

Effects of CPT and Lorentz Invariance Violation on Pulsar Kicks

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The breakdown of Lorentz's and CPT invariance, as described by the Extension of the Standard Model, gives rise to a modification of the dispersion relation of particles. Consequences of such a modification are reviewed in the framework of pulsar kicks induced by neutrino oscillations (active-sterile conversion). A peculiar feature of the modified energy-momentum relations is the occurrence of terms of the form $\delta\mathbf{\Pi} \cdot \hat{\mathbf{p}}$, where $\delta\mathbf{\Pi}$ accounts for the difference of spatial components of flavor depending coefficients which lead to the departure of the Lorentz symmetry, and $\hat{\mathbf{p}} = \mathbf{p}/p$, being \mathbf{p} the neutrino momentum. Owing to the relative orientation of \mathbf{p} with respect to $\delta\mathbf{\Pi}$, the coupling $\delta\mathbf{\Pi} \cdot \hat{\mathbf{p}}$ may induce the mechanism to generate the observed pulsar velocities. Topics related to the velocity distribution of pulsars are also discussed.

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The studies of a possible breakdown of the fundamental symmetries in physics represent a very active research area. As suggested by Kostelecký and Samuel, String/M theory provides a scenario in which a departure of the Lorentz invariance might manifest [1]. Recently, these investigations have been reconsidered in the context of D -branes [2], Loop Quantum Gravity [3–5], Non-Commutative Geometry [6], and through the spacetime variation of fundamental coupling constants [8]. For modern tests on Lorentz invariance, see [7]

According to Ref. [1], Lorentz's invariance violation (due to non trivial solution of (open) string field theory) follows from the observation that the vacuum solution of the theory could spontaneously violate the Lorentz and CPT invariance, even though such symmetries are satisfied by the underlying theory. The breakdown of these fundamental symmetries occurs in the Extension of the Standard Model, and only operators of mass with dimension four or less [9,10] are involved in order that the standard model power-counting renormalizability is preserved. In a recent work by Kostelecký and Mewes [11], it has been studied the general formalism for violations of Lorentz and CPT symmetry in the neutrino sector. The generalized equation of motion for free fermions (neutrinos in our case) is given by (we shall use natural units $c = 1 = \hbar$)

$$(i\Gamma_{AB}^\nu \partial_\nu - M_{AB})\psi_B = 0, \quad (1)$$

where the spinor ψ_B contains all the fields and their conjugates, the indices A and B range over all $2N$ possibility $\{f, f^C\}$, being $f = e, \mu, \tau, \dots$ the neutrino flavors and $f_A^C = C_{AB}f_B$. C is the symmetric matrix with non zero components $C_{ff^c} = 1$. Γ_{AB}^ν and M_{AB} are 4×4 matrices in the spinor space, and can be decomposed using the basis of γ matrices

$$\Gamma_{AB}^\nu = \gamma_{AB}^\nu + c_{AB}^{\mu\nu}\gamma_\mu + d_{AB}^{\mu\nu}\gamma_5\gamma_\mu + e_{AB}^\nu + if_{AB}^\nu\gamma_5 + \frac{1}{2}g_{AB}^{\lambda\mu\nu}\sigma_{\lambda\mu}, \quad (2)$$

and

$$M_{AB} = m_{AB} + im_5\gamma_5 + a_{\mu AB}\gamma^\mu + b_{\mu AB}\gamma_5\gamma^\mu + \frac{1}{2}H_{AB}^{\mu\nu}\sigma_{\mu\nu}. \quad (3)$$

The coefficients $a_{\mu AB}$, $b_{\mu AB}$, $c_{AB}^{\mu\nu}, \dots$ are constants and in general they are flavor depending, and $\sigma^{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu]$. $c_{AB}^{\mu\nu}$, $d_{AB}^{\mu\nu}$, and $H_{AB}^{\mu\nu}$ preserve the CPT invariance, while $a_{\mu AB}$, $b_{\mu AB}$, $e_{\mu AB}$, $f_{\mu AB}$ violate CPT and Lorentz invariance. Finally m and m_5 are Lorentz and CPT conserving.

The time evolution of neutrinos is governed by the effective Hamiltonian

$$(H_{eff})_{ab} = p\delta_{ab} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2p} \begin{pmatrix} (\tilde{m}^2)_{ab} & 0 \\ 0 & (\tilde{m}^2)_{ab}^* \end{pmatrix} + \frac{1}{p} \begin{pmatrix} [(a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu]_{ab} & -i\sqrt{2}p_\mu(\epsilon_+)^\nu [(g^{\mu\nu\sigma} p_\sigma - H^{\mu\nu})\mathcal{C}]_{ab} \\ i\sqrt{2}p_\mu(\epsilon_+)^\nu [(g^{\mu\nu\sigma} p_\sigma + H^{\mu\nu})\mathcal{C}]_{ab}^* & [-(a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu]_{ab}^* \end{pmatrix}, \quad (4)$$

where $(c_L)_{ab}^{\mu\nu} = (c + d)_{ab}^{\mu\nu}$ and $(a_L)_{ab}^\mu = (a + b)_{ab}^\mu$, the vector $(\epsilon_+)^\nu = \frac{1}{\sqrt{2}}(0, \epsilon_1 + i\epsilon_2)$ is related to the helicity of neutrinos (ϵ_1 and ϵ_2 are two real vectors), while $(\tilde{m}^2)_{ab}$ is related to the usual neutrino masses. Details of properties

of the coefficients entering in the effective Hamiltonian can be found in [11]. It is worth to note that Eq. (4) involves only the coefficients a , c , g and H , since the other coefficients breaking the Lorentz invariance can be removed, at the leading order, with an appropriate redefinition of fields.

Bounds on parameters entering in the Extension of the Standard Model which involve neutrino oscillations have been discussed in [11–15]. Stringent bounds on the coefficients $(a_L)^\mu$ and $(c_L)^{\mu\nu}$ have been obtained in [11–13] by analyzing various type of experiments (solar neutrino experiments [SK, SNO]; atmospheric neutrinos experiments [SK, KamLAND, LSND, K2K]; reactor experiments [CHOOZ, Palo Verde]; short-baseline experiments [CHORUS, NOMAD, KARMEN]; long-baseline experiments [ICARUS, MINOS, OPERA]). Estimations of the attainable sensitivities to the dimensionless $(\delta c_L = c_L^f - c_L^{f'})$ and dimension-one $(\delta a_L = a_L^f - a_L^{f'})$ coefficients are [11]

$$10^{-26} \lesssim \delta c_L^{\mu\nu} \lesssim 10^{-17}, \quad 10^{-19} \text{eV} \lesssim \delta a_L^\mu \lesssim 10^{-9} \text{eV}. \quad (5)$$

The aim of this paper is to investigate the consequences of the Lorentz invariance breakdown in relation to pulsar kicks induced by neutrino oscillations in matter. The origin of the pulsar velocity is till now an open issue of the modern astrophysics. As follows from the observations, pulsars have a very high velocity, in comparison with the surrounding stars, which may varies from $450 \pm 90 \text{Km/sec}$ up to values greater than 1000Km/sec [16,17]. This suggests that nascent pulsars undergo to some kind of kick. After the supernova collapse of a massive star, neutrinos carry away almost all (99%) the gravitational binding energy ($3 \times 10^{53} \text{erg}$). The momentum taken by them is about 10^{43}gr cm/sec . A fractional anisotropy of the order $\sim 1\%$ of the outgoing neutrino momenta would suffice to account for the neutron star recoil of 300Km/sec . Many mechanisms have been proposed to solve such a issue, but a definitive solution is still laking.

An elegant and interesting mechanism to generate the pulsar velocity, which involves the neutrino oscillation physics, has been proposed by Kusenko and Segré [18]. Under suitable conditions, a resonant oscillation $\nu_e \rightarrow \nu_{\mu,\tau}$ may occur in the region between the corresponding (ν_e and $\nu_{\mu,\tau}$) neutrinospheres. The ν_e are trapped by the medium (due to neutral and charged interactions) but neutrinos $\nu_{\mu,\tau}$ generate via oscillations can escape from the protostar being outside of their neutrinosphere. Thus, the surface of the resonance acts as an effective muon/tau neutrinosphere. In the presence of a magnetic field \mathbf{B} , the surface of resonance is distorted with the ensuing that the energy flux turns out to be generated anisotropically (neutrinos generated in regions with different temperatures are emitted with different energies). Indeed, in a magnetized medium, the resonance condition in matter is modified by a term $\sim \mathbf{B} \cdot \mathbf{p}$, where \mathbf{p} the neutrino momentum. The relative orientation of the neutrino momentum with respect to the magnetic field generate the asymmetry in the neutrino emission. The case in which active-sterile neutrinos are involved has been studied in [19]. Papers dealing with the problem of the origin of pulsar velocities, can be found in Refs. [20–30].

The effective Hamiltonian (4) suggests a further mechanism to generate pulsar kicks. As we will see, terms breaking the Lorentz invariance give rise to a coupling of the form $\sim \delta \mathbf{\Pi} \cdot \hat{\mathbf{p}}$, where $\delta \mathbf{\Pi}$ is the difference of spatial components c_L^{0i} , a_L^i referred to different flavors, and $\hat{\mathbf{p}} = \mathbf{p}/p$, where $p = |\mathbf{p}|$. The relative orientation of neutrino momenta with respect to $\delta \mathbf{\Pi}$ determines an asymmetry in the neutrino emission, hence can generate the pulsar kicks. This effect might provide a signature of a possible violation of the Lorentz invariance.

As observed in [19], oscillations of active neutrinos could be a plausible explanation of the observed pulsar velocities if the resonant conversion $\nu_{\tau,\mu} \leftrightarrow \nu_e$ occurs between two different neutrinospheres. However, the resonant transition between two neutrinospheres leads to neutrino masses which do not agree with the present limits on the masses of standard electroweak neutrinos. These limits do not apply to sterile neutrinos that may have only a small mixing angle with the ordinary neutrinos [19]. Thus we shall confine ourselves to active-sterile conversion of neutrinos.

Eq. (4) implies that the energy of neutrinos with a given flavor is (we shall consider only the diagonal terms)

$$E \simeq p + \frac{m^2}{2p} + \Omega, \quad (6)$$

with

$$\Omega = -\frac{c_L^{\mu\nu} p_\mu p_\nu}{p} + \frac{a_{L\mu} p^\mu}{p}. \quad (7)$$

For our purpose, it is convenient to rewrite Eq. (7) in the form

$$\Omega \approx \Pi_0 + \mathbf{\Pi} \cdot \hat{\mathbf{p}}, \quad (8)$$

where

$$\Pi_0 = -c_L^{00} p + \frac{c_L^{ij} p_i p_j}{p} + a_{L0}, \quad (9)$$

$$\mathbf{\Pi} = -2p \mathbf{c}_L + \mathbf{a}_L, \quad (10)$$

and the relations $p^0 \simeq p$ and $c^i = c^{0i}$ have been used.

To estimate the anisotropy of the outgoing neutrinos, one needs to evaluate the energy flux \mathbf{F}_s emitted by nascent stars. According to Ref. [23], the asymmetry of the neutrino momentum is

$$\frac{\Delta p}{p} \simeq \frac{1}{3} \frac{\int_0^\pi \mathbf{F}_s(\vartheta) \cdot \delta \hat{\mathbf{\Pi}} da}{\int_0^\pi \mathbf{F}_s(\vartheta) \cdot \hat{\mathbf{n}} da} \sim -\frac{1}{9} \frac{\varrho}{r_{res}}. \quad (11)$$

The factor 1/3 takes into account of the fact that only one neutrinos species is responsible for the anisotropy (thus it carries out only 1/3 of the total energy). \mathbf{F}_s is the outgoing neutrino flux through the element area da of the emission surface. $\delta \hat{\mathbf{\Pi}} = \mathbf{\Pi}/|\mathbf{\Pi}|$ is the direction of the vector $\delta \mathbf{\Pi} = \mathbf{\Pi}^{(\nu_f)} - \mathbf{\Pi}^{(\nu_s)}$, whereas $\hat{\mathbf{n}}$ is the unity vector orthogonal to the element area da . The parameter ϱ is the radial deformation of the effective surface of resonance generated by the term breaking the Lorentz invariance $\mathbf{\Pi}$. It shifts the resonance point r_{res} to $r(\phi) = r_{res} + \varrho \cos \phi$, with $\varrho \ll r_{res}$ and $\cos \phi = \delta \hat{\mathbf{\Pi}} \cdot \hat{\mathbf{p}}$. The resonance point r_{res} is determined by the usual resonance condition (MSW effect [31])

$$2\delta c_2 - V_{\nu_f}(r_{res}) = 0, \quad (12)$$

where

$$\delta = \frac{\Delta m^2}{4p}, \quad c_2 = \cos 2\theta. \quad (13)$$

$\Delta m^2 = m_2^2 - m_1^2$ is the mass squared difference (in the notation of (13), we shall also indicate $\sin 2\theta = s_2$). In (12), the potential V_{ν_f} is defined as follows

$$V_{\nu_e} = -V_{\bar{\nu}_e} = V_0(3Y_e - 1 + 4Y_{\nu_e}), \quad (14)$$

$$V_{\nu_{\mu,\tau}} = -V_{\bar{\nu}_{\mu,\tau}} = V_0(Y_e - 1 + 2Y_{\nu_e}), \quad (15)$$

where Y_e (Y_{ν_e}) represents the ratio between the number density of electrons (neutrinos), and

$$V_0 = \frac{G_F \rho}{\sqrt{2} m_n} = \frac{\rho}{10^{14} \text{gr/cm}^3} 3.8 \text{ eV}. \quad (16)$$

$m_n = 938 \text{ MeV}$ is the nucleon mass and ρ the matter density. For sterile neutrinos one has $V_{\nu_s} = 0$.

The equation of evolution describing the conversion between two neutrino flavors (we consider the conversion $\nu_f \leftrightarrow \nu_s$, $f = e, \mu, \tau$) is

$$i \frac{d}{dr} \begin{pmatrix} \nu_f \\ \nu_s \end{pmatrix} = \mathcal{H} \begin{pmatrix} \nu_f \\ \nu_s \end{pmatrix}, \quad (17)$$

where the matrix \mathcal{H} is the effective Hamiltonian defined as

$$\mathcal{H} = \begin{bmatrix} V_{\nu_f} - c_2 \delta + \mathbf{\Pi}^{(\nu_f)} \cdot \hat{\mathbf{p}} & s_2 \delta \\ s_2 \delta & c_2 \delta + \mathbf{\Pi}^{(\nu_s)} \cdot \hat{\mathbf{p}} \end{bmatrix}, \quad (18)$$

up to terms proportional to identity matrix¹.

¹A comment is in order. The Lorentz invariance violation implies that the propagation of neutrinos depends, in general, on their flavors through the parameters $c_L^{\mu\nu}$, $a_{L\mu}$ (for the moment, we shall neglect neutrino masses). The velocity is

$$v_f = \frac{dE}{dp} \approx 1 - \frac{d\Omega}{dp}.$$

Different neutrino species may have different maximum attainable velocities [32,33]. This occurs if neutrino flavor eigenstates are distinct from neutrino velocity eigenstates, being the two eigenstates related linearly by the mixing angle θ_v . Thus, the diagonal terms $\mathbf{\Pi}^{(\nu_f, \nu_s)} \cdot \hat{\mathbf{p}}$ in (18) should be replaced by $\mathbf{\Pi}^{(\nu_f, \nu_s)} \cdot \hat{\mathbf{p}} \cos 2\theta_v$, whereas the term $\mathbf{\Pi}^{(\nu_f, \nu_s)} \cdot \hat{\mathbf{p}} \sin 2\theta_v$ should appear in the off-diagonal. For simplicity we have taken $\theta_v = 0$.

As follows from (18), the resonance condition reads

$$2\delta c_2 = V_{\nu_f} + \delta \mathbf{\Pi} \cdot \hat{\mathbf{p}} + \delta \Pi_0. \quad (19)$$

where

$$\begin{aligned} \delta \mathbf{\Pi} \cdot \hat{\mathbf{p}} &= (\delta \mathbf{a} - 2p\delta \mathbf{c}) \cdot \hat{\mathbf{p}} \\ &= |\delta \mathbf{a} - 2p\delta \mathbf{c}| \cos \phi, \end{aligned} \quad (20)$$

and $\delta \mathbf{c} = \mathbf{c}^{(\nu_f)} - \mathbf{c}^{(\nu_s)}$, $\delta \mathbf{a} = \mathbf{a}^{(\nu_f)} - \mathbf{a}^{(\nu_s)}$. A similar expression holds for $\delta \Pi_0$. Notice that $\delta \Pi_0(\mathbf{p}) = \delta \Pi_0(-\mathbf{p})$ and, for typical values of $\rho \sim (10^{11} - 10^{14})\text{gr/cm}^3$ in the protostar, $\delta \Pi_0 \ll 2\delta c_2$ according to constraints (5). Moreover, since $\delta \mathbf{\Pi} \cdot \hat{\mathbf{p}}$ changes sign under the transformation $\mathbf{p} \rightarrow -\mathbf{p}$, it may deform the resonance surface, a condition necessary in order that the asymmetry of the outgoing neutrino momenta may occur.

Let us now evaluate ϱ . Expanding the terms in (19) about to $r = r_{res} + \varrho \cos \phi$, with $p \rightarrow p + \delta_p$, $V_{\nu_f} \rightarrow V_{\nu_f} + \delta_{V_{\nu_f}}$ [23], in which

$$\delta_p = \frac{d \ln p}{dr} p \varrho \cos \phi = h_p^{-1} p \varrho \cos \phi, \quad (21)$$

$$\delta_{V_{\nu_f}} = \frac{d \ln V_{\nu_f}}{dr} V_{\nu_f} \varrho \cos \phi = h_{V_{\nu_f}}^{-1} V_{\nu_f} \varrho \cos \phi, \quad (22)$$

and using the resonance condition (12), one infers (all quantities are evaluated at the resonance)

$$\varrho \approx -\frac{\Sigma}{V_{\nu_f}} \frac{1}{h_p^{-1} + h_{V_{\nu_f}}^{-1}}, \quad (23)$$

where

$$\Sigma \equiv 2p|\delta \mathbf{c}_L| + |\delta \mathbf{a}_L|. \quad (24)$$

Inserting Eq. (23) into Eq. (11) one gets

$$\frac{\Delta p}{p} = \frac{1}{9} \frac{\Sigma}{V_{\nu_f}} \frac{1}{r_{res}(h_p^{-1} + h_{V_{\nu_f}}^{-1})}, \quad (25)$$

which implies that the fractional asymmetry $\Delta p/p$ is $\sim 1\%$ provided

$$\Sigma \sim 0.09 V_{\nu_f} r_{res} (h_p^{-1} + h_{V_{\nu_f}}^{-1}). \quad (26)$$

To compute $h_p^{-1} + h_{V_{\nu_f}}^{-1}$ one has to specify a model for the protostar. To this aim, we assume that the inner core of a protostar is consistently described by a polytropic gas of relativistic nucleon with adiabatic index $\Gamma = 4/3$ [34]. The pressure P and matter density ρ are related by [34,23]

$$P = K \rho^\Gamma, \quad (27)$$

where $K = T_c/m_n \rho_c^{1/3} \simeq 5.6 \times 10^{-5} \text{MeV}^{-4/3}$. Here $T_c = 40 \text{MeV}$ and $\rho_c = 2 \times 10^{14} \text{gr/cm}^3$ are the temperature and the matter density of the core, respectively.

The matter density $\rho(r)$ can be expressed in the form [23] (see Appendix)

$$\rho^{\Gamma-1}(x) = \rho_c^{\Gamma-1} [a'x^2 + b'x + c'], \quad (28)$$

where

$$x = \frac{r_c}{r}, \quad a' = (1 - \mu)\lambda_\Gamma, \quad b' = (2\mu - 1)\lambda_\Gamma, \quad c' = 1 - \mu\lambda_\Gamma, \quad (29)$$

and $r_c = 10 \text{km}$ is the core radius. The parameter μ is determined by setting $\rho(R_s) = 0$ (R_s the radius of the star)

$$\mu = \left[\frac{R_s}{\lambda_\Gamma(R_s - r_c)} - \frac{r_c}{R_s} \right] \frac{R_s}{R_s - r_c}. \quad (30)$$

λ_Γ in (30) and (29) is given by

$$\lambda_\Gamma = \frac{G_N M_c \Gamma - 1}{r_c \rho_c^{\Gamma-1} K \Gamma} \simeq 0.29 \frac{2G_N M_c}{\text{km}} \frac{10\text{km}}{r_c} \frac{40\text{MeV}}{T_c}, \quad (31)$$

where $M_c \simeq M_\odot$ is the mass of the core (M_\odot is the solar mass). The temperature profile $T(r)$ is related to matter density $\rho(r)$ through the relation [23] (see Appendix)

$$\frac{dT^2}{dr} = -\frac{9\kappa L_c}{\pi r^2} \rho, \quad (32)$$

where L_c is the core luminosity, $L_c \sim 9.5 \times 10^{51}$ erg/sec, and $\kappa = 5.6 \times 10^{-9}$ cm⁴/erg³sec² $\sim 6.2 \times 10^{-56}$ eV⁻⁵. Eq. (32) can be immediately integrate by using (28)

$$T(r) = T_c \sqrt{2\lambda_c[\chi(x) - \chi(1)] + 1}, \quad (33)$$

where

$$\chi(x) = c'^3 x + \frac{3}{2} b' c'^2 x^2 + c'(a' c' + b'^2) x^3 + \frac{b'}{4} (6a' c' + b'^2) x^4 + \frac{3a'}{5} (a' c' + b'^2) x^5 + \frac{b' a'^2}{2} x^6 + \frac{a'^3}{7} x^7, \quad (34)$$

and

$$\lambda_c = \frac{9}{2\pi} \frac{\kappa L_c \rho_c}{T_c^2 r_c} \sim 1.95 \frac{\rho_c}{10^{14} \text{gr/cm}^3} \frac{10\text{km}}{r_c} \left(\frac{40\text{MeV}}{T_c} \right)^2. \quad (35)$$

Since $p \sim T$ (it is assumed the thermal equilibrium between neutrinos and the medium, so that the average energy of the emitted neutrino is proportional to the temperature at the emission point [23]) and $V_{eff} \sim \rho$, one can rewrite the inverse characteristic lengths h_p^{-1} and $h_{V_{\nu_f}}^{-1}$ as $h_p^{-1} \equiv h_T^{-1}$ and $h_{V_{\nu_f}}^{-1} \equiv h_\rho^{-1}$ [23]. Eqs. (28) and (33) imply (at the resonance)

$$h_T^{-1} = \frac{d \ln T}{dr} = -\lambda_c \frac{\rho(r_{res})}{\rho_c} \left(\frac{T_c}{T(r_{res})} \right)^2 \frac{x_{res}}{r_{res}}, \quad (36)$$

$$h_\rho^{-1} = \frac{d \ln \rho}{dr} = -3 \left(\frac{\rho_c}{\rho(r_{res})} \right)^{1/3} (2a x_{res} + b) \frac{x_{res}}{r_{res}}, \quad (37)$$

so that Eq. (26) reads

$$\Sigma \sim 0.09 V_{\nu_f} \lambda_\Gamma x_{res} \eta, \quad (38)$$

where

$$\eta \equiv \varepsilon^2 \frac{\lambda_c}{\lambda_\Gamma} + 3(2\mu - 1) - 6(\mu - 1)x_{res}, \quad (39)$$

and the parameter $\varepsilon = T_c/T(r_{res})$ has been introduced. Eq. (39) gives a constraint on possible values of μ , ε and x_{res} .

The Lorentz and CPT symmetry breakdown is relevant (on pulsar kicks) for resonances occurring at $\rho(r_{res}) = \rho_c$. In such a case, Eq. (15) reduces to $V_{\nu_f} \simeq 0.7V_0$ (we use $Y_e \sim Y_\nu \sim \mathcal{O}(10^{-1})$). The mass of the sterile neutrino ($m_{\nu_s} \gg m_{\nu_f}$) is derived through Eq. (12) for small mixing angle and $p \sim 20\text{MeV}$. One gets $m_{\nu_s} \sim \text{few keV}$, according to Ref. [22]. Eq. (38) then becomes

$$\Sigma \sim 2 \times 10^{-2} x_{res} \eta \text{eV}. \quad (40)$$

Eqs. (24) and (40), and $x_{res} \sim \mathcal{O}(1)$ (as we shall determine below) yield

$$|\delta \mathbf{c}_L| \lesssim 4 \times 10^{-10} \eta, \quad |\delta \mathbf{a}_L| \lesssim 1.6 \times 10^{-2} \eta \text{eV}. \quad (41)$$

The constraints (5) provide $\eta \lesssim 10^{-7}$.

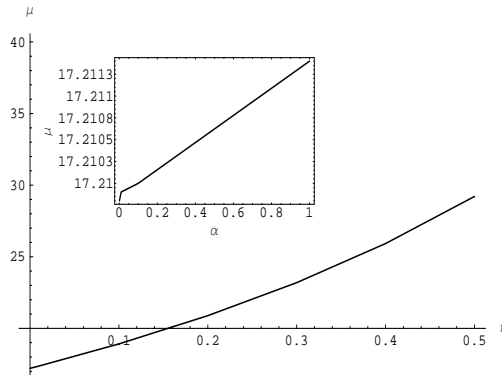


FIG. 1. The parameter μ , solution of Eq. (42), vs the parameter η . α is defined as $\alpha = \eta/10^{-4}$, and, in the small panel it varies from 1 to 10^{-3} .

Eq. (28) admits the solutions $x_{res} = 1$ and $x_{res} = \mu/(\mu - 1)$. The first solution is not compatible with (39) since it implies $\eta = \varepsilon^2 \lambda_c / \lambda_\Gamma + 3$, so that no real solution there exists for ε . The second one inserted into Eq. (39) yields

$$\varepsilon = \sqrt{\frac{\lambda_\Gamma(3 + \eta)}{\lambda_c}} \simeq 0.82.$$

Finally, the parameter μ is determined via Eq. (33)

$$\chi\left(\frac{\mu}{\mu - 1}\right) - \chi(1) + \frac{1}{2\lambda_c} - \frac{1}{2\lambda_\Gamma(3 + \eta)} = 0, \quad (42)$$

from which one gets $\mu \simeq 17.21$ (hence $x_{res} \sim 1.1 \sim \mathcal{O}(1)$). Fig. 1 shows the values of the parameter μ for varying η . As one can see, μ approximates to 17.21 as $\eta \lesssim 10^{-7}$.

From the above results, one infers

$$T(r_{res}) \sim 1.2 T_c, \quad r_{res} \sim 0.9 r_c, \quad R_s \sim 1.4 r_c.$$

Let us now discuss the obtained results in relation to the velocity distribution of pulsars. At the moment, the statistical analysis of pulsar population neither support nor rule out any model/mechanism proposed to explain pulsar kicks. This is essentially due to the lacking of correlation between neutron star velocity and the other properties of neutron stars (see for example [35] for a general discussion and references therein).

Concerning the coefficients for the Lorentz violation, it is convenient to adopt a standard inertial frame with respect to which experimental measurements of these coefficients are reported [36]. In this frame the coefficients \mathbf{a}_L and \mathbf{c}_L (hence $\delta\mathbf{a}_L$ and $\delta\mathbf{c}_L$) point along all possible orientation². Usually, it is assumed that Lorentz violating parameters are independent on position [11–13]. Nevertheless, as discussed by Kostelecký, Lehnert, and Perry in [8], they may also exhibit a space-dependence, such that they may take *arbitrary* different values for pulsars at different places.

These considerations entail two basic features:

- A priori there is no favored direction for the Lorentz violating coefficients. This results in a uniform distribution of pulsars, in agreement with statistical analysis performed in Refs. [17,35,37].
- The mechanism proposed in this paper does not require any correlation between the velocity V and other physical parameters of pulsars, such as for example, the magnetic field B . The reason is essentially due to the fact that in the Standard Model Extension, the coefficients a_L^μ and $c_L^{\mu\nu}$ describe the most general renormalizable effects that are possible in the gauge-invariant neutrino sector, no matter what are their origin at the Planck

²In particular, in the papers [11,12] it has been studied the general theory with a_L^μ and $c_L^{\mu\nu}$ which include all orientations, whereas in [13] the a_L^μ coefficients point along the north direction, and $c_L^{\mu\nu}$ are isotropic.

scale (they can be regarded as vacuum expectation values of tensor operators arising from the spontaneous breaking mechanism). As a consequence, a correlation among the coefficients a_L^μ and $c_L^{\mu\nu}$ and the parameters characterizing the pulsar properties, in particular the magnetic field, is not necessarily expected. Such a result, indeed, seems to be corroborated by recent analysis of Refs. [17,35,37] in which it is pointed out the apparent lack of evidence in favor of the $B - V$ correlation³.

In conclusion, the origin of pulsar velocities has been studied in the framework of Standard Model Extension. The effective Hamiltonian for the time evolution of neutrinos (see Eq. (4)) gives rise to a term of the form $\sim \delta\mathbf{\Pi} \cdot \mathbf{p}$. The relative orientation of neutrino momenta with respect to $\delta\mathbf{\Pi}$ leads to the fractional asymmetry necessary to generate the observed pulsar motion. Future observations and statistical analysis on the velocity distribution of pulsars might provide a scenario in which the Lorentz and CPT invariance violation, as suggested by Kostelecký, might be tested on astrophysical scales.

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APPENDIX: THE POLYTROPIC MODEL

The aim of this Appendix is to recall the main topics of the polytropic model which describes the inner core of a protostar. To this end, we shall refer to the paper by Barkovich, D'Olivo, Montemayor and Zanella [23]. The relevant equations for the description of an isotropic neutrinosphere are the equation for the hydrodynamical equilibrium

$$\frac{dP(r)}{dr} = -\rho(r) \frac{GM(r)}{r^2}, \quad (\text{A1})$$

where $M(r) = 4\pi \int_0^r dr' r'^2 \rho_T(r')$ (ρ_T is the total density of matter), the equation for the energy transport

$$F(r) = -\frac{1}{36} \frac{1}{\kappa\rho(r)} \frac{dT^2}{dr}, \quad (\text{A2})$$

and the equation for the flux conservation

$$F(r) = \frac{L_c}{4\pi r^2}, \quad (\text{A3})$$

where L_c is the luminosity of the protostar. The equation of state with adiabatic index Γ is [34,23]

$$P(r) = K\rho^\Gamma, \quad K = \frac{T_c}{m_n \rho_c^{1/3}} \quad (\text{A4})$$

where T_c and ρ_c are the temperature and the matter density of the core. From Eqs. (A4) and (A1) one gets

$$\frac{d\rho^{\Gamma-1}}{dr} = -\frac{\lambda_\Gamma r_c \rho_c^{\Gamma-1}}{M_c} \frac{M(r)}{r^2}, \quad (\text{A5})$$

³It should be noted that the analysis of Refs. [17,35,37] does not imply that the Kusenko-Segré mechanism for generate the pulsar kicks, as well as those mechanisms that also rely on magnetic fields, do not work. As pointed out by Kusenko in [20], such mechanisms do not predict a $B - V$ correlation since the relevant magnetic field for the kick velocity is the magnetic field *inside* the hot neutron star during the first seconds after the Supernova collapse. Astronomical observations allow to infer the surface magnetic field some millions of years later. These two fields are not trivially correlated each other because of the complex evolution of the magnetic field during the cooling process of neutron star, which lead to a final (surface) magnetic field whose configuration is different from the initial one, i.e. few seconds after the onset of the Supernova (see [20] for a detailed discussion).

where M_c is the core mass. A function which represents an extremely good approximation for ρ is given by [23]

$$\rho^{\Gamma-1}(r) = \rho_c^{\Gamma-1} \left[\lambda_\Gamma \left(\frac{r_c}{r} - 1 \right) m(r) + 1 \right], \quad (\text{A6})$$

where $m(r) = \mu + (1-\mu)r_c/r$. Notice that $m(r_c) = 1$. The parameter μ is determined by using the equation $\rho(R_s) = 0$ (R_s is the radius of the star), i.e.

$$\mu = \left[\frac{R_s}{\lambda_\Gamma(R_s - r_c)} - \frac{r_c}{R_s} \right] \frac{R_s}{R_s - r_c}. \quad (\text{A7})$$

Eq. (A6) can be easily recast in the form

$$\rho^{\Gamma-1}(r) = \rho_c^{\Gamma-1} \left[a' \left(\frac{r_c}{r} \right)^2 + b' \frac{r_c}{r} + c' \right], \quad (\text{A8})$$

where the coefficients a', b', c' are defined in (29). Eqs. (A3) and (A2) allow to infer the temperature profile

$$\frac{dT^2}{dr} = -\frac{9\kappa L_c}{\pi r^2} \rho, \quad (\text{A9})$$

with ρ given by (A8). For $\Gamma = 4/3$ the solution of the integro-differential equation (A9) is

$$T(r) = T_c \sqrt{2\lambda_c [\chi(r_c/r) - \chi(1)] + 1}, \quad (\text{A10})$$

with $\chi(r_c/r)$ defined in (34).

- [1] V.A. Kostelecký, S. Samuel, Phys. Rev. D **39**, 683 (1989); Phys. Rev. Lett. **63**, 224 (1989).
V.A. Kostelecký, R. Potting, Nucl. Phys. B **359**, 545 (1991); Phys. Rev. D **51**, 3923 (1995).
- [2] J. Ellis, K. Farakos, N.E. Mavromatos, V.A. Mitsou, D.V. Nanopoulos, Astrophys. J. **535**, 139 (2000).
J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, G. Volkov, Gen. Rel. Grav. **32**, 1777 (2000).
J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, Phys. Rev. D **61**, 027503 (2000).
- [3] J. Alfaro, H.A. Morales-Técolt, L.F. Urrutia, Phys. Rev. Lett. **84**, 2318 (2000).
- [4] J. Alfaro, H. Morales-Técolt, L.F. Urrutia, Phys. Rev. D **65**, 103509 (2002).
J. Alfaro, H. Morales-Técolt, L.F. Urrutia, Phys. Rev. D **66**, 124006 (2002).
- [5] R. Gambini, J. Pullin, Phys. Rev. D **59**, 124021 (1999).
J. Pullin, *Canonical Quantization of General Relativity: The Last 18 Years in a Nutshell*, [arXiv:gr-qc/0209008].
For more details on Loop Quantum Gravity formalism, see also:
C. Rovelli, Living Reviews, Vol. 1 *Loop Quantum Gravity*, at <http://www.livingreviews.org/articles>.
T. Thiemann, *Lectures on Loop Quantum Gravity*, [arXiv: gr-qc/0210094].
R. Gambini, J. Pullin, *Loops, Knots, Gauge Theories and Quantum Gravity*, Cambridge University Press, 1996.
A. Ashtekar, *Lectures on New Perturbative Canonical Gravity*, World Scientific, 1991.
C. Rovelli, S. Speziale, Phys. Rev. D **67**, 064019 (2003).
- [6] G. Amelino-Camelia, S. Majid, Int. J. Mod. Phys. A **15**, 4301 (2000).
G. Amelino-Camelia, Int. J. Mod. Phys. D **11**, 35 (2002).
A. Matusis, L. Sussking, N. Toumbas, JHEP 0012, 02 (2000).
S.M. Carroll, J. Harvey, V.A. Kostelecký, C.D. Lane, T. Okamoto, Phys. Rev. Lett. **87**, 141601 (2001).
Z. Guralnik, R. Jackiw, S.Y. Pi, A.P. Polychronakos, Phys. Lett. B **517**, 450 (2001).
C.E. Carlson, C.D. Carone, R.F. Lebed, Phys. Lett. B **518**, 201 (2001).
A. Anisimov, T. Banks, M. Dine, M. Graesser, Phys. Rev. D **65**, 085032 (2002).
I. Mocioiu, M. Pospelov, R. Roiban, Phys. Rev. D **65**, 107702 (2002).
M. Chaichian, M.M. Sheikh-Jabbari, A. Tureanu, hep-th/0212259.
J.L. Hewett, F.J. Petriello, T.G. Rizzo, Phys. Rev. D **66**, 036001 (2002).
- [7] D. Mattingly, gr-qc/0502097.
H. Vucetich, gr-qc/0502093
- [8] V.A. Kostelecký, R. Lehnert, M.J. Perry, Phys. Rev. D **68**, 123511 (2003).
O. Bertolami, R. Lehnert, R. Potting, A. Ribeiro, Phys. Rev. D **69**, 083513 (2004).

- [9] D. Colladay, V.A. Kostelecký, Phys. Rev. D **58**, 116002 (1998); Phys. Rev. D **55**, 6760 (1997).
V.A. Kostelecký, Phys. Rev. D **69**, 105009 (2004).
- [10] V.A. Kostelecký, R. Lehnert, Phys. Rev. D **63**, 065088 (2001).
- [11] V.A. Kostelecký, M. Mewes, Phys. Rev. D **69**, 016005 (2004).
- [12] V.A. Kostelecký, M. Mewes, hep-ph/0406255.
- [13] V.A. Kostelecký, M. Mewes, Phys. Rev. D **70**, 031902 (2004).
- [14] V.A. Kostelecký, M. Mewes, Phys. Rev. D **66**, 056005 (2002).
- [15] G. Lambiase, Phys. Lett. B **560**, 1 (2003).
- [16] A.G. Lyne, D.R. Lorimer, Nature **369**, 127 (1994).
J.M. Corder, D.F. Chernoff, Astrophys. J. **505**, 315 (1998).
B.M.S. Hansen, E.S. Phinney, Mon. Not. R. Astron. Soc. **291**, 569 (1997).
- [17] Z. Arzoumanian, D.F. Chernoff, J.M. Cordes, Astrophys. J. **568**, 289 (2002).
- [18] A. Kusenko, G. Segre', Phys. Rev. Lett. **77**, 4872 (1996).
- [19] A. Kusenko, G. Segre', Phys. Lett. B **396**, 197 (1997).
A. Kusenko, G. Segre', Phys. Rev. D **59**, 061302 (1999).
- [20] A. Kusenko, astro-ph/0409521.
- [21] Y.-Z. Qian, Phys. Rev. Lett. **79**, 2750 (1997).
E.Kh. Akhmedov, A. Lanza, D.W. Sciama, Phys. Rev. D **56**, 6117 (1997).
D. Grasso, H. Nunokawa, J.W.F. Valle, Phys. Rev. Lett. **81**, 2412 (1998).
R. Horvat, Mod. Phys. Lett. A **13**, 2379 (1998).
H.-Th. Janka, G.G. Raffelt, Phys. Rev. D **59**, 023005 (1998).
- [22] G.M. Fuller, A. Kusenko, I. Macioiu, S. Pascoli, Phys. Rev. D **68**, 103002 (2003).
- [23] M. Barkovich, J.C. D'Olivo, R. Montemayor, J.F. Zanella, Phys. Rev. D **66**, 123005 (2002).
- [24] M. Barkovich, J.C. D'Olivo, R. Montemayor, Phys. Rev. D **70**, 043005 (2004).
- [25] M. Barkovich, H. Casini, J.C. D'Olivo, R. Montemayor, Phys. Lett. B **506**, 20 (2001).
- [26] E. Nardi, J.I. Zuluaga, Astrophys. J. **549**, 1076 (2001).
- [27] A. Burrows, J. Hayes, Phys. Rev. Lett. **76**, 352 (1996).
- [28] H.J. Mosquera Cuesta, Astrophys. J. **544**, L61 (2000); Phys. Rev. D **65**, 061503 (2002).
H.J. Mosquera Cuesta, K. Fiuzza, Eur. Phys. J. C **35**, 543 (2004).
- [29] J.R. Gott, J.E. Gunn, J.P. Ostriker, Astrophys. J. Lett. **201**, L91 (1970).
E.R. Harrison, E.P. Tademaru, Astrophys. J. **201**, 447 (1975).
- [30] G. Lambiase, astro-ph/0411242, astro-ph/0412408.
- [31] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); Phys. Rev. D **20**, 2634 (1979).
S.P. Mikheyev, A.Yu Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)], Nuovo Cimento C **9**, 17 (1986).
- [32] S. Coleman, S.L. Glashow, Phys. Lett. B **405**, 249 (1997).
- [33] S.L. Glashow, A. Halprin, P.I. Krastev, C.N. Leung, J. Pantaleone, Phys. Rev. D **56**, 2433 (1977).
- [34] S.L. Shapiro, S.A. Teukolsky, *Black Holes, White Dwarfs, and Neutron Stars*, John Wiley and Sons, New York, 1983.
S. Weinberg, *Gravitation and Cosmology*, John Wiley and Sons, New York, 1972.
M. Harwit, *Astrophysical Concepts*, Springer, New York, 1998.
- [35] D. Lai, astro-ph/0212140, astro-ph/0312542.
- [36] Let us discuss in some detail this point. For the effective Hamiltonian (4) the choice of observer reference frames is irrelevant owing to the fact that the physics is coordinate invariant, and in particular is observer Lorentz invariant [9]. In the presence of particle Lorentz symmetry breakdown [9], it is advisable to adopt a standard inertial frame with respect to which experimental measurements of the coefficients for Lorentz violation are reported. It is conventionally taken as a Sun-centered celestial equatorial frame (T, X, Y, Z), with Z -axis oriented along the Earth rotating axis, and X -axis directed along the vernal equinox [11,14]. The coefficients for the Lorentz violation in any inertial frame can be related to those in the standard Sun centered frame by an appropriate Lorentz transformation [14]. Since neutrino oscillations in the presence of Lorentz violation can exhibit orientation-dependent effects (as discussed in this paper, Lorentz and CPT invariance violation induce a dependence on the relative orientation of the neutrino momenta p^i with respect to vectors δa_L^i and δc_L^i (or their combinations)), it is also convenient to define a standard parameterization in the Sun centered frame for the direction of neutrino propagation $\hat{\mathbf{p}}$, and the ϵ_1 and ϵ_2 vectors [11,14]

$$\begin{aligned}\hat{\mathbf{p}} &= (\sin \Theta \cos \Phi, \sin \Theta \sin \Phi, \cos \Theta), \\ \epsilon_1 &= (\cos \Theta \cos \Phi, \cos \Theta \sin \Phi, -\sin \Theta), \\ \epsilon_2 &= (-\sin \Phi, \cos \Phi, 0),\end{aligned}$$

where Θ and Φ are the celestial colatitude and longitude of propagation, respectively.

- [37] J.M. Cordes, D.F. Chernoff, Astrophys. J. **505**, 315 (1998).
D.R. Lorimer, M. Bailes, P.A. Harrison, Mon. Not. Astron. Soc. **289**, 592 (1997).
E. De Donder, D. Vanbeveren, New Astronomy **4**, 167 (1999).

B.M.S. Hansen, S. Phinney, *Mon. Not. R. Astron. Soc.* **291**, 569 (1997).
M. Toscano et al., *Mon. Not. R. Astron. Soc.* **307**, 925 (1999).
N. Wex, V. Kalogera, M. Kramer, *Astrophys. J.* **528**, 401 (2000).