

# Higgs Boson Mass and Electroweak-Gravity Hierarchy from Dynamical Gauge-Higgs Unification in the Warped Spacetime

Yutaka Hosotani\* and Mitsuru Mabe†

*Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*

## Abstract

Dynamical electroweak symmetry breaking by the Hosotani mechanism in the Randall-Sundrum warped spacetime is examined, relations among the W-boson mass ( $m_W$ ), the Kaluza-Klein mass scale ( $M_{KK}$ ), and the Higgs boson mass ( $m_H$ ) being derived. It is shown that  $M_{KK}/m_W \sim (2\pi kR)^{1/2}(\pi/\theta_W)$  and  $m_H/m_W \sim 0.058 \cdot kR(\pi/\theta_W)$ , where  $k^2$ ,  $R$ , and  $\theta_W$  are the curvature and size of the extra-dimensional space and the Wilson line phase determined dynamically. For typical values  $kR = 12$  and  $\theta_W = (0.2 \sim 0.4)\pi$ , one finds that  $M_{KK} = (1.7 \sim 3.5)$  TeV,  $k = (1.3 \sim 2.6) \times 10^{19}$  GeV, and  $m_H = (140 \sim 280)$  GeV.

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\*hosotani@het.phys.sci.osaka-u.ac.jp

†mitsuru@het.phys.sci.osaka-u.ac.jp

Although the standard model of the electroweak interactions has been successful to account for all the experimental data so far observed, there remain a few major issues to be settled. First of all, Higgs particles are yet to be discovered. The Higgs sector of the standard model is for the most part unconstrained unlike the gauge sector where the gauge principle regulates the interactions among matter. Secondly, the origin of the scale of the electroweak interactions characterized by the W-boson mass  $m_W \sim 80 \text{ GeV}$  or the vacuum expectation value of the Higgs field  $v \sim 246 \text{ GeV}$  becomes mysterious once one tries to unify the electroweak interactions with the strong interactions in the framework of grand unified theory, or with gravity, where the energy scale is given by  $M_{\text{GUT}} \sim 10^{15} - 10^{17} \text{ GeV}$  or  $M_{\text{Pl}} \sim 10^{19} \text{ GeV}$ , respectively. The natural explanation of such hierarchy in the energy scales is desirable. In this paper we show that the Higgs sector of the electroweak interactions can be integrated in the gauge sector, and the electroweak energy scale is naturally placed with the gravity scale within the framework of dynamical gauge-Higgs unification in the Randall-Sundrum warped spacetime.

The scheme of dynamical gauge-Higgs unification was put forward long time ago in the context of higher dimensional non-Abelian gauge theory with non-simply connected extra-dimensional space.[1, 2] In non-simply connected space there appear non-Abelian Aharonov-Bohm phases, or Wilson line phases, which can dynamically induce gauge symmetry breaking even within configurations of vanishing field strengths. The extra-dimensional components of gauge potentials play a role of Higgs fields in four dimensions. The Higgs fields are unified with the gauge fields and the gauge symmetry is dynamically broken at the quantum level. It was originally designed that Higgs fields in the adjoint representation in  $SU(5)$  grand unified theory are unified with the gauge fields.

The attempt to identify scalar fields as parts of gauge fields was made earlier by utilizing symmetry reduction. Witten observed that gauge theory in four-dimensional Minkowski spacetime with spherical symmetry reduces to a system of gauge fields and scalar fields in two-dimensional curved spacetime.[3] This idea was extended to six-dimensional gauge theory by Fairlie[4] and by Forgacs and Manton[5] to accommodate the electroweak theory in four dimensions. It was recognized there that to yield  $SU(2)_L \times U(1)_Y$  symmetry of electroweak interactions in four dimensions one need start with a larger gauge group such as  $SU(3)$ ,  $SO(5)$  or  $G_2$ . The reduction of the symmetry to  $SU(2)_L \times U(1)_Y$  was made by an ad hoc ansatz for field configurations in the extra-dimensional space. For instance, Manton assumed spherically symmetric configurations in the extra-dimensional space  $S^2$ . As was

pointed out later,[6] such a configuration can be realized by a monopole configuration on  $S^2$ .<sup>‡</sup> However, classical non-vanishing field strengths in the background would lead to the instability of the system. In this regard gauge theory defined on non-simply connected spacetime has big advantage in the sense that even with vanishing field strengths Wilson line phases become dynamical and can induce symmetry breaking at the quantum level by the Hosotani mechanism.

Recently significant progress has been achieved along this line by considering gauge theory on orbifolds which are obtained by modding out non-simply connected space by discrete symmetry such as  $Z_n$ . [7]-[21] With the orbifold symmetry breaking induced from boundary conditions at fixed points of the orbifold, a part of light modes in the Kaluza-Klein tower expansion of fields are eliminated from the spectrum at low energies so that chiral fermions in four dimensions naturally emerge.[7] Further, in  $SU(5)$  grand unified theory (GUT) on orbifolds the triplet-doublet mass splitting problem of the Higgs fields [10] and the gauge hierarchy problem[8] can be naturally solved.

The orbifold symmetry breaking, however, accompanies indeterminacy in theory. It poses the arbitrariness problem of boundary conditions.[15] One needs to show how and why a particular set of boundary conditions is chosen naturally or dynamically, which is achieved, though partially, in the scheme of dynamical gauge-Higgs unification.

Quantum dynamics of Wilson line phases in GUT on orbifolds was first examined in ref. [14] where it was shown that the physical symmetry is determined by the matter content. Several attempts to implement dynamical gauge-Higgs unification in the electroweak theory have been made since then. The most intriguing among those is the  $U(3) \times U(3)$  model of Antoniadis, Benakli and Quiros.[9] The effective potential of the Wilson line phases in this model has been recently evaluated to show that the electroweak symmetry breaking dynamically takes place with minimal addition of heavy fermions.[20] The model is restrictive enough to predict the Kaluza-Klein mass scale ( $M_{KK}$ ) and the Higgs boson mass ( $m_H$ ) with the  $W$ -boson mass ( $m_W$ ) as an input. It turned out that  $M_{KK} \sim 10 m_W$  and  $m_H \sim \sqrt{\alpha_w} m_W$ , which contradicts with the observation.

We argue that this is not a feature of the specific model examined, but is a general feature of orbifold models in which extra-dimensional space is flat. Unless tuning of matter content is enforced, the relation  $m_H \sim \sqrt{\alpha_w} m_W$  is unavoidable in flat space as shown below. To circumvent this difficulty, it is necessary to have curved extra-dimensional space.

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<sup>‡</sup>The monopole configuration for  $A_M^8$  of the  $SU(3)$  gauge fields on  $S^2$  realizes the envisaged symmetry reduction to  $SU(2) \times U(1)$  in ref. [5].

Randall and Sundrum introduced warped spacetime with an extra-dimensional space having topology of  $S^1/Z_2$  which is five-dimensional anti-de Sitter spacetime with boundaries of two flat four-dimensional branes.[22] It was argued there that the standard model of electroweak interactions is placed on one of the branes such that the electroweak scale becomes natural compared with the Planck scale characterizing gravity. Since then many variations of the Randall-Sundrum model have been investigated. The standard model can be placed in the bulk five-dimensional spacetime, not being restricted on one of the branes.[23] However, fine-tuning of the Higgs potential remains necessary.

More promising is to consider dynamical gauge-Higgs unification in the Randall-Sundrum background where gauge theory is defined in the bulk five-dimensional spacetime without five-dimensional scalar fields. The first step in this direction has been made by Oda and Weiler who evaluated the 1-loop effective potential for Wilson line phases in the  $SU(N)$  gauge theory.[24] We will show in the present paper that the electroweak symmetry breaking can be naturally implemented in dynamical gauge-Higgs unification on the Randall-Sundrum background to avoid the aforementioned difficulty concerning  $M_{KK}$  and  $m_H$ . We show that in this scheme the Higgs mass  $m_H$  should be between 140 GeV and 280 GeV, and the Kaluza-Klein mass scale  $M_{KK}$  must be between 1.7 TeV and 3.5 TeV. It is exciting that the predicted ranges of  $m_H$  and  $M_{KK}$  fall in the region where experiments at LHC can explore in the near future.<sup>§</sup>

We consider gauge theory in the Randall-Sundrum warped spacetime whose metric is given by

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (1)$$

( $\mu, \nu = 0, 1, 2, 3$ ). Here  $\sigma(y) = k|y|$  for  $|y| \leq \pi R$ ,  $\sigma(y + 2\pi R) = \sigma(y)$  and  $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ . Points  $(x^\mu, -y)$  and  $(x^\mu, y + 2\pi R)$  are identified with  $(x^\mu, y)$ . The resultant spacetime is an anti-de Sitter spacetime ( $0 < y < \pi R$ ) sandwiched by four-dimensional spacetime branes at  $y = 0$  and  $y = \pi R$ . It has topology of  $R^4 \times (S^1/Z_2)$ . The curvature is given by  $k^2$ .

As a prototype of the models we take the  $U(3)_S \times U(3)_W$  gauge theory[9], though the results do not depend on the details of the model. Weak  $W$  bosons reside in the  $U(3)_W$  gauge group. The  $U(3)_W$  part of the action is  $I = \int d^5x \sqrt{-\det g} \{ -\frac{1}{2} \text{Tr} F_{MN} F^{MN} +$

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<sup>§</sup>Cosmological consequences of the Hosotani mechanism in curved spacetime has been previously investigated in ref. [25]. The Hosotani mechanism in the Randall-Sundrum warped spacetime has been applied to the electroweak symmetry breaking in ref. [26].

$\mathcal{L}_{\text{matter}}$  where the five-dimensional coordinates are  $x^M = (x^\mu, y)$  and  $\mathcal{L}_{\text{matter}}$  represents the part for quarks and leptons. Five-dimensional scalar fields are not introduced. The zero modes of the extra-dimensional components of the vector potentials,  $A_y^a$ , generate non-Abelian Aharonov-Bohm phases (Wilson line phases) and serve as four-dimensional Higgs fields effectively.

To see it more clearly, it is convenient to work in a new coordinate system  $x^M = (x^\mu, w)$  where  $w = e^{2ky}$  for  $0 \leq y \leq \pi R$ . The metric becomes

$$ds^2 = \frac{1}{w} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{1}{4k^2 w^2} dw^2 . \quad (2)$$

Boundary conditions for the gauge potentials in the original coordinate system  $(x^\mu, y)$  are given in the form  $(A_\mu, A_y)(x, y_j - y) = P_j (A_\mu, -A_y)(x, y_j + y) P_j^\dagger$  where  $y_0 = 0$ ,  $y_1 = \pi R$ ,  $P_j \in U(3)$  and  $P_j^2 = 1$  ( $j = 0, 1$ ). [14, 18, 20] They follow from the  $S^1/Z_2$  nature of the spacetime. In the new coordinate system  $(x^\mu, w)$ , the boundary conditions are summarized as

$$\begin{aligned} \begin{pmatrix} A_\mu \\ A_w \end{pmatrix} (x, w_j) &= P_j \begin{pmatrix} A_\mu \\ -A_w \end{pmatrix} (x, w_j) P_j^\dagger , \\ \begin{pmatrix} \partial_w A_\mu \\ \partial_w A_w \end{pmatrix} (x, w_j) &= P_j \begin{pmatrix} -\partial_w A_\mu \\ \partial_w A_w \end{pmatrix} (x, w_j) P_j^\dagger , \end{aligned} \quad (3)$$

where  $w_0 = 1$  and  $w_1 = e^{2\pi kR}$ . Similarly, for a fermion in the fundamental representation

$$\begin{aligned} \psi(x, w_j) &= \eta_j P_j \gamma^5 \psi(x, w_j) , \\ \partial_w \psi(x, w_j) &= -\eta_j P_j \gamma^5 \partial_w \psi(x, w_j) , \end{aligned} \quad (4)$$

where  $\eta_j = \pm 1$ . We take

$$P_0 = P_1 = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix} \quad (5)$$

to ensure the electroweak symmetry.

The advantage of the  $w$  coordinate over the  $y$  coordinate lies in the fact that zero modes of  $A_w(x, w)$  become independent of  $w$ . In the  $y$  coordinate  $A_y(x, y)$  has cusp singularities at  $y = 0$  and  $y = \pi R$ . To observe it explicitly, we specify the gauge-fixing term in the action. A general procedure in curved spacetime has been given in ref. [2]. It is convenient to adopt the prescription for gauge-fixing given in ref. [24]. As is justified a posteriori, the

effective potential is evaluated in the background field method with a constant background  $A_M^c = \delta_{Mw} A_w^c$ . The gauge fixing term  $\int d^4x dw \sqrt{-g} \mathcal{L}_{g.f.}$  is chosen to be

$$\mathcal{L}_{g.f.} = -w^2 \text{Tr} (D_\mu^c A^\mu + 4k^2 w D_w^c A_w)^2 \quad (6)$$

where  $D_M^c A_N \equiv \partial_M A_N + ig[A_M^c, A_N]$  and  $D_\mu^c A^\mu \equiv \eta^{\mu\nu} D_\mu^c A_\nu$ . In the path integral formula we write  $A_M = A_M^c + A_M^q$  and expand the action in  $A_M^q$ . The bilinear part of the action including the ghost part is given by

$$\begin{aligned} I_{\text{eff}} = - \int d^4x \int_{w_0}^{w_1} dw \left\{ \frac{1}{2kw} \text{Tr} A_\nu^q (\partial^\mu \partial_\mu + 4k^2 w D_w^c D_w^c) A^{q\nu} \right. \\ \left. + 2k \text{Tr} A_w^q (\partial^\mu \partial_\mu + 4k^2 D_w^c w D_w^c) A_w^q \right. \\ \left. - \frac{1}{2kw^2} \text{Tr} \bar{\eta} (\partial^\mu \partial_\mu + 4k^2 w D_w^c D_w^c) \eta \right\}. \quad (7) \end{aligned}$$

Partial integration necessary in deriving (7) is justified as  $\text{Tr} A_\mu \partial_w A^\mu$  and  $\text{Tr} A_w \partial_w A_w$  vanish at  $w = w_0, w_1$  with the boundary conditions (3).

Let us denote  $A_M = \sum_{a=0}^8 \frac{1}{2} \lambda^a A_M^a$  with the standard Gell-Mann matrices  $\lambda^a$ . ( $\lambda^0$  represents the  $U(1)$  part.) With (3) and (5),  $A_\mu^a$  ( $a = 0, 1, 2, 3, 8$ ) and  $A_w^b$  ( $b = 4, 5, 6, 7$ ) satisfy Neumann boundary conditions at  $w = w_0, w_1$ , whereas  $A_\mu^a$  ( $a = 4, 5, 6, 7$ ) and  $A_w^b$  ( $b = 0, 1, 2, 3, 8$ ) satisfy Dirichlet boundary conditions. Zero modes independent of  $w$  are allowed for  $A_\mu^a$  ( $a = 0, 1, 2, 3, 8$ ) and  $A_w^b$  ( $b = 4, 5, 6, 7$ ). It is found from (7) that they indeed constitute massless particles in four dimensions when  $A_w^c = 0$ . Gauge fields of  $SU(2)_L \times U(1)_Y$  are in  $A_\mu^a$  ( $a = 0, 1, 2, 3, 8$ ), whereas doublet Higgs fields are in  $A_w^b$  ( $b = 4, 5, 6, 7$ ). We note that in the  $y$  coordinate system  $A_y^i = 2ke^{2ky} A_w$  so that the zero modes are not constant in  $y$ , which gives rise to unphysical cusp singularities at  $y = 0, \pi R$ .

Mode expansion for  $A_\mu(x, w)$  is inferred from (7) to be

$$\begin{aligned} A_\mu^a(x, w) &= \sum_n A_{\mu,n}^a(x) f_n(w), \\ -4k^2 w \frac{d^2}{dw^2} f_n(w) &= \lambda_n f_n(w), \\ \int_{w_0}^{w_1} dw \frac{1}{2kw} f_n(w) f_m(w) &= \delta_{nm}. \quad (8) \end{aligned}$$

For  $A_w(x, w)$  one finds

$$A_w^a(x, w) = \sum_n A_{w,n}^a(x) h_n(w),$$

$$\begin{aligned}
-4k^2 \frac{d}{dw} w \frac{d}{dw} h_n(w) &= \hat{\lambda}_n h_n(w) , \\
\int_{w_0}^{w_1} dw \, 2k h_n(w) h_m(w) &= \delta_{nm} .
\end{aligned} \tag{9}$$

Given boundary conditions,  $(\lambda_n, f_n(w))$  and  $(\hat{\lambda}_n, h_n(w))$  are determined.  $A_\mu^a$  ( $A_w^a$ ) has a zero mode  $\lambda_0 = 0$  ( $\hat{\lambda}_0 = 0$ ) only with Neumann boundary conditions at  $w = w_j$ . For the zero modes  $f_0(w) = 1/\sqrt{\pi R}$  and  $h_0(w) = 1/\sqrt{2k(w_1 - w_0)}$ . [27]

Except for the zero modes, both  $\lambda_n$  and  $\hat{\lambda}_n$  are positive. Apart from the normalization factors eigen functions are given by  $f_n(w) = \sqrt{w} Z_1(\sqrt{\lambda_n w}/k)$  and  $h_n(w) = Z_0(\sqrt{\hat{\lambda}_n w}/k)$  where  $Z_\nu(z)$  is a linear combination of Bessel functions  $J_\nu(z)$  and  $Y_\nu(z)$  of order  $\nu$ .  $(\lambda_n, f_n)$  with the Neumann boundary conditions and  $(\hat{\lambda}_n, h_n)$  with the Dirichlet boundary conditions are determined by

$$\frac{J_0(\beta_n \sqrt{w_0})}{Y_0(\beta_n \sqrt{w_0})} = \frac{J_0(\beta_n \sqrt{w_1})}{Y_0(\beta_n \sqrt{w_1})} , \tag{10}$$

whereas  $(\lambda_n, f_n)$  with the Dirichlet boundary conditions and  $(\hat{\lambda}_n, h_n)$  with the Neumann boundary conditions are determined by

$$\frac{J_1(\beta_n \sqrt{w_0})}{Y_1(\beta_n \sqrt{w_0})} = \frac{J_1(\beta_n \sqrt{w_1})}{Y_1(\beta_n \sqrt{w_1})} . \tag{11}$$

Here  $\beta_n = \sqrt{\lambda_n}/k$  or  $\sqrt{\hat{\lambda}_n}/k$ . For  $\beta_n \gg 1$ ,  $\beta_n = \pi n / (\sqrt{w_1} - \sqrt{w_0})$ . For  $w_1^{-1/2} \ll \beta_n \ll 1$ ,  $\beta_n = (n - \frac{1}{4})\pi/\sqrt{w_1}$  or  $(n + \frac{1}{4})\pi/\sqrt{w_1}$  for the case (10) or (11), respectively. The first excited state is given by  $\beta_1 \sqrt{w_1} \sim 2.6$  or  $3.8$ . Hence, the Kaluza-Klein mass scale is given by

$$M_{KK} = \frac{\pi k}{\sqrt{w_1} - \sqrt{w_0}} = \begin{cases} R^{-1} & \text{for } k \rightarrow 0, \\ \pi k e^{-\pi k R} & \text{for } e^{\pi k R} \gg 1. \end{cases} \tag{12}$$

With  $P_j$  in (5), the W boson and the weak Higgs doublet  $\Phi$  are contained in the zero modes of  $(A_\mu^1 \pm iA_\mu^2)(x, w)$  and  $A_w^b(x, w)$  ( $b = 4, 5, 6, 7$ );

$$\begin{aligned}
\frac{1}{\sqrt{2}}(A_\mu^1 + iA_\mu^2)(x, w) &\Rightarrow \frac{1}{\sqrt{2}}(A_{\mu,0}^1 + iA_{\mu,0}^2)(x) f_0(w) = \frac{1}{\sqrt{\pi R}} W_\mu(x) , \\
\frac{1}{\sqrt{2}} \begin{pmatrix} A_w^4 - iA_w^5 \\ A_w^6 - iA_w^7 \end{pmatrix} (x, w) &\Rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} A_{w,0}^4 - iA_{w,0}^5 \\ A_{w,0}^6 - iA_{w,0}^7 \end{pmatrix} (x) h_0(w) = \frac{\Phi(x)}{\sqrt{2k(w_1 - w_0)}} .
\end{aligned} \tag{13}$$

There is no potential term for  $\Phi$  at the classical level, but nontrivial effective potential is generated at the quantum level. As in the model discussed in ref. [20], the effective potential is supposed to have a global minimum at  $\Phi \neq 0$ , inducing dynamical electroweak

symmetry breaking. Making use of the residual  $SU(2) \times U(1)$  invariance, we need to evaluate the effective potential for the configuration

$$A_w = A_w^c = \alpha \Lambda \quad , \quad \Lambda = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} . \quad (14)$$

Note that  $v = \sqrt{2} \langle \Phi^0 \rangle = 2\sqrt{2k(w_1 - w_0)} \alpha$ .

The Randall-Sundrum warped spacetime has topology of  $R^4 \times (S^1/Z_2)$ . As  $S^1$  is not simply connected, there arise Aharonov-Bohm phases, or Wilson line phases, which become physical degrees of freedom.[1, 2] The Wilson line phases are defined by eigenvalues of  $P \exp \left\{ ig \int_C dw A_w \right\} \cdot U$  where the path  $C$  is a closed non-contractible loop along  $S^1$  and  $U = P_1 P_0$ . In the present case  $U = I$  so that all gauge potentials are periodic on  $S^1$ . It follows that  $\alpha$  in (14) is related to the Wilson line phase by

$$\theta_W = 2g\alpha(w_1 - w_0) . \quad (15)$$

It will be shown below that  $\theta_W$  and  $\theta_W + 2\pi$  are gauge equivalent. The  $SU(2)_L$  gauge coupling constant in four dimensions,  $g_4$ , is easily found by inserting  $A_\mu(x, w) \sim (\pi R)^{-1/2} A_{\mu,0}(x)$  into  $F_{\mu\nu}$ ;

$$g_4 = \frac{g}{\sqrt{\pi R}} . \quad (16)$$

Nonvanishing  $\theta_W$  or  $v$  gives the  $W$  boson a mass  $m_W$ . In our scheme the mass term for  $W$  arises from the term  $-\int dw 2k \text{Tr} A_\nu^a D_w^c D_w^c A^{a\nu}$  in (7). The resultant relation is the standard one,  $m_W = \frac{1}{2} g_4 v$ . Thus one finds

$$m_W = \frac{g_4 v}{2} = \left[ \frac{\pi k}{2R(w_1 - w_0)} \right]^{1/2} \frac{\theta_W}{\pi} = \begin{cases} \frac{1}{2} \frac{\theta_W}{\pi} M_{KK} & \text{for } k \rightarrow 0 , \\ \frac{1}{\sqrt{2\pi k R}} \frac{\theta_W}{\pi} M_{KK} & \text{for } e^{\pi k R} \gg 1 , \end{cases} \quad (17)$$

where  $M_{KK}$  is given in (12).

The precise value of  $\theta_W$  depends on the details of the model. If the effective potential is minimized at  $\theta_W = 0$ , then the electroweak symmetry breaking does not occur. If it occurs,  $\theta_W$  takes a value typically around  $0.2\pi$  to  $0.4\pi$ , unless artificial tuning of matter content is made. As an example, in the model discussed in ref. [20] in flat space,  $\theta_W \sim 0.25\pi$ , which, with  $m_W = 80.4 \text{ GeV}$  inserted, yielded too small  $M_{KK} \sim 640 \text{ GeV}$ .



In the present case, with the value of  $\theta_W$  given,  $kR$  determines  $M_{KK}$  and  $k$ . Recall that the four- and five-dimensional Planck constants  $M_{\text{pl}}$  and  $M_{5\text{d}}$  are related by  $M_{\text{pl}}^2 k \sim M_{5\text{d}}^3$ . To have a natural relation  $M_{5\text{d}} \sim M_{\text{pl}}$ ,  $kR$  must be in the range  $11 < kR < 13$ . To confirm it, take  $\theta_W = 0.25\pi$  as an example. For  $kR = 12$ , one finds  $M_{KK} = 2.8 \text{ TeV}$  and  $k = 2.1 \times 10^{19} \text{ GeV}$ . However, for  $kR = 6$  and  $24$  one finds  $k = 9.7 \times 10^{10} \text{ GeV}$  and  $7.0 \times 10^{35} \text{ GeV}$ , respectively. In (17),  $M_{KK}/m_W \propto \sqrt{kR}$ , and  $kR$  is about 12 if there is only one gravity scale ( $M_{5\text{d}} \sim k$ ). Thus the value of  $M_{KK}$  is predicted to be  $1.7 \text{ TeV} < M_{KK} < 3.5 \text{ TeV}$  for  $0.2\pi < \theta_W < 0.4\pi$  in the present scenario.

How about the Higgs boson mass? The finite mass of the Higgs field  $\Phi$  is generated by quantum effects.[14] One needs to evaluate the effective potential for the Wilson line phase,  $V_{\text{eff}}(\theta_W)$ . The Higgs mass is determined from the curvature at the minimum, with the substitution  $\theta_W = g[(w_1 - w_0)k^{-1}\Phi^\dagger\Phi]^{1/2}$ . Its magnitude is estimated reliably thanks to the phase nature of  $\theta_W$ .

To prove that  $\theta_W$  and  $\theta_W + 2\pi$  are physically equivalent, we go back to the boundary conditions (3) and (4) with general  $P_j$ . Let us perform a gauge transformation  $A'_M = \Omega A_M \Omega^\dagger - (i/g)\Omega \partial_M \Omega^\dagger$ .  $A'_M$  does not satisfy the same boundary conditions as  $A_M$  in general. Instead

$$\begin{aligned} \begin{pmatrix} A'_\mu \\ A'_w \end{pmatrix} (x, w_j) &= P'_j \begin{pmatrix} A'_\mu \\ -A'_w \end{pmatrix} (x, w_j) P_j'^\dagger \quad , \\ \begin{pmatrix} \partial_w A'_\mu \\ \partial_w A'_w \end{pmatrix} (x, w_j) &= P'_j \begin{pmatrix} -\partial_w A'_\mu \\ \partial_w A'_w \end{pmatrix} (x, w_j) P_j'^\dagger \quad , \\ P'_j &= \Omega(x, w_j) P_j \Omega(x, w_j)^\dagger \quad , \end{aligned} \tag{18}$$

provided

$$\begin{aligned} [P'_j, \partial_\mu \Omega \Omega^\dagger(x, w_j)] &= \{P'_j, \partial_w \Omega \Omega^\dagger(x, w_j)\} = 0 \quad , \\ \{P'_j, \partial_\mu (\Omega \partial_w \Omega^\dagger)(x, w_j)\} &= [P'_j, \partial_w (\Omega \partial_w \Omega^\dagger)(x, w_j)] = 0 \quad . \end{aligned} \tag{19}$$

In general,  $P'_j$  differs from  $P_j$ . When the conditions in (19) are satisfied, the two sets of the boundary conditions are said to be in the equivalence relation  $\{P_0, P_1\} \sim \{P'_0, P'_1\}$ , which defines equivalence classes of boundary conditions. Extensive analysis of the equivalence classes of boundary conditions has been given in refs. [2, 14, 16]. It was shown there that physics is the same in each equivalence class of boundary conditions.

In the present context we are interested in the residual gauge invariance which preserves the boundary conditions. In particular we would like to know  $\Omega(x, w)$  which satisfies (19) and yields  $P'_j = P_j$ , but shifts  $\theta_W$ . Take  $(P_0, P_1)$  in (5). We perform a gauge transformation

$$\Omega(x, w) = e^{i\beta(w-w_0)\Lambda} \quad (20)$$

where  $\Lambda$  is defined in (14) and satisfies  $\{\Lambda, P_j\} = 0$ . Note that  $P'_0 = P_0$ ,  $P'_1 = e^{2i\beta(w_1-w_0)\Lambda}P_1$ , and  $\partial_w\Omega\Omega^\dagger = -\Omega\partial_w\Omega^\dagger = i\beta\Lambda$ . All the conditions in (19) are satisfied. Further, for  $\beta = n\pi/(w_1 - w_0)$  ( $n$ : an integer),  $P'_j = P_j$ , i.e. the boundary conditions are preserved. For the configuration  $A_w$  in (14), the new gauge potential is  $A'_w = (\alpha - [n\pi/g(w_1 - w_0)])\Lambda$ .  $\theta_W = 2g\alpha(w_1 - w_0)$  is shifted, under the gauge transformation (20), to  $\theta'_W = \theta_W - 2n\pi$ .  $\theta_W$  and  $\theta_W + 2\pi$  are related by a large gauge transformation so that they are physically equivalent.

Having established the phase nature of  $\theta_W$ , we estimate  $V_{\text{eff}}(\theta_W)$ .  $V_{\text{eff}}(\theta_W)$  in the models in flat orbifolds has been evaluated well.[12, 14, 16, 18, 20]  $V_{\text{eff}}(\theta_W)$  in the Randall-Sundrum spacetime in the  $SU(N)$  gauge theory has been evaluated by Oda and Weiler.[24] With the background  $A_w^c$  or  $\theta_W$ , the spectrum  $\lambda_n$  of each field degree of freedom depends on  $\theta_W$  as well as on the boundary conditions of the field. Its contribution to four-dimensional  $V_{\text{eff}}(\theta_W)$  at the one loop level is summarized as

$$V_{\text{eff}}(\theta_W) = \mp \frac{i}{2} \int \frac{d^4p}{(2\pi)^4} \sum_n \ln \left\{ -p^2 + \lambda_n(\theta_W) \right\} , \quad (21)$$

where  $- (+)$  sign is for a boson (fermion). The spectrum  $\lambda_n$  for  $\theta_W = 0$  is determined as described in the discussions from Eq. (7) to (11). It is found there that  $\lambda_n \sim M_{KK}^2 n^2$  for large  $n$ . Hence one can write, after making a Wick rotation, as

$$V_{\text{eff}}(\theta_W) = \pm \frac{1}{2} M_{KK}^4 \int \frac{d^4q_E}{(2\pi)^4} \sum_n \ln \left\{ q_E^2 + \rho_n(\theta_W) \right\} + \text{constant} , \quad (22)$$

where  $\rho_n(\theta_W) = \lambda_n/M_{KK}^2$ . It is known that on an orbifold with topology of  $S^1/Z_2$ , fields form a  $Z_2$  doublet pair to have an interaction with  $\theta_W$ . [14] The resultant spectrum for a  $Z_2$  doublet is cast in the form where the sum in (22) extends over from  $n = -\infty$  to  $n = +\infty$ . Further  $\rho_n(\theta_W + 2\pi) = \rho_{n+\ell}(\theta_W)$  ( $\ell$ : an integer), and  $\rho_n(\theta_W) \sim [n + \gamma(\theta_W)]^2$  for large  $|n|$  where  $\gamma(\theta_W + 2\pi) = \gamma(\theta_W) + \ell$ . For instance, in the  $U(3) \times U(3)$  model in flat space,  $\rho_n(\theta_W) = [n + \ell\theta_W/2\pi + (\text{const})]^2$  with  $\ell = 0, \pm 1, \pm 2$ . [20] The important feature is that as  $\theta_W$  is shifted to  $\theta_W + 2\pi$  by a large gauge transformation, each eigen mode is shifted to the next KK mode in general, but the spectrum as a whole remains the same.

Recall the formula

$$\frac{1}{2} \int \frac{d^4 q_E}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \ln \{q_E^2 + (n+x)^2\} = -\frac{3}{64\pi^6} h(x) + \text{constant} ,$$

$$h(x) = \sum_{n=1}^{\infty} \frac{\cos 2n\pi x}{n^5} . \quad (23)$$

The  $x$ -dependent part is finite. In the present case we have  $\sum(\pm)h[\gamma(\theta_W)]$ . The total effective potential takes the form

$$V_{\text{eff}}(\theta_W) = N_{\text{eff}} \frac{3}{128\pi^6} M_{KK}^4 f(\theta_W) \quad (24)$$

where  $f(\theta_W + 2\pi) = f(\theta_W)$  and its amplitude is normalized to be an unity. Once the matter content of the model is specified, the coefficient  $N_{\text{eff}}$  is determined. In the minimal model or its minimal extension,  $N_{\text{eff}} = O(1)$  as supported by examples.

When  $V_{\text{eff}}(\theta_W)$  has a global minimum at a nontrivial  $\theta_W = \theta_W^{\text{min}}$ , dynamical electroweak symmetry breaking takes place. It typically happens at  $\theta_W^{\text{min}} = (0.2 \sim 0.3)\pi$ . [20] It is possible to have a very small  $\theta_W^{\text{min}} \sim 0.01\pi$  by fine-tuning of the matter content as shown in ref. [19], which, however, is eliminated in the present consideration for the artificial nature. The mass  $m_H$  of the neutral Higgs boson is found by expanding  $V_{\text{eff}}(\theta_W)$  around  $\theta_W^{\text{min}}$  and using  $\theta_W = g[(w_1 - w_0)k^{-1}\Phi^\dagger\Phi]^{1/2}$ . One finds

$$m_H^2 = N_{\text{eff}} f''(\theta_W^{\text{min}}) \frac{3\alpha_w}{64\pi^4} \frac{R(w_1 - w_0)}{k} M_{KK}^4 \quad (25)$$

where  $\alpha_w = g_4^2/4\pi$ . In a generic model  $f''(\theta_W^{\text{min}}) \sim 1$ . Making use of (12) and (17), one finds

$$m_H = \begin{cases} c \left( \frac{3\alpha_w}{32\pi^3} \right)^{1/2} M_{KK} & = c \left( \frac{3\alpha_w}{8\pi^3} \right)^{1/2} \frac{\pi}{\theta_W^{\text{min}}} m_W & \text{for } k \rightarrow 0 , \\ c \left( \frac{3\alpha_w}{64\pi^2} \right)^{1/2} \sqrt{kR} M_{KK} & = c \left( \frac{3\alpha_w}{32\pi} \right)^{1/2} kR \frac{\pi}{\theta_W^{\text{min}}} m_W & \text{for } e^{\pi kR} \gg 1 , \end{cases} \quad (26)$$

where  $c = [N_{\text{eff}} f''(\theta_W^{\text{min}})]^{1/2}$ . The values of  $\theta_W^{\text{min}}$  and  $c$  depend on details of the model. In the models analysed in ref. [20],  $(\theta_W^{\text{min}}, c)$  ranges from  $(0.269\pi, 2.13)$  to  $(0.224\pi, 1.63)$ , which justifies our estimate. Hereafter we set  $c = 1.9$ , understanding 20% uncertainty. Inserting  $\alpha_w = 0.032$  and  $kR = 12$ , we obtain that  $m_H = 0.70(\pi/\theta_W^{\text{min}})m_W$  and  $M_{KK} = 12.4m_H$ . In flat space (in the  $k \rightarrow 0$  limit),  $m_H = 0.037(\pi/\theta_W^{\text{min}})m_W$  and  $M_{KK} = 53.9m_H$ ,

which yielded too small  $m_H$ . There appears a large enhancement factor  $kR$  in the relation connecting  $m_H$  and  $m_W$  in the Randall-Sundrum warped spacetime. For a typical value  $\theta_W^{\min} = (0.2 \sim 0.4)\pi$ , the mass of the Higgs boson and the Kaluza-Klein mass scale are given by  $m_H = (140 \sim 280)$  GeV and  $M_{KK} = (1.7 \sim 3.5)$  TeV, respectively.

The relations (17) and (26) reveal many remarkable facts. First of all, only the parameter  $kR$  in the Randall-Sundrum spacetime appears in the relations connecting  $m_W$ ,  $m_H$  and  $M_{KK}$ . Secondly, if one supposes that  $k = O(M_{\text{pl}})$ , then  $kR = 12 \pm 1$  to have the observed value for  $m_W$ . The electroweak-gravity hierarchy is accounted for by a moderate value for  $kR$ . Thirdly, another quantity  $\theta_W^{\min}$  involved in those relations is dynamically determined, once the matter content of the model is specified. In case the electroweak symmetry breaking takes place, it typically takes  $(0.2 \sim 0.4)\pi$ .  $m_H$  and  $M_{KK}$  are predicted up to the factor  $\theta_W^{\min}$ . Fourthly and most remarkably, the predicted value for  $m_H$ ,  $140 \sim 280$  GeV, is exactly in the range which can be explored in the experiments at LHC and other planned facilities in the near future. In conjunction with it, we recall that in the minimal supersymmetric standard model the Higgs boson mass is predicted in the range  $100 < m_H < 130$  GeV.[28] Experimentally preferred value is  $m_H = 126^{+73}_{-48}$  GeV.[29]

In the dynamical gauge-Higgs unification the Higgs field in four dimensions is identified with the extra-dimensional component of the gauge fields. The Hosotani mechanism induces dynamical electroweak symmetry breaking, giving both weak gauge bosons and Higgs boson finite masses. The desirable enhancement factor for  $m_H$  originates from the property that the Higgs field is a part of five-dimensional vector, not a scalar, whose coupling to gravity and matter differs from those of four-dimensional gauge fields in the Randall-Sundrum warped spacetime.

Our scenario significantly differs from the Higgsless model where four-dimensional Higgs fields are eliminated from the spectrum by ad hoc boundary conditions on orbifolds.[30] In our scenario there is a Higgs boson with  $m_H = (140 \sim 280)$  GeV. Its mass is generated by radiative corrections. There is no quadratic divergence associated with  $m_H^2$  thanks to the gauge invariance in five dimensions. As in supersymmetric theories the unitarity is expected to be assured by the existence of light Higgs boson.

The scenario of the dynamical gauge-Higgs unification in the warped spacetime is promising. In the present paper we focused on  $m_H$  and  $M_{KK}$ . There are many issues to be examined. Yukawa couplings among fermions and the Higgs boson, couplings of fermions to Kaluza-Klein excitations of the gauge and Higgs bosons, and self-couplings of

the Higgs boson can be also explored in the forthcoming experiments. It is also interesting to extend our analysis to supersymmetric (SUSY) theories in the Randall-Sundrum spacetime.[31] SUSY breaking scale  $M_{\text{SUSY}} \sim 1 \text{ TeV}$  is not far from  $M_{KK}$  in the present paper. We shall come back to these issues in separate publications.

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