

Lepton masses and mixing angles from heterotic orbifold models

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Abstract

We systematically study the possibility for realizing realistic values of lepton mass ratios and mixing angles by using only renormalizable Yukawa couplings derived from the heterotic Z_6 -I orbifold. We assume one pair of up and down sector Higgs fields. We consider both the Dirac neutrino mass scenario and the seesaw scenario with degenerate right-handed majorana neutrino masses. It is found that realistic values of the charged lepton mass ratios, m_e/m_τ and m_μ/m_τ , the neutrino mass squared difference ratio, $\Delta m_{31}^2/\Delta m_{21}^2$, and the lepton mixing angles can be obtained in certain cases.

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1 Introduction

Understanding the origin of fermion masses and mixing angles is one of the most important issues in particle physics. Quark masses and mixing angles, and charged lepton masses are well known. They have hierarchical structures. Recently, neutrino oscillation experiments have provided us with information about neutrino mass squared differences and mixing angles [1]. Two of these mixing angles are large in contrast to the small quark mixing angles. Within the framework of the standard model and its extensions, a fermion acquires its mass from the Yukawa coupling with the electroweak Higgs fields. For right-handed neutrinos, majorana masses are additional sources of masses and mixing angles. Thus, it is important to study Yukawa couplings and right-handed majorana neutrino masses derived from an underlying theory.

Superstring theory is a promising candidate for a unified theory including gravity. For consistency, superstring theory requires 10D space-time. That is, it predicts 6D extra space other than our 4D space-time, and such a space must be compact. This 6D compact space is very important from the viewpoint of string phenomenology. In general, it is an origin of fermion flavor structure. That is, the flavor structure derived from a string model depends on the geometrical aspects of the 6D compact space, and in principle one can calculate Yukawa couplings for a given compact space with known geometry. Geometrical aspects of the 6D compact space also provide us with selection rules for allowed Yukawa couplings. In several cases, such selection rules are so severe that off-diagonal Yukawa couplings are not allowed for one Higgs field, because different families are discriminated by quantum numbers, which originate from geometrical aspects of the 6D compact space.

Among several types of string models, heterotic orbifold models [2] and intersecting D-brane models are particularly interesting,⁴ to realize realistic Yukawa couplings. They have localized modes in the 6D compact space, and their Yukawa couplings can be calculated [4, 5, 6, 7], because their 6D geometry is not complicated.⁵ Indeed, a Yukawa coupling among localized modes has a suppression factor depending on their distances [4, 5, 11, 6, 7]. That can explain suppressed Yukawa couplings. Therefore, it is very important to study possibilities for deriving realistic fermion masses and mixing angles from heterotic orbifold models and intersecting D-brane models. The number

⁴See for a review of intersecting D-brane models, e.g. Ref. [3] and references therein.

⁵See for calculations of Yukawa couplings in intersecting D-brane models Refs. [8, 9, 10].

of 6D Z_N orbifolds [2, 12, 13] and $Z_N \times Z_M$ orbifolds [14], especially with 4D $N = 1$ supersymmetry, is finite, i.e. Z_3, Z_4, Z_6 -I, Z_6 -II, Z_7, Z_8 -I, Z_8 -II, Z_{12} -I, Z_{12} -II, and $Z_N \times Z_M$ for $(N, M) = (2, 2), (2, 3), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)$. (See Refs. [15] for examples of explicit models, and Refs. [16, 17, 18, 19] for recent model buildings.) In particular, non-prime order orbifolds would be interesting, because they allow off-diagonal couplings [20, 21]. On each orbifold, all the fixed points are known, where light modes can be localized. Thus, a systematical analysis on Yukawa matrices, which can be derived from heterotic orbifold models, would be possible, while the number of intersecting D-brane configurations seems to be infinity. Indeed, such studies have been done for the quark sector [22, 23]. In particular, in Ref. [23] possibilities for leading to the realistic mixing angle V_{cb} and mass ratios m_c/m_t and m_s/m_b from the Z_6 -I orbifold have been shown in the case with the minimal number of Higgs fields.

However, such a study considering mixing angles has not been done for the lepton sector. In this paper, we study lepton mass ratios and mixing angles derived from the Z_6 -I orbifold. Commonly, the smallness of neutrino masses is explained in two ways. One is the Dirac neutrino mass scenario, that is, Yukawa couplings between neutrino and Higgs fields are strongly suppressed for some reason. Since a Yukawa coupling is suppressed depending on the size of extra dimensional space in an heterotic orbifold model, strongly suppressed Yukawa couplings can be obtained in the case that an extra dimensional space is large compared with the string scale. The other is the seesaw scenario [24], which requires right-handed majorana neutrino masses at the intermediate scale between the weak scale and the Planck scale.⁶ Within the framework of string theory, the natural mass scale is just the string scale, and such an intermediate mass scale should be obtained through vacuum expectation values (VEVs) of some scalar fields. However, that is quite model-dependent. Here we study both the Dirac scenario and the seesaw scenario, and for the latter case we assume the right-handed majorana neutrino mass matrix to be proportional to the identity matrix with a universal mass scale. With this, we consider all of the possible assignments of leptons to fixed points, examine Yukawa terms allowed by the selection rule and calculate their Yukawa matrices varying orbifold moduli parameters. In practice, the number of relevant moduli parameters is two in our models.

⁶In Ref. [25], realization of the minimal seesaw mechanism has been examined in explicit Z_3 orbifold models, and its difficulty has been shown.

Then we try to fit the Yukawa matrices by these two moduli parameters in order to get realistic values of six observables, that is, lepton mass ratios and mixing angles.

This paper is organized as follows. In section 2, we give a brief review on fixed points on the Z_6 -I orbifold and the corresponding twisted states. Also, their selection rules for allowed Yukawa couplings and the strength of Yukawa couplings are reviewed. In section 3, we study systematically the possibility for realizing realistic lepton masses and mixing angles by using only renormalizable Yukawa couplings derived from Z_6 -I orbifold models. We assume one pair of up and down sector Higgs fields. We consider the Dirac neutrino mass scenario in section 3.1. In section 3.2, we perform the same analysis for the seesaw scenario. Section 4 is devoted to conclusion and discussions.

2 Orbifold models and selection rule

Here we give a brief review on heterotic orbifold models, that is, the structure of fixed points on orbifolds, the selection rule for allowed Yukawa couplings and their Yukawa coupling strength. In particular, we concentrate our attention on Z_6 -I orbifold models. (See for details of generic Z_N orbifolds Refs. [20, 21].)

2.1 Fixed points and twisted sectors

An orbifold is defined as a division of a torus by a discrete rotation, i.e., a twist θ . The 6D Z_6 -I orbifold is obtained by dividing T^6 by the twist θ , whose eigenvalues are $\text{diag}(e^{2\pi i/6}, e^{2\pi i/6}, e^{2\pi i/3})$, that is, a direct product of two 2D Z_6 orbifolds and a 2D Z_3 orbifold. The 2D Z_6 orbifold is obtained e.g. through dividing R^2 by the G_2 lattice and its automorphism, that is, the Coxeter element of G_2 algebra, which transforms the G_2 simple roots e_1 and e_2 ,

$$\theta e_1 \rightarrow -e_1 - e_2, \quad \theta e_2 \rightarrow 3e_1 + 2e_2, \quad (1)$$

i.e., the Z_6 twist. Similarly, we can obtain the 2D Z_3 orbifold by dividing R^2 by the $SU(3)$ root lattice and its Coxeter element, which transforms the $SU(3)$ simple roots e_5 and e_6 ,

$$\theta e_5 \rightarrow e_6, \quad \theta e_6 \rightarrow -e_5 - e_6, \quad (2)$$

that is, the Z_3 rotation.

There are two types of closed strings on an orbifold. One is the untwisted string, which is already closed on a torus before orbifold twisting. The other is the twisted string, which is a localized mode we are interested in. A twisted string has the following boundary condition,

$$X^i(\sigma = 2\pi) = (\theta^k X)^i(\sigma = 0) + v^i, \quad (3)$$

where the shift vector v^i is on the torus lattice Λ . That is, the center of mass of a θ^k twisted string is localized at a fixed point f , which is defined as

$$f^i = (\theta^k f)^i + v^i. \quad (4)$$

The fixed point f is represented by the corresponding space group element (θ^k, v^i) . The fixed points, which differ by lattice vectors, correspond to equivalent fixed points. That implies that (θ^k, v^i) is equivalent to $(\theta^k, v^i + (1 - \theta^k)\Lambda)$. Hereafter the θ^k -twisted sector is denoted by \hat{T}_k .

The 2D Z_3 orbifold has the following three fixed points under θ ,

$$g_{Z_3,1}^{(0)} = (0, 0), \quad g_{Z_3,1}^{(1)} = (2/3, 1/3), \quad g_{Z_3,1}^{(2)} = (1/3, 2/3), \quad (5)$$

in the $SU(3)$ simple root basis, and each of these is represented by the space group element as

$$g_{Z_3,1}^{(n)} : (\theta, ne^1), \quad (6)$$

where $n = 0, 1, 2$, up to $(1 - \theta)\Lambda_{SU(3)}$. The corresponding twisted ground states are denoted by $|g_{Z_3,1}^{(n)}\rangle$ with $n = 0, 1, 2$.

Similarly, we can obtain fixed points on the 2D Z_6 orbifold. The θ twisted sector on the 2D Z_6 orbifold has only one fixed point,

$$g_{Z_6,1}^{(0)} = (0, 0), \quad (7)$$

in the G_2 simple root basis. The θ^2 twisted sector has three fixed points,

$$g_{Z_6,2}^{(0)} = (0, 0), \quad g_{Z_6,2}^{(1)} = (0, 1/3), \quad g_{Z_6,2}^{(2)} = (0, 2/3). \quad (8)$$

Note that these fixed points are defined up to the G_2 lattice. For example, one can use the fixed point $g_{Z_6,2}^{(2)} = (1, 2/3)$, which is equivalent to $(0, 2/3)$. The corresponding three twisted ground states are denoted by $|g_{Z_6,2}^{(i)}\rangle$. However, not all of the three points $g_{Z_6,2}^{(i)}$ are fixed points of θ . While $g_{Z_6,2}^{(0)}$ is also a fixed

point of the twist θ , the other two fixed points $g_{Z_6,2}^{(1)}$ and $g_{Z_6,2}^{(2)}$ are transformed to each other by θ . Since physical states are constructed as θ -eigenstates, we take linear combinations of states corresponding to $g_{Z_6,2}^{(1)}$ and $g_{Z_6,2}^{(2)}$ as [26, 20]

$$|g_{Z_6,2}^{(1)}; \pm 1\rangle \equiv \frac{1}{\sqrt{2}} \left(|g_{Z_6,2}^{(1)}\rangle \pm |g_{Z_6,2}^{(2)}\rangle \right), \quad (9)$$

with the eigenvalues $\gamma = \pm 1$, while the state $|g_{Z_6,2}^{(0)}\rangle$ corresponding to the fixed point $g_{Z_6,2}^{(0)}$ is by itself a θ -eigenstate.

The θ^3 twisted sector has four fixed points,

$$\begin{aligned} g_{Z_6,3}^{(0)} &= (0, 0), & g_{Z_6,3}^{(1)} &= (0, 1/2), \\ g_{Z_6,3}^{(2)} &= (1/2, 0), & g_{Z_6,3}^{(3)} &= (1/2, 1/2). \end{aligned} \quad (10)$$

Recall that these fixed points are defined up to the G_2 lattice. For instance, the fixed point $g_{Z_6,3}^{(1)} = (1, 1/2)$ is equivalent to $(0, 1/2)$. Not all of the four points are fixed points of the twist θ . The θ -eigenstates for each G_2 part are obtained as

$$|g_{Z_6,3}^{(0)}\rangle, \quad |g_{Z_6,3}^{(1)}; \gamma\rangle \equiv \frac{1}{\sqrt{3}} \left(|g_{Z_6,3}^{(1)}\rangle + \gamma |g_{Z_6,3}^{(2)}\rangle + \gamma^2 |g_{Z_6,3}^{(3)}\rangle \right), \quad (11)$$

where $\gamma = 1, \omega, \omega^2$ with $\omega = e^{2\pi i/3}$.

A fixed point on the 6D Z_6 -I orbifold is obtained as a direct product of three fixed points, each coming from one of the two 2D Z_6 orbifolds and the 2D Z_3 orbifold. The corresponding twisted ground state is obtained in the same manner. The θ twisted sector has the following ground states,

$$|g_{Z_6,1}^{(0)}\rangle \otimes |g_{Z_6,1}^{(0)}\rangle \otimes |g_{Z_3,1}^{(i)}\rangle, \quad (12)$$

for $i = 0, 1, 2$. The θ^2 twisted sector has the following ground states,

$$\begin{aligned} &|g_{Z_6,2}^{(0)}\rangle \otimes |g_{Z_6,2}^{(0)}\rangle \otimes |g_{Z_3,2}^{(j)}\rangle, \\ &|g_{Z_6,2}^{(1)}; \gamma\rangle \otimes |g_{Z_6,2}^{(0)}\rangle \otimes |g_{Z_3,2}^{(j)}\rangle, \\ &|g_{Z_6,2}^{(0)}\rangle \otimes |g_{Z_6,2}^{(1)}; \gamma'\rangle \otimes |g_{Z_3,2}^{(j)}\rangle, \\ &|g_{Z_6,2}^{(1)}; \gamma\rangle \otimes |g_{Z_6,2}^{(1)}; \gamma'\rangle \otimes |g_{Z_3,2}^{(j)}\rangle, \end{aligned} \quad (13)$$

for $\gamma, \gamma' = \pm 1$ and $j = 0, 1, 2$. The θ^3 twisted sector has the following ground states,

$$\begin{aligned}
& |g_{Z_6,3}^{(0)}\rangle \otimes |g_{Z_6,3}^{(0)}\rangle, \\
& |g_{Z_6,3}^{(1)}; \gamma\rangle \otimes |g_{Z_6,3}^{(0)}\rangle, \\
& |g_{Z_6,3}^{(0)}\rangle \otimes |g_{Z_6,3}^{(1)}; \gamma'\rangle, \\
& |g_{Z_6,3}^{(1)}; \gamma\rangle \otimes |g_{Z_6,3}^{(1)}; \gamma'\rangle,
\end{aligned} \tag{14}$$

where $\gamma, \gamma' = 1, \omega, \omega^2$.

2.2 Selection rule

Here we give a brief review on the selection rule for Yukawa couplings in orbifold models. (See for their details Refs. [20, 21].) The fixed point f of the θ^k twisted sector is denoted by its space group element, $(\theta^k, (1 - \theta^k)f)$, as said in the previous subsection. Thus, the three states corresponding to the three fixed points $(\theta^{k_i}, (1 - \theta^{k_i})f_i)$ for $i = 1, 2, 3$ can couple if the product of their space group elements $\prod_i (\theta^{k_i}, (1 - \theta^{k_i})f_i)$ is equivalent to identity. That implies the space group selection rule for allowed Yukawa couplings requires [4]

$$\prod_i (\theta^{k_i}, (1 - \theta^{k_i})(f_i + \Lambda)) = (1, 0), \tag{15}$$

because the fixed point $(\theta^k, (1 - \theta^k)f)$ is equivalent to $(\theta^k, (1 - \theta^k)(f + \Lambda))$. This space group selection rule includes the point group selection rule, that is, the product of twists must be identity, $\prod_i \theta^{k_i} = 1$. In Z_6 -I orbifold models, the point group selection rule and H -momentum conservation [27] allow only the following couplings [26],

$$\hat{T}_1 \hat{T}_2 \hat{T}_3, \quad \hat{T}_2 \hat{T}_2 \hat{T}_2. \tag{16}$$

The space group selection rule for the 2D Z_3 orbifold is simple. For a $\hat{T}_2 \hat{T}_2 \hat{T}_2$ coupling, three states corresponding to fixed points $g_{Z_3,2}^{(i_1)}$, $g_{Z_3,2}^{(i_2)}$ and $g_{Z_3,2}^{(i_3)}$, can couple when the following equation is satisfied,

$$i_1 + i_2 + i_3 = 0 \pmod{3}. \tag{17}$$

Also, for a $\hat{T}_1 \hat{T}_2 \hat{T}_3$ coupling, three states can couple when the fixed point of \hat{T}_1 is the same as that of \hat{T}_2 . Thus, the space group selection rule of the 2D

Z_3 part allows only diagonal couplings, that is, this part is not relevant to our purpose of deriving realistic Yukawa matrices with non-vanishing mixing angles in the case with the minimal number of Higgs fields. For a while, we will assume that all relevant states correspond to the same fixed point on the 2D Z_3 orbifold. There is another possibility for a $\hat{T}_2\hat{T}_2\hat{T}_2$ coupling, that is, the three states correspond to different fixed points on the 2D Z_3 orbifold, and in this case, we have a suppressed Yukawa coupling depending on the volume of the 2D Z_3 orbifold. We will give a comment on it later.

We can discuss the space group selection rule for the 2D Z_6 part. For $\hat{T}_2\hat{T}_2\hat{T}_2$ couplings, the space group selection rule on the 2D Z_6 orbifold is exactly the same as that on the 2D Z_3 orbifold, i.e. eq.(17), when we consider the basis of twisted states corresponding directly to fixed points. However, in Z_6 -I orbifold models, we take linear combinations as in Eq. (9). The space group selection rule for $\hat{T}_1\hat{T}_2\hat{T}_3$ couplings is non-trivial. All couplings on the 2D Z_6 orbifold are allowed by the space group selection rule, because $(1 - \theta)\Lambda = \Lambda$. Thus, off-diagonal couplings are allowed for $\hat{T}_1\hat{T}_2\hat{T}_3$ couplings in the 2D Z_6 orbifold part. Furthermore, the selection rule requires that the product of θ -eigenvalues of the coupling states must be equal to identity, that is, $\prod \gamma = 1$. Therefore, the twisted states, which are relevant to our purpose, are the single \hat{T}_1 state,

$$|g_{Z_6,1}^{(0)}\rangle \otimes |g_{Z_6,1}^{(0)}\rangle, \quad (18)$$

and the five \hat{T}_2 states,

$$\begin{aligned} \hat{T}_2^{(1)} &\equiv |g_{Z_6,2}^{(0)}\rangle \otimes |g_{Z_6,2}^{(0)}\rangle, \\ \hat{T}_2^{(2)} &\equiv |g_{Z_6,2}^{(0)}\rangle \otimes |g_{Z_6,2}^{(1)}; +1\rangle, \\ \hat{T}_2^{(3)} &\equiv |g_{Z_6,2}^{(1)}; +1\rangle \otimes |g_{Z_6,2}^{(0)}\rangle, \\ \hat{T}_2^{(4,\gamma)} &\equiv |g_{Z_6,2}^{(1)}; \gamma\rangle \otimes |g_{Z_6,2}^{(1)}; \gamma^{-1}\rangle, \end{aligned} \quad (19)$$

where $\gamma = \pm 1$, and the six \hat{T}_3 states,

$$\begin{aligned} \hat{T}_3^{(1)} &\equiv |g_{Z_6,3}^{(0)}\rangle \otimes |g_{Z_6,3}^{(0)}\rangle, \\ \hat{T}_3^{(2)} &\equiv |g_{Z_6,3}^{(0)}\rangle \otimes |g_{Z_6,3}^{(1)}; +1\rangle, \\ \hat{T}_3^{(3)} &\equiv |g_{Z_6,3}^{(1)}; +1\rangle \otimes |g_{Z_6,3}^{(0)}\rangle, \\ \hat{T}_3^{(4,\gamma)} &\equiv |g_{Z_6,3}^{(1)}; \gamma\rangle \otimes |g_{Z_6,3}^{(1)}; \gamma^{-1}\rangle, \end{aligned} \quad (20)$$

where $\gamma = 1, \omega, \omega^2$. We have omitted the index for the fixed point on the 2D Z_3 orbifold, because we assume all states sit on the same fixed point on the 2D Z_3 orbifold for the moment.

2.3 Yukawa couplings

The strength of a Yukawa coupling has been calculated by use of 2D conformal field theory. It depends on distances between fixed points. The Yukawa coupling strength of the $\hat{T}_1\hat{T}_2\hat{T}_3$ coupling in Z_6 -I orbifold models is obtained for the $G_2 \times G_2$ part as [4, 5, 6, 21]

$$Y = \sum_{f_{23}=f_2-f_3+\Lambda} \exp\left[-\frac{\sqrt{3}}{4\pi} f_{23}^T M f_{23}\right], \quad (21)$$

up to an overall normalization factor, where

$$M = \begin{pmatrix} R_1^2 & -\frac{3}{2}R_1^2 & 0 & 0 \\ -\frac{3}{2}R_1^2 & 3R_1^2 & 0 & 0 \\ 0 & 0 & R_2^2 & -\frac{3}{2}R_2^2 \\ 0 & 0 & -\frac{3}{2}R_2^2 & 3R_2^2 \end{pmatrix}, \quad (22)$$

in the $G_2 \times G_2$ root basis. Here, f_2 and f_3 denote fixed points of the \hat{T}_2 and \hat{T}_3 sectors, respectively, and R_i corresponds to the radius of the i -th torus, which can be written as the real part of the i -th Kähler modulus T_i up to a constant factor. Here we follow Ref. [21] for the normalization of R_i . (See Ref. [28] for the normalization of the moduli such that the transformation $T_\ell \rightarrow T_\ell + i$ is a symmetry.) The imaginary parts of T_i also contribute to mass matrices, i.e., eigenvalues, mixing angles and CP violating phases. However, here we consider only the real parts R_i for simplicity. Since the states with fixed points in the same conjugacy class contribute to the Yukawa coupling, we take summation of these contributions in eq. (21). However, the states corresponding to the nearest fixed points (f_2, f_3) contribute dominantly to the Yukawa coupling for a large value of R_i . Hence, we calculate Yukawa couplings only by the contribution due to the nearest fixed points (f_2, f_3). Indeed such an approximation is valid when R_i is sufficiently large as in the cases we will study in the following sections.

Similarly, the strength of $\hat{T}_2\hat{T}_2\hat{T}_2$ Yukawa couplings is obtained in the

basis of twisted states corresponding directly to fixed points as

$$Y = \sum_{f_{23}=f_2-f_3+\Lambda} \exp\left[-\frac{\sqrt{3}}{16\pi} f_{23}^T M f_{23}\right], \quad (23)$$

where M is the same matrix as Eq.(22). Here, f_2 and f_3 denote two of the three fixed points in the \hat{T}_2 sector. Recall that when we choose two states, the other state, which is allowed to couple, is uniquely fixed in the basis of states corresponding directly to fixed points.

Here, we give a comment on the Kähler metric. For a given k , the θ^k twisted states have the same Kähler metric, even if they correspond to different fixed points. Thus, the Kähler metric is irrelevant to mass ratios or mixing angles when the three families of leptons with the same $SU(3) \times SU(2) \times U(1)_Y$ quantum numbers are assigned to states in a single \hat{T}_k sector. Indeed this type of assignment is required by the point group selection rule in order that non-vanishing mixing angles can be realized.

3 Lepton masses and mixing angles

Here we systematically study the possibility for deriving realistic lepton masses and mixing angles by use of twisted states, their selection rules and the Yukawa couplings, which are shown in the previous section. We assume that we obtain the standard gauge group $SU(3) \times SU(2) \times U(1)_Y$ and three families of leptons in a Z_6 -I orbifold model. Indeed it is quite a nontrivial issue to construct a realistic model including the gauge group $SU(3) \times SU(2) \times U(1)_Y$ and three families of quarks and leptons, but here we just assume them, because our purpose is not to construct an explicit model, but to show the possibility for leading to realistic mixing angles and mass ratios. We also assume one pair of up and down Higgs fields.

Charged lepton masses are well known and shown in Table 1 [29]. The table also shows the neutrino mass squared differences and mixing angles, which are consistent with recent experiments on neutrino oscillations [1].

3.1 Dirac neutrino mass scenario

First we consider the Dirac neutrino mass scenario. The relevant terms in the superpotential are

$$W \supset H_u L_i (Y_\nu)_{ij} N_j - H_d L_i (Y_e)_{ij} e_j^c, \quad (24)$$

Parameter	Experimental value
m_e	$(0.51099892 \pm 0.00000004) \text{ MeV}$
m_μ	$(105.658369 \pm 0.000009) \text{ MeV}$
m_τ	$(1776.99 \pm 0.275) \text{ MeV}$
Δm_{21}^2	$(8.1 \pm 0.3) \times 10^{-5} \text{ eV}^2$
Δm_{31}^2	$(2.2 \pm 0.3) \times 10^{-3} \text{ eV}^2$
$\sin^2 \theta_{12}$	0.30 ± 0.025
$\sin^2 \theta_{23}$	0.50 ± 0.065
$\sin^2 \theta_{13}$	≤ 0.014

Table 1: Experimental values of lepton masses and mixing angles.

where H_u and H_d are the up- and down-sector Higgs fields, L_i are the SU(2) doublet leptons, N_i are the gauge singlet neutrinos, and e_i^c are the SU(2) singlet charged leptons. After replacing the Higgs fields with their VEVs, v_u and v_d , leptons gain the mass terms,

$$W \supset -\nu_i(m_D)_{ij}N_j - e_i(m_e)_{ij}e_j^c, \quad (25)$$

where $(\nu_i, e_i) = L_i$, $(m_D)_{ij} = (v_u Y_\nu)_{ij}$, and $(m_e)_{ij} = (v_d Y_e)_{ij}$. The mass matrices can be diagonalized as

$$m_D = U_\nu m_D^{\text{diag}} V_\nu^\dagger, \quad m_e = U_e m_e^{\text{diag}} V_e^\dagger. \quad (26)$$

In terms of these unitary matrices, the lepton mixing matrix is defined as

$$U_{\text{MNS}} \equiv U_e^T U_\nu^*. \quad (27)$$

We consider all possible assignments of L_i, N_i, e_i^c and $H_{u,d}$ to the twisted states shown in the previous section. The point group selection rule implies that we can obtain non-vanishing mixing angles only when each species of L_i, N_i , and e_i^c , belongs to the same twisted sector. The \hat{T}_1 sector has a single state for the $T^2/Z_6 \times T^2/Z_6$ part. Hence, we have to assign L_i, N_i and e_i^c to \hat{T}_2 or \hat{T}_3 to obtain non-vanishing mixing angles, while the Higgs fields can be assigned to \hat{T}_1 . Therefore, there are five classes of assignments, which are shown in Table 2. For each of possible assignments, we examine the selection rule for allowed Yukawa couplings and calculate Yukawa couplings as functions of R_1 and R_2 . Then, varying these two parameters R_1 and R_2 , we try to fit the charged lepton mass ratios m_e/m_τ and m_μ/m_τ , and the

Class	L	N	e^c	H_u	H_d
Assignment 1	\hat{T}_2	\hat{T}_3	\hat{T}_3	\hat{T}_1	\hat{T}_1
Assignment 2	\hat{T}_3	\hat{T}_2	\hat{T}_2	\hat{T}_1	\hat{T}_1
Assignment 3	\hat{T}_2	\hat{T}_3	\hat{T}_2	\hat{T}_1	\hat{T}_2
Assignment 4	\hat{T}_2	\hat{T}_2	\hat{T}_3	\hat{T}_2	\hat{T}_1
Assignment 5	\hat{T}_2	\hat{T}_2	\hat{T}_2	\hat{T}_2	\hat{T}_2

Table 2: Five classes of assignments

ratio of neutrino mass squared difference $\Delta m_{31}^2/\Delta m_{21}^2$ and mixing angles θ_{12} , θ_{23} , θ_{13} . For the ease of presentation, we display the ratios of experimental values, which can be immediately obtained from Table 1.

$$(m_e/m_\tau, m_\mu/m_\tau) = (0.000288, 0.0595), \quad (28)$$

$$\Delta m_{31}^2/\Delta m_{21}^2 = 27. \quad (29)$$

In particular, we are interested in deriving their orders, but not precise values, because those values are obtained at the string scale. However, it is quite non-trivial to fit these six observables, m_e/m_τ , m_μ/m_τ , $\Delta m_{31}^2/\Delta m_{21}^2$, $\sin^2 \theta_{12}$, $\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$, only by two parameters, R_1 and R_2 . It may be most important to realize the mass ratios of charged leptons m_e/m_τ and m_μ/m_τ , because their values are measured very precisely by experiments. Hence, the two parameters, R_1 and R_2 , are almost fixed in order to fit m_e/m_τ and m_μ/m_τ . In all the five classes of assignments, we can find cases that fit the charged lepton mass ratios, but not in all of them can we fit the neutrino oscillation data because mass matrix patterns are limited.

We show our results in what follows.

Assignment 1

In this class of assignments, we can fit only the mass ratios m_e/m_τ and m_μ/m_τ properly. For example, we take $(R_1^2, R_2^2) = (26, 33)$ in the following assignment,

$$\begin{aligned} (L_1, L_2, L_3) &= (\hat{T}_2^{(1)}, \hat{T}_2^{(2)}, \hat{T}_2^{(4,1)}), & (N_1, N_2, N_3) &= (\hat{T}_3^{(1)}, \hat{T}_3^{(2)}, \hat{T}_3^{(3)}), \\ (e_1^c, e_2^c, e_3^c) &= (\hat{T}_3^{(1)}, \hat{T}_3^{(2)}, \hat{T}_3^{(4,1)}), & (H_u, H_d) &= (\hat{T}_1, \hat{T}_1). \end{aligned} \quad (30)$$

Then, we can realize the experimental values of m_e/m_τ and m_μ/m_τ . However, the neutrino oscillation data cannot be well accommodated. In this

particular assignment, we obtain

$$\begin{aligned} \frac{\Delta m_{31}^2}{\Delta m_{21}^2} &= 110, & \sin^2 \theta_{12} &= 5 \times 10^{-5}, \\ \sin^2 \theta_{23} &= 6 \times 10^{-3}, & \sin^2 \theta_{13} &= 6 \times 10^{-11}. \end{aligned} \quad (31)$$

Thus, the mass squared difference ratio $\Delta m_{31}^2/\Delta m_{21}^2$ is large, and what is worse is that the mixing angles θ_{12} and θ_{23} are too small. This is the character of this class of assignments. Namely, we can fit the charged lepton mass ratios m_e/m_τ and m_μ/m_τ , but the mixing angles are too small except the trivial cases. This character is shown in Table 3. Here and hereafter, by a trivial result we mean that at least one of charged leptons is massless, charged leptons or neutrinos have degenerate masses, or the mixing matrix is trivially the identity matrix.

Assignment 2

In this class of assignments, we can fit only the mass ratios m_e/m_τ and m_μ/m_τ properly. For example, we take $(R_1^2, R_2^2) = (21, 26)$ in the following assignment,

$$\begin{aligned} (L_1, L_2, L_3) &= (\hat{T}_3^{(1)}, \hat{T}_3^{(2)}, \hat{T}_3^{(3)}), & (N_1, N_2, N_3) &= (\hat{T}_2^{(1)}, \hat{T}_2^{(2)}, \hat{T}_2^{(4,1)}), \\ (e_1^c, e_2^c, e_3^c) &= (\hat{T}_2^{(1)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,1)}), & (H_u, H_d) &= (\hat{T}_1, \hat{T}_1). \end{aligned} \quad (32)$$

Then, we can realize the proper mass ratios m_e/m_τ and m_μ/m_τ . The neutrino masses and mixing angles are not so satisfactory, however. We obtain

$$\begin{aligned} \frac{\Delta m_{31}^2}{\Delta m_{21}^2} &= 150, & \sin^2 \theta_{12} &= 4 \times 10^{-7}, \\ \sin^2 \theta_{23} &= 2 \times 10^{-3}, & \sin^2 \theta_{13} &= 9 \times 10^{-8}. \end{aligned} \quad (33)$$

Thus, the mass squared difference ratio $\Delta m_{31}^2/\Delta m_{21}^2$ is large, and the mixing angles θ_{12} and θ_{23} are too small. That is the character of this class of assignments. Namely, we can fit the charged lepton mass ratios m_e/m_τ and m_μ/m_τ , but the mixing angles are too small except the trivial cases. This character is shown in Table 3.

Assignment 3

In this class of assignments, we can fit only m_e/m_τ , m_μ/m_τ , and $\Delta m_{31}^2/\Delta m_{21}^2$ properly. For example, we take $(R_1^2, R_2^2) = (205, 481)$ in the following assignment,

$$(L_1, L_2, L_3) = (\hat{T}_2^{(2)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,1)}), \quad (N_1, N_2, N_3) = (\hat{T}_3^{(1)}, \hat{T}_3^{(2)}, \hat{T}_3^{(4,1)}),$$

$$(e_1^c, e_2^c, e_3^c) = (\hat{T}_2^{(1)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,1)}), \quad (H_u, H_d) = (\hat{T}_1, \hat{T}_2^{(2)}). \quad (34)$$

Then, we can realize the proper mass ratios m_e/m_τ and m_μ/m_τ . On the other hand, we obtain

$$\begin{aligned} \frac{\Delta m_{31}^2}{\Delta m_{21}^2} &= 19, & \sin^2 \theta_{12} &= 3 \times 10^{-4}, \\ \sin^2 \theta_{23} &= 1.0, & \sin^2 \theta_{13} &= 3 \times 10^{-10}. \end{aligned} \quad (35)$$

Thus, the mass squared difference ratio $\Delta m_{31}^2/\Delta m_{21}^2$ is compatible with the data, but the mixing angle θ_{12} (θ_{23}) is too small (too big). The sizes of $\sin^2 \theta_{12,23,13}$ vary from case to case, but they turn out to be either too big $\gtrsim 0.9$ or too small $\lesssim 0.1$ in most cases. Sometimes they are 0.3 or 0.5, but no case leads to the right values of the three mixing angles at the same time. That is the character of this class of assignments. Namely, we can fit the charged lepton mass ratios m_e/m_τ and m_μ/m_τ , and the neutrino mass squared difference ratio, $\Delta m_{31}^2/\Delta m_{21}^2$, but the mixing angles are either too big or too small. This character is shown in Table 3.

Assignment 4

In this class of assignments, we can fit the neutrino oscillation data as well as the mass ratios m_e/m_τ and m_μ/m_τ . For example, we take $(R_1^2, R_2^2) = (26, 21)$ in the following assignment,

$$\begin{aligned} (L_1, L_2, L_3) &= (\hat{T}_2^{(2)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,-1)}), & (N_1, N_2, N_3) &= (\hat{T}_2^{(2)}, \hat{T}_2^{(4,1)}, \hat{T}_2^{(4,-1)}), \\ (e_1^c, e_2^c, e_3^c) &= (\hat{T}_3^{(1)}, \hat{T}_3^{(2)}, \hat{T}_3^{(4,1)}), & (H_u, H_d) &= (\hat{T}_2^{(2)}, \hat{T}_1). \end{aligned} \quad (36)$$

Then, we can realize the experimental values of m_e/m_τ and m_μ/m_τ . Moreover, in this assignment we obtain

$$\begin{aligned} \frac{\Delta m_{31}^2}{\Delta m_{21}^2} &= 14, & \sin^2 \theta_{12} &= 0.38, \\ \sin^2 \theta_{23} &= 0.70, & \sin^2 \theta_{13} &= 6.3 \times 10^{-6}. \end{aligned} \quad (37)$$

Thus, this assignment can realize orders of six observables only by two parameters. Almost the same result can be derived from the following assignment,

$$\begin{aligned} (L_1, L_2, L_3) &= (\hat{T}_2^{(2)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,-1)}), & (N_1, N_2, N_3) &= (\hat{T}_2^{(2)}, \hat{T}_2^{(4,1)}, \hat{T}_2^{(4,-1)}), \\ (e_1^c, e_2^c, e_3^c) &= (\hat{T}_3^{(2)}, \hat{T}_3^{(3)}, \hat{T}_3^{(4,1)}), & (H_u, H_d) &= (\hat{T}_2^{(2)}, \hat{T}_1). \end{aligned} \quad (38)$$

There are also several other assignments leading to similar results with smaller values of $\Delta m_{31}^2/\Delta m_{21}^2$ in the range of

$$1 < \frac{\Delta m_{31}^2}{\Delta m_{21}^2} < 14. \quad (39)$$

In Table 3, the best fitting result is shown.

Assignment 5

In this class of assignments, we can fit only m_e/m_τ , m_μ/m_τ , and $\Delta m_{31}^2/\Delta m_{21}^2$ properly. For example, we take $(R_1^2, R_2^2) = (243, 419)$ in the following assignment,

$$\begin{aligned} (L_1, L_2, L_3) &= (\hat{T}_2^{(1)}, \hat{T}_2^{(2)}, \hat{T}_2^{(4)}), & (N_1, N_2, N_3) &= (\hat{T}_2^{(1)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,1)}), \\ (e_1^c, e_2^c, e_3^c) &= (\hat{T}_2^{(2)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,1)}), & (H_u, H_d) &= (\hat{T}_2^{(2)}, \hat{T}_2^{(4,1)}). \end{aligned} \quad (40)$$

Then, we can realize the proper mass ratios m_e/m_τ and m_μ/m_τ . On the other hand, we obtain

$$\begin{aligned} \frac{\Delta m_{31}^2}{\Delta m_{21}^2} &= 28, & \sin^2 \theta_{12} &= 1 \times 10^{-3}, \\ \sin^2 \theta_{23} &= 0.06, & \sin^2 \theta_{13} &= 3 \times 10^{-14}. \end{aligned} \quad (41)$$

Thus, the mass squared difference ratio $\Delta m_{31}^2/\Delta m_{21}^2$ is compatible with the data, but the mixing angles θ_{12} and θ_{23} are too small. The size of $\sin^2 \theta_{23}$ varies from case to case, but it turns out to be either too big ~ 0.9 or too small ~ 0.1 in any case. That is the character of this class of assignments. Namely, we can fit the charged lepton mass ratios m_e/m_τ and m_μ/m_τ , but θ_{12} is too small and θ_{23} is either too big or too small. This character is shown in Table 3.

As a result, we can realize the charged lepton mass ratios, the ratio of neutrino mass squared differences and the lepton mixing angles in certain cases of Assignment 4, but not in the other classes of assignments. Let us also note that the neutrino mass spectrum has normal hierarchy with vanishing or very small lightest neutrino mass in every case leading to reasonable value of $\Delta m_{31}^2/\Delta m_{21}^2$.

So far, we have assumed that all states correspond to the same fixed point on the 2D Z_3 orbifold. However, that case leads to $(Y_\nu)_{33} = O(1)$ which is not realistic. We need a suppression factor of $O(10^{-12} - 10^{-13})$ to fit the overall

Class	$\Delta m_{31}^2/\Delta m_{21}^2$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{23}$	$\sin^2 \theta_{13}$
Assignment 1	~ 100	$\lesssim 10^{-5}$	$\lesssim 10^{-2}$	$\lesssim 10^{-7}$
Assignment 2	~ 100	$\gtrsim 10^{-5}$	$\gtrsim 10^{-2}$	$\gtrsim 10^{-7}$
Assignment 3	$\gtrsim 1.4$			
Assignment 4	14	0.38	0.70	6.3×10^{-6}
Assignment 5	~ 28	≤ 0.09		$\lesssim 10^{-2}$
Experimental values	27	0.30	0.50	0.000

Table 3: Characteristics of each assignment 1–5 in the Dirac neutrino case. Typical behavior of each value is described for combinations resulting in relatively good fits in a given assignment except Assignment 4. The row corresponding to Assignment 4 shows the best fit. We omit m_e/m_τ and m_μ/m_τ because they can be fit in all the assignments.

magnitude. In Assignment 4, the neutrino Yukawa couplings originate from $\hat{T}_2\hat{T}_2\hat{T}_2$ couplings. On the 2D Z_3 orbifold, Yukawa couplings corresponding to three different fixed points are allowed, too. Here, we assume that L_i , N_i and H_u sit at three different fixed points on the 2D Z_3 orbifold. In this case, the neutrino Yukawa couplings universally have an exponential suppression factor like eqs.(21), (22) and (23). Then, we can derive $(Y_\nu)_{33} = O(10^{-12})$, when we take the radius of the third torus as $R_3^2 = O(1000)$. That implies that the compactification scale is smaller by $O(10^{-1} - 10^{-2})$ than the string scale. (See for phenomenological aspects of such a scenario e.g. Refs. [16, 17, 18, 19].)

3.2 Seesaw scenario

In this subsection, considering the seesaw scenario of the neutrino mass matrix, we systematically carry out the same analysis as in the previous subsection. The relevant terms in the superpotential are

$$W \supset H_u L_i (Y_\nu)_{ij} N_j - H_d L_i (Y_e)_{ij} e_j^c - \frac{1}{2} N_i (M_N)_{ij} N_j, \quad (42)$$

where the third term is the right-handed majorana neutrino mass term. After replacing the Higgs fields with their VEVs, we obtain the mass terms,

$$W \supset -\nu_i (m_D)_{ij} N_j - e_i (m_e)_{ij} e_j^c - \frac{1}{2} N_i (M_N)_{ij} N_j. \quad (43)$$

We assume $M_N \gg m_D$, and in this case the heavy fields N_i can be integrated out. Then the lighter degrees of freedom have the effective mass terms,

$$W \supset -\frac{1}{2}\hat{\nu}_i(\hat{m}_\nu)_{ij}\hat{\nu}_j, \quad (44)$$

where

$$\hat{m}_\nu = -m_D M_N^{-1} m_D^T. \quad (45)$$

In string models, the natural order of mass is the string scale. On the other hand, the mass scale of M_N is phenomenologically required to be an intermediate scale between the string scale and the weak scale. Such an intermediate scale may be obtained by VEVs of some fields, as a comment will be given at the end of this section. However, such a scenario is quite model-dependent. Thus, we assume that M_N is proportional to the identity for simplicity. Under such an assumption, we obtain

$$\hat{m}_\nu = M_N^{-1} m_D m_D^T = (U_\nu) \hat{m}_\nu^{\text{diag}} (U_\nu)^T, \quad (46)$$

$$\hat{m}_\nu^{\text{diag}} = M_N^{-1} (m_D^{\text{diag}})^2, \quad (47)$$

where U_ν is the same diagonalizing matrix as the one in the Dirac neutrino mass scenario. Hence, the mixing angles are almost the same as those in the Dirac neutrino mass scenario. On the other hand, neutrino mass hierarchy is enhanced compared with that in the Dirac neutrino mass scenario. The results are as follows.

Assignment 1

Like the Dirac neutrino mass scenario, we can fit only the charged lepton mass ratios properly. The neutrino mass squared difference ratio is too large, and mixing angles are too small. Those results are shown in Table 4 .

Assignment 2

We can fit only the charged lepton mass ratios. The neutrino mass squared difference ratio is too large, and mixing angles are too small, as shown in Table 4.

Assignment 3

We can fit the charged lepton mass ratios. However, the neutrino mass squared difference ratio turns out to be rather large $\gtrsim 50$ except for the case that gives $\Delta m_{31}^2 / \Delta m_{21}^2 = 2$. Also, the mixing angles are typically either too big or too small.

Class	$\Delta m_{31}^2/\Delta m_{21}^2$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{23}$	$\sin^2 \theta_{13}$
Assignment 1	~ 6000	$\lesssim 10^{-5}$	$\lesssim 10^{-2}$	$\lesssim 10^{-7}$
Assignment 2	~ 7000	$\lesssim 10^{-5}$	$\lesssim 10^{-2}$	$\lesssim 10^{-7}$
Assignment 3	$\gtrsim 2$			
Assignment 4	29	0.32	0.48	3.6×10^{-6}
Assignment 5	~ 28	≤ 0.09		$\lesssim 10^{-2}$
Experimental values	27	0.30	0.50	0.000

Table 4: Characteristics of each assignment 1–5 in the seesaw case. Typical behavior of each parameter is described for combinations resulting in relatively good fits in a given assignment. The row corresponding to Assignment 4 shows the best fit. We omit m_e/m_τ and m_μ/m_τ because they can be fit in all the assignments.

Assignment 4

Like the Dirac neutrino mass scenario, we can properly fit both the charged lepton mass ratios and the neutrino oscillation data. For example we take $(R_1^2, R_2^2) = (23, 26)$ in the following assignment,

$$\begin{aligned}
(L_1, L_2, L_3) &= (\hat{T}_2^{(1)}, \hat{T}_2^{(2)}, \hat{T}_2^{(4,1)}), & (N_1, N_2, N_3) &= (\hat{T}_2^{(2)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,1)}), \\
(e_1^c, e_2^c, e_3^c) &= (\hat{T}_3^{(1)}, \hat{T}_3^{(2)}, \hat{T}_3^{(4,1)}), & (H_u, H_d) &= (\hat{T}_2^{(2)}, \hat{T}_1).
\end{aligned} \tag{48}$$

Then, we can realize the experimental values of m_e/m_τ and m_μ/m_τ . Furthermore, in this assignment we obtain

$$\begin{aligned}
\frac{\Delta m_{31}^2}{\Delta m_{21}^2} &= 29, & \sin^2 \theta_{12} &= 0.32, \\
\sin^2 \theta_{23} &= 0.48, & \sin^2 \theta_{13} &= 3.6 \times 10^{-6}.
\end{aligned} \tag{49}$$

Thus, this assignment can realize orders of six observables only by two parameters. This class of assignments include several other cases leading to almost the same results.

Assignment 5

We can fit the charged lepton mass ratios and $\Delta m_{31}^2/\Delta m_{21}^2$. However, θ_{12} is too small and θ_{23} is either too big or too small, as shown in Table 4.

As a result, we can realize the charged lepton mass ratios, the ratio of neutrino mass squared differences and the lepton mixing angles in certain

cases of Assignment 4, when we assume that the right-handed majorana neutrino mass matrix M_N is proportional to the identity matrix. As was in the Dirac scenario, the neutrino mass spectrum has normal hierarchy with vanishing lightest neutrino mass. The neutrino Yukawa coupling $(Y_\nu)_{33}$ is $O(1)$. We need $M_N = O(10^{15})$ GeV for the seesaw mechanism to produce the correct neutrino mass scale.

In the above analysis, we have assumed $(M_N)_{ij} = M_N \delta_{ij}$ for simplicity. It is nontrivial to realize such a right-handed neutrino majorana mass matrix. Majorana mass terms can be generated by the $(n + 2)$ -point couplings

$$Y_{ij} N_i N_j (\phi_1 \cdots \phi_n), \quad (50)$$

after ϕ_i develop their VEVs.⁷ Thus, the right-handed neutrino mass matrix depends on VEVs of scalar fields and the selection rule for higher dimensional operators [31], that is, quite a model-dependent feature. In certain cases, we may obtain $(M_N)_{ij} = M_N \delta_{ij}$. However, we, in general, obtain nontrivial majorana matrix $(M_N)_{ij}$ if it is derived from $Y_{ij} N_i N_j (\phi_1 \cdots \phi_n)$ through VEVs of $(\phi_1 \cdots \phi_n)$. In such a case, assignments other than assignment 4 might lead to realistic results. Thus, it would be interesting to study such case somehow systematically. However, that is beyond our scope. We leave it for future study.

4 Conclusion

We have systematically studied the possibility for realizing lepton masses and mixing angles by use of only renormalizable couplings derived from Z_6 -I heterotic orbifold models. We have assumed one pair of up and down type Higgs fields. We have found Assignment 4 has such a possibility in both the Dirac neutrino mass scenario and the simple seesaw scenario where the right-handed majorana mass matrix is proportional to identity. The resulting neutrino mass spectrum shows normal hierarchy.

It is quite non-trivial to fit six observables only by two parameters R_1 and R_2 . However, how to stabilize these moduli at proper values is an important issue to study further. Moreover, if F-components of moduli fields contribute to SUSY breaking, models corresponding to Assignment 4 may show a certain pattern of soft SUSY breaking parameters. It is interesting to study their effects on flavor violation. (See e.g. [32] and references therein.)

⁷Such VEVs may be given e.g. through anomalous U(1) breaking [30].

In addition to the real parts R_i of the moduli T_i , mass matrices also depend on imaginary parts of T_i , although we have fixed them to vanish in our analysis. When such imaginary parameters are included, other types of assignments may lead to realistic results. Thus, it is interesting to carry out the same analysis including imaginary parts of T_i .

We have systematically studied all possible assignments of leptons and Higgs fields to twisted states, but not constructed an explicit heterotic orbifold model by fixing gauge shifts and Wilson lines. It is also important to construct explicitly heterotic models corresponding to Assignment 4.

In addition, it is interesting to extend our analysis to other Z_N and $Z_N \times Z_M$ orbifold models including the quark sector. In principle, systematical studies as in this paper are possible for other orbifold models. Such a study will be done elsewhere.

Although we have discussed the possibility for obtaining realistic Yukawa matrices from only stringy renormalizable couplings in this paper, another possibility for realistic Yukawa matrices is that higher dimensional operators play roles to generate effective Yukawa couplings after symmetry breaking. (See e.g. [33].) Particle mixing through symmetry breaking is also one possibility [34]. It would also be interesting to study such a possibility somehow systematically. In the former case, it is quite important to study symmetries to control higher dimensional operators in string models.

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