0 & \hat{\boldsymbol{\sigma}} \end{pmatrix}, \quad (12)$$

where the values $s = \pm 1$ specify the two neutrino helicity states, ν_+ and ν_- . In the relativistic limit the negative-helicity neutrino state is dominated by the left-handed chiral state ($\nu_- \approx \nu_L$), whereas the positive-helicity state is dominated by the right-handed chiral state ($\nu_+ \approx \nu_R$).

For the stationary states of the equation (5) we get

$$\Psi(\mathbf{r}, t) = e^{-i(E_\varepsilon t - \mathbf{p}\mathbf{r})} u(\mathbf{p}, E_\varepsilon), \quad (13)$$

where $u(\mathbf{p}, E_\varepsilon)$ is independent on the coordinates and time. Upon the condition that the equation (5) has a non-trivial solution, we arrive to the energy spectrum of a neutrino moving in the background matter:

$$E_\varepsilon = \varepsilon \sqrt{\mathbf{p}^2 \left(1 - s\alpha \frac{m}{p}\right)^2 + m^2 + \alpha m}, \quad (14)$$

where we use the notation

$$\alpha = \frac{1}{2\sqrt{2}} \tilde{G}_F \frac{n}{m}. \quad (15)$$

The quantity $\varepsilon = \pm 1$ splits the solutions into the two branches that in the limit of the vanishing matter density, $\alpha \rightarrow 0$, reproduce the positive and negative-frequency solutions, respectively. It is also important to note that the neutrino energy in the background matter depends on the state of the neutrino longitudinal polarization, i.e. in the relativistic case the left-handed and right-handed neutrinos with equal momenta have different energies.

The procedure, similar to one used for derivation of the solution of the Dirac equation in vacuum, can be adopted for the case of a neutrino moving in matter. We apply this procedure to the equation (5) and arrive to the final form of the wave function of a neutrino moving in the background matter:

$$\Psi_{\varepsilon, \mathbf{p}, s}(\mathbf{r}, t) = \frac{e^{-i(E_\varepsilon t - \mathbf{p}\mathbf{r})}}{2L^{\frac{3}{2}}} \begin{pmatrix} \sqrt{1 + \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 + s \frac{p_3}{p}} \\ s \sqrt{1 + \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 - s \frac{p_3}{p}} e^{i\delta} \\ s\varepsilon \sqrt{1 - \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 + s \frac{p_3}{p}} \\ \varepsilon \sqrt{1 - \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 - s \frac{p_3}{p}} e^{i\delta} \end{pmatrix} \quad (16)$$

where the energy E_ε is given by (14), L is the normalization length and $\delta = \arctan p_2/p_1$. In the limit of vanishing density of matter, when $\alpha \rightarrow 0$, the wave function (16) transforms to the vacuum solution of the Dirac equation.

The quantum equation (5) for a neutrino in the background matter with the obtained exact solution (16) and energy spectrum (14) establish a basis for a very effective method (similar to the Furry representation of quantum electrodynamics) in investigations of different phenomena that can appear when neutrinos are moving in the media.

Let us now consider in some detail the properties of a neutrino energy spectrum (14) in the background matter that are very important for understanding of the mechanism of the neutrino spin light phenomena. For the fixed magnitude of the neutrino momentum p there are the two values for the "positive sign" ($\varepsilon = +1$) energies

$$\begin{aligned} E^{s=+1} &= \sqrt{\mathbf{p}^2 \left(1 - \alpha \frac{m}{p}\right)^2 + m^2 + \alpha m}, \\ E^{s=-1} &= \sqrt{\mathbf{p}^2 \left(1 + \alpha \frac{m}{p}\right)^2 + m^2 + \alpha m}, \end{aligned} \quad (17)$$

that determine the positive- and negative-helicity eigenstates, respectively. The energies in Eq.(17) correspond to the particle (neutrino) solutions in the background matter. The

two other values for the energy, corresponding to the negative sign $\varepsilon = -1$, are for the antiparticle solutions. As usual, by changing the sign of the energy, we obtain the values

$$\begin{aligned}\tilde{E}^{s=+1} &= \sqrt{\mathbf{p}^2 \left(1 - \alpha \frac{m}{p}\right)^2 + m^2} - \alpha m, \\ \tilde{E}^{s=-1} &= \sqrt{\mathbf{p}^2 \left(1 + \alpha \frac{m}{p}\right)^2 + m^2} - \alpha m,\end{aligned}\tag{18}$$

that correspond to the positive- and negative-helicity antineutrino states in the matter. The expressions in Eqs.(17) and (18) would reproduce the neutrino dispersion relations of [18] (see also [20]), if the contribution of the neutral-current interaction to the neutrino potential were left out.

3 Quantum theory of $SL\nu$ with exact account for matter density

In this section we should like to use the obtained solutions (16) of the equation (5) for a neutrino moving in the background matter for the study of the spin light of neutrino ($SL\nu$) in the matter. We develop below the *quantum* theory of this effect exactly accounting for the matter number density n . Note that in our previous studies [16] the $SL\nu$ rate and radiation power were derived in the limit of not very high densities of the background matter (see also our comments to Eq.(31) below).

Within the quantum approach, the corresponding Feynman diagram of the $SL\nu$ in the matter is the standard one-photon emission diagram with the initial and final neutrino states described by the "broad lines" that account for the neutrino interaction with the matter. It follows from the usual neutrino magnetic moment interaction with the quantized photon field, that the amplitude of the transition from the neutrino initial state ψ_i to the final state ψ_f , accompanied by the emission of a photon with a momentum $k^\mu = (\omega, \mathbf{k})$ and polarization \mathbf{e}^* , can be written in the form

$$S_{fi} = -\mu\sqrt{4\pi} \int d^4x \bar{\psi}_f(x) (\hat{\Gamma} \mathbf{e}^*) \frac{e^{ikx}}{\sqrt{2\omega L^3}} \psi_i(x),\tag{19}$$

where μ is the neutrino magnetic moment, ψ_i and ψ_f are the corresponding exact solutions of the equation (5) given by (16), and

$$\hat{\Gamma} = i\omega \{ [\boldsymbol{\Sigma} \times \boldsymbol{\varkappa}] + i\gamma^5 \boldsymbol{\Sigma} \}.\tag{20}$$

Here $\boldsymbol{\varkappa} = \mathbf{k}/\omega$ is the unit vector pointing the direction of the emitted photon propagation. Note that the developed approach to the considered process in the presence of matter is similar to the Furry representation in studies of quantum processes in external electromagnetic fields.

The integration in (21) with respect to time yields

$$S_{fi} = -\mu \sqrt{\frac{2\pi}{\omega L^3}} 2\pi \delta(E_f - E_i + \omega) \int d^3x \bar{\psi}_f(\mathbf{r})(\hat{\Gamma} \mathbf{e}^*) e^{i\mathbf{k}\mathbf{r}} \psi_i(\mathbf{r}), \quad (21)$$

where the delta-function stands for the energy conservation. Performing the integrations over the spatial co-ordinates, we can recover the delta-functions for the three components of the momentum. Finally, we get the law of the energy-momentum conservation for the considered process,

$$E_i = E_f + \omega, \quad \mathbf{p}_i = \mathbf{p}_f + \boldsymbol{\varkappa}. \quad (22)$$

where E_i and E_f are the energies of the initial and final neutrino states in matter. From (22) it follows that the emitted photon energy ω exhibits the critical dependence on the helicities of the initial and final neutrino states. In the case of electron neutrino moving in matter composed of electrons α is positive. Thus, it follows that the only possibility for the $SL\nu$ to appear is provided in the case when the neutrino initial and final states are characterized by $s_i = -1$ and $s_f = +1$, respectively. We conclude that in the considered process the relativistic left-handed neutrino is converted to the right-handed neutrino. A discussion on the main properties of the $SL\nu$ emitted by different flavour neutrinos moving in matter composed of electrons, protons and neutrons can be found in [16] (see also [21]).

The emitted photon energy in the considered case ($s_i = -s_f = -1$) obtained as an exact solution of the equations (22) reads

$$\omega = \frac{2\alpha m p [(E - \alpha m) - (p + \alpha m) \cos \theta]}{(E - \alpha m - p \cos \theta)^2 - (\alpha m)^2}, \quad (23)$$

where θ is the angle between $\boldsymbol{\varkappa}$ and the direction of the initial neutrino propagation.

In the case of not very high density of matter, when the parameter $\tilde{G}_F n/m \ll 1$, we can expand the photon energy (23) over the above mentioned parameter and in the liner approximation get the result of [16]:

$$\omega = \frac{1}{1 - \beta \cos \theta} \omega_0, \quad (24)$$

where

$$\omega_0 = \frac{\tilde{G}_F}{\sqrt{2}} n \beta, \quad (25)$$

and β is the neutrino speed in vacuum.

Using the wave functions (16) for the neutrino initial and final states in matter we calculate the spin light transition rate exactly accounting for the matter density parameter and get

$$\Gamma = \mu^2 \int_0^\pi \omega^3 [(\tilde{\beta} \tilde{\beta}' + 1)(1 - y \cos \theta) - (\tilde{\beta} + \tilde{\beta}')(\cos \theta - y)] \frac{\sin \theta}{1 + \tilde{\beta}' y} d\theta, \quad (26)$$

Here we use the notations

$$\tilde{\beta} = \frac{p + \alpha m}{E - \alpha m}, \quad \tilde{\beta}' = \frac{p' - \alpha m}{E' - \alpha m}, \quad (27)$$

where the final neutrino energy and momentum are

$$E' = E - \omega, \quad p' = K\omega - p, \quad (28)$$

respectively, and

$$y = \frac{\omega - p \cos \theta}{p'}, \quad K = \frac{E - \alpha m - p \cos \theta}{\alpha m}. \quad (29)$$

Performing the integration in Eq.(26), we obtain for the $SL\nu$ rate in matter

$$\begin{aligned} \Gamma = & \frac{1}{2(E-p)^2(E+p-2\alpha m)^2(E-\alpha m)p^2} \\ & \times \left\{ (E^2 - p^2)^2 (p^2 - 6\alpha^2 m^2 + 6E\alpha m - 3E^2) ((E - 2\alpha m)^2 - p^2)^2 \right. \\ & \times \ln \left[\frac{(E+p)(E-p-2\alpha m)}{(E-p)(E+p-2\alpha m)} \right] + 4\alpha m p [16\alpha^5 m^5 E (3E^2 - 5p^2) \\ & - 8\alpha^4 m^4 (15E^4 - 24E^2 p^2 + p^4) + 4\alpha^3 m^3 E (33E^4 - 58E^2 p^2 + 17p^4) \\ & - 2\alpha^2 m^2 (39E^2 - p^2) (E^2 - p^2)^2 + 12\alpha m E (2E^2 - p^2) (E^2 - p^2)^2 \\ & \left. - (3E^2 - p^2) (E^2 - p^2)^3 \right\}, \quad (30) \end{aligned}$$

where the energy of the initial neutrino is given by (14).

As it follows from (30), the $SL\nu$ rate is a rather complicated function of the neutrino momentum p and mass m , it is also non-trivially dependent on the matter density parameter α . In the limit of low densities of matter, $\alpha \ll 1$, we get

$$\Gamma \simeq \frac{64}{3} \frac{\mu^2 \alpha^3 p^3 m}{E_0}, \quad (31)$$

where $E_0 = \sqrt{p^2 + m^2}$. The obtained expression is in agreement with our results of refs. [10, 16]. Note that the considered limit of $\alpha \ll 1$ can be appropriate even for very dense media of neutron stars with $n \sim 10^{33} \text{ cm}^{-3}$ because $\frac{1}{2\sqrt{2}} \tilde{G}_F n \sim 1 \text{ eV}$ for a medium characterized by $n = 10^{37} \text{ cm}^{-3}$.

Let us consider the $SL\nu$ rate for the different limiting values of the neutrino momentum p and matter density parameter α . In the relativistic case $p \gg m$ from (30) we get

$$\Gamma = \begin{cases} \frac{64}{3} \mu^2 \alpha^3 p^2 m, & \text{for } \alpha \ll \frac{m}{p}, \\ 4\mu^2 \alpha^2 m^2 p, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ 4\mu^2 \alpha^3 m^3, & \text{for } \alpha \gg \frac{p}{m}. \end{cases} \quad (32)$$

In the opposite case $p \ll m$ we get

$$\Gamma = \begin{cases} \frac{64}{3}\mu^2\alpha^3p^3, & \text{for } \alpha \ll 1, \\ \frac{512}{5}\mu^2\alpha^6p^3, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^3m^3, & \text{for } \alpha \gg \frac{m}{p}. \end{cases} \quad (33)$$

On the basis of (21) we also derive the $SL\nu$ radiation power:

$$I = \mu^2 \int_0^\pi \omega^4 [(\tilde{\beta}\tilde{\beta}' + 1)(1 - y \cos \theta) - (\tilde{\beta} + \tilde{\beta}')(\cos \theta - y)] \frac{\sin \theta}{1 + \tilde{\beta}'y} d\theta.$$

Performing the integration and considering the case $\alpha \ll 1$, we get for the total power

$$I \simeq \frac{128}{3}\mu^2\alpha^4p^4 \quad (34)$$

in agreement with refs. [10, 16].

Let us now consider the $SL\nu$ radiation power for the different limiting values of the neutrino momentum p and matter density parameter α . In the relativistic case $p \gg m$ from (34) we get

$$I = \begin{cases} \frac{128}{3}\mu^2\alpha^4p^4, & \text{for } \alpha \ll \frac{m}{p}, \\ \frac{4}{3}\mu^2\alpha^2m^2p^2, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ 4\mu^2\alpha^4m^4, & \text{for } \alpha \gg \frac{p}{m}. \end{cases} \quad (35)$$

In the opposite case $p \ll m$ we get

$$I = \begin{cases} \frac{128}{3}\mu^2\alpha^4p^4, & \text{for } \alpha \ll 1, \\ \frac{1024}{3}\mu^2\alpha^8p^4, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^4m^4, & \text{for } \alpha \gg \frac{m}{p}. \end{cases} \quad (36)$$

Note that the obtained $SL\nu$ rate and radiation power in the case $p \gg m$ for $\alpha \gg \frac{m}{p}$ are in agreement with [21].

From the obtained expressions for the $SL\nu$ rate and total power it is possible to get an estimation for the emitted photons average energy:

$$\langle \omega \rangle = \frac{I}{\Gamma}. \quad (37)$$

In the case $p \gg m$ we get

$$\langle \omega \rangle \simeq \begin{cases} 2\alpha\frac{p^2}{m}, & \text{for } \alpha \ll \frac{m}{p}, \\ \frac{1}{3}p, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ \alpha m, & \text{for } \alpha \gg \frac{p}{m}. \end{cases} \quad (38)$$

In the opposite case $p \ll m$ for the emitted photons overage energy we have

$$\langle \omega \rangle \simeq \begin{cases} 2p\alpha, & \text{for } \alpha \ll 1, \\ \frac{10}{3}p\alpha^2, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ \alpha m, & \text{for } \alpha \gg \frac{m}{p}. \end{cases} \quad (39)$$

4 Conclusion

We should like to note that for a wide range of the neutrino momentum p and density parameter α the $SL\nu$ power is collimated along the direction of the neutrino propagation. The form of the radiation power spatial distribution calculated with use of Eq.(34) in the case $p > m$ for low and high density of matter are shown in Figs. 1 and 2, respectively. As it follows from these figures, the form of the distribution depends on the density of matter. The form of the spatial distribution of the radiation moves from the projector-like shape to the cap-like one with increase of the matter density. From (34) we derive, that in the case of $p \gg m$ for a wide range of the matter densities, $\alpha \ll \frac{p}{m}$, the direction of the maximum in the spatial distribution of the radiation power is characterized by the angle

$$\cos \theta_{max} \simeq 1 - \frac{2}{3} \alpha \frac{m}{p}. \quad (40)$$

It follows that in a dense matter the $SL\nu$ radiation in the direction of the initial neutrino motion is strongly suppressed, whereas there is a lighted ring on the plane perpendicular to the neutrino motion.

The quantum calculation of the radiation power with the photon's circular polarization being accounted for (see also [16]) shows another very important property of the $SL\nu$ in matter. In the limit of low matter density, $\alpha \ll 1$, we get for the power

$$I^{(l)} \simeq \frac{64}{3} \mu^2 \alpha^4 p^4 \left(1 - l \frac{p}{2E_0} \right), \quad (41)$$

where $l = \pm 1$ corresponds to the right and left photon circular polarizations, respectively. In this limiting case the power of radiation of the left-polarized photons exceeds that of the right-polarized photons. In particular, this result is also valid for the non-relativistic case $p \ll m$ for low density with $\alpha \ll 1$. It is remarkable that in the most interesting case of a rather dense matter ($\alpha \gg \frac{m}{p}$ for $p \gg m$ and $\alpha \gg 1$ for $p \ll m$), the main contribution to the power is provided by the right-polarized photons, whereas the emission of the left-polarized photons is suppressed:

$$I^{(+1)} \simeq I, \quad (42)$$

$$I^{(-1)} \simeq 0. \quad (43)$$

We conclude that in a dense matter the $SL\nu$ photons are emitted with nearly total right-circular polarization.

Finally, we should like to discuss in some detail restrictions on the propagation of the $SL\nu$ photons that are set by the presence of the background electron plasma in the case of $p \gg m$ for the density parameter $\frac{m}{p} \ll \alpha \ll \frac{p}{m}$. Only the photons with energy that exceeds the plasmon frequency

$$\omega_{pl} = \sqrt{\frac{4\pi e^2}{m_e} n}, \quad (44)$$

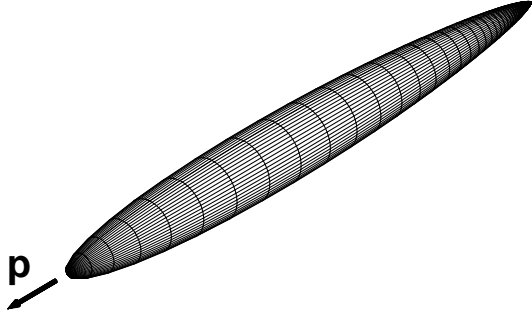


Figure 1: The spatial distribution of the $SL\nu$ radiation power for the case $p/m = 5$, $\alpha m = 0.01$.

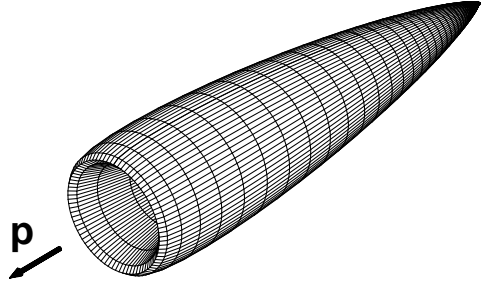


Figure 2: The spatial distribution of the $SL\nu$ radiation power for the case $p/m = 10^3$, $\alpha m = 100$.

can propagate in the plasma (here $e^2 = \alpha_{QED}$ is the fine-structure constant and m_e is the mass of the electron). From (23) and (40) it follows that, on the one hand, the photon's energy and the radiation power depend on the direction of the radiation. On the other hand, we can conclude that the maximal value of the photon's energy,

$$\omega_{max} = p, \quad (45)$$

and the energy the photon emitted in the direction of the maximum radiation power,

$$\omega(\theta_{max}) = \frac{3}{4}p, \quad (46)$$

are of the same order in the considered case. For the relativistic neutrino and rather dense matter the angle θ_{max} , at which the radiation power has its maximum, is very close to zero when the photon energy reaches the maximal value (45). Therefore, the condition for the effective $SL\nu$ production and propagation through the plasma can be written in the form

$$p \gg p_{min} = 3.5 \times 10^4 \left(\frac{n}{10^{30} cm^{-3}} \right)^{1/2} eV. \quad (47)$$

For $n \sim 10^{33} cm^{-3}$ we get $p_{min} \sim 1 MeV$.

The investigated properties of the neutrino spin light in matter (i.e., the spatial distribution of the radiation power and the angular dependence of the emitted photon's energy, as well as nearly total circular polarization of the radiation) might be important for the experimental identification of the $SL\nu$ from different astrophysical and cosmology objects and media.

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