Lepton Flavor Violation as a Probe of Quark-Lepton Unification

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Abstract

The recent measurements of the solar neutrino mixing angle θ_{sol} and the Cabibbo mixing angle θ_C reveal a surprising relation, $\theta_{sol} + \theta_C \simeq \frac{\pi}{4}$. Interpreting this empirical relation as a support of the quark-lepton unification, we find that the PMNS mixing matrix can be decomposed into a CKM-like matrix and maximal mixing matrices, which can give profound implications on the quark-lepton unification. We explore a possibility to probe the implication of quark-lepton unification by considering the relative sizes of branching ratios for the lepton flavor violating radiative decay processes, $l_i \to l_j \gamma$, in the context of the supersymmetric standard model with heavy right-handed Majorana neutrinos.

Neutrino studies will enter a new era when the MINOS experiment starts firing a neutrino beam toward the Soudan mine in March 2005. Until now, while the atmospheric neutrino deficit still points toward a maximal mixing between the tau and muon neutrinos, however the solar neutrino problem favors a not-so-maximal mixing between the electron and muon neutrinos. Surprisingly, it has recently been noted that the solar neutrino mixing angle θ_{sol} required for a solution of the solar neutrino problem and the Cabibbo angle θ_C reveal a striking relation [1]

$$\theta_{sol} + \theta_C \simeq \frac{\pi}{4}, \tag{1}$$

which is satisfied by the experimental results within a few percent accuracy $\theta_{sol} + \theta_C = 45.4° \pm 1.7°$ [2, 3, 4]. This quark-lepton complementarity (QLC) relation (1) has been simply interpreted as an evidence for certain quark-lepton symmetry or quark-lepton unification as shown in Refs. [1, 5, 6, 7].

To effectively describe the deviation from maximal mixing of solar neutrino as well as a small mixing element U_{e3} and possible deviation from maximal mixing of atmospheric neutrino, three possible combinations of maximal mixing and a certain mixing matrix $U(\lambda)$ parameterized in terms of a small parameter $\lambda \sim \sin\theta_C$ have been proposed as parametrization of U_{PMNS} [8, 9, 10, 11]:

$$
\begin{align}
\text{(a)} \quad & U^\dagger(\lambda)U_{\text{bimax}}, \notag \\
\text{(b)} \quad & U_{\text{bimax}}U^\dagger(\lambda), \tag{2} \\
\text{(c)} \quad & U_{23}^m U^\dagger(\lambda)U_{12}^m. \notag
\end{align}
$$

Here U_{bimax} corresponds to the bi-maximal lepton mixing matrix [12], and U_{23}^m, U_{12}^m denote the rotation matrices with (2,3) and (1,2) maximal mixing, respectively. Even though the present data is not sufficient to determine which combination can give correct flavor structure in lepton sectors, it is very important to single them out because the QLC relation is strongly correlated to each combination differently. As extensively studied in [5], the QLC relation can be derived from the parametrization given above but up to some corrections. These corrections to the QLC relation can be compensated with renormalization effects [7].

In this Letter, we will show that among possible forms of $U(\lambda)$ which are consistent with the neutrino experimental results, the "CKM-like" form of $U(\lambda)$ has profound implication on the quark-lepton unification. Motivated by this observation, we will study the implication

of the parametrization composed of bi-maximal mixing and U_{CKM} reflecting quark-lepton unification by considering the lepton flavor violating (LFV) decays particularly in the context of supersymmetric standard model (SSM). We also examine a possibility to differentiate the above combinations by considering the relative size of branching ratios of the radiative LFV decays, $Br(l_i \to l_j\gamma)$ $(i, j = e, \mu, \tau)$. While the LFV processes have tiny rates in the minimal extensions of the standard model (SM) with heavy right-handed Majorana neutrinos, the supersymmetric extensions of the SM can lead to sizable effects on the LFV processes due to new sources of lepton flavor violation. As is well known, the LFV decays in SSM can be caused by the misalignment of lepton and slepton mass matrices [13] and the branching ratios of the LFV decays depend on the specific structure of the neutrino Dirac Yukawa matrix Y_ν. Therefore, we expect that a specific structure of Y_ν reflecting quark-lepton unification can lead to distinctive predictions for the branching ratios of the LFV decays. However, the branching ratios of the LFV decays in SSM strongly depend on several parameters which make it difficult to probe the structure of Y_ν. Instead of considering the branching ratios of each LFV process, we can rely on the relative size of $Br(l_i \to l_j\gamma)$ among the three different flavors, because the relative size is almost free from arbitrary supersymmetric parameters. These ratios of $Br(l_i \to l_j\gamma)$ can be useful to probe the structure of Y_ν with the help of the parametrization of U_{PMNS} given in Eq. (2). In particular, we expect that a hierarchical structure of Y_ν predicted by quark-lepton unification may be responsible for the hierarchy of $Br(l_i \to l_j\gamma)$ if they are observed in the future experiments. In such a way, the quark-lepton unification could be tested from the determination of the relative size of the branching ratios in future experiments.

Let us begin by considering how the parametrization given by the forms of Eq. (2) can be realized in the framework of the quark-lepton unification. For our purpose, it is useful to work in a basis where the quark and lepton Yukawa matrices are related to each other by a certain symmetry. In general, the quark Yukawa matrices Y_u, Y_d are given by $Y_u = U_u Y_u^{diag} V_u^\dagger$, $Y_d = U_d Y_d^{diag} V_d^\dagger$, from which the observable CKM quark mixing matrix is described as $U_{\text{CKM}} = U_u^\dagger U_d$. For the lepton sector, we consider the following leptonic superpotential, which implements the seesaw mechanism:

$$W_{\text{lepton}} = Y_l \widehat{L}\widehat{l_L^c} \widehat{H}_d + Y_\nu \widehat{L}\widehat{N}_L^c \widehat{H}_u - \frac{1}{2}\widehat{N}_L^{cT} M_R \widehat{N}_L^c, \qquad (3)$$

where the family indices have been suppressed and $\widehat{L}_j, \ j = e, \mu, \tau \equiv 1, 2, 3$, represent the

chiral super-multiplets of the $SU(2)_L$ doublet lepton fields, $\widehat{N}^c_{jL}, \widehat{l}^c_{jL}$ are the super-multiplet of the $SU(2)_L$ singlet neutrino and charged lepton field, respectively. In the superpotential W_{lepton}, M_R is the heavy Majorana neutrino mass matrix. Y_l and Y_ν are the 3×3 charged lepton and neutrino Dirac Yukawa matrices, respectively and can be parameterized as

$$Y_l = U_l Y_l^{diag} V_l^\dagger, \quad Y_\nu = U_0 Y_\nu^{diag} V_0^\dagger. \tag{4}$$

We note that in the framework of the minimal unification and the symmetric basis where the quarks and leptons are interrelated, M_R is generally not diagonal. The light neutrino mass matrix can be generated through the seesaw mechanism after the breaking of the electroweak symmetry as

$$M_\nu = \left(U_0 M_{\text{Dirac}}^{diag} V_0^\dagger \right) M_R^{-1} \left(V_0^* M_{\text{Dirac}}^{diag} U_0^T \right), \tag{5}$$

where $M_{\text{Dirac}} = Y_\nu v_u / \sqrt{2}$ with $v_u = v \sin\beta$. We can then rewrite M_ν as follows

$$M_\nu = U_0 V_M M_\nu^{diag} V_M^T U_0^T, \tag{6}$$

where V_M represents the diagonalizing matrix of

$$M_{\text{Dirac}}^{diag} V_0^\dagger M_R^{-1} V_0^* M_{\text{Dirac}}^{diag}.$$

Then, the observable PMNS mixing matrix can be written as

$$U_{\text{PMNS}} = U_l^\dagger U_\nu = U_l^\dagger U_0 V_M. \tag{7}$$

Now, let us consider how U_{PMNS} given by Eq. (7) can be related with U_{CKM} in the context of quark-lepton unification. The quark-lepton unification based on the minimal $SU(5)$ leads to the following simple relations,

$$Y_e = Y_d^T, \quad Y_u = Y_u^T. \tag{8}$$

Then, we deduce that $U_l = V_d^*$ from which

$$U_{\text{PMNS}} = V_d^T U_0 V_M. \tag{9}$$

As one can easily see, the contribution of U_{CKM} may appear in U_{PMNS} if we further assume that the Yukawa matrix of the up-type quark sector is related with that of Dirac-type

neutrinos such as $Y_\nu = Y_u$ which can be realized in some larger unified gauge group such as $SO(10)$. Then, the lepton flavor mixing matrix can be written as

$$U_{\text{PMNS}} = V_d^T U_d U_{\text{CKM}}^\dagger V_M. \tag{10}$$

In addition, requiring symmetric form of the down-type quark Yukawa matrix, we obtain

$$U_{\text{PMNS}} = U_{\text{CKM}}^\dagger V_M, \tag{11}$$

where the mixing matrix V_M should have two almost maximal mixings so as to account for the solar and atmospheric neutrino oscillations. This expression for U_{PMNS} corresponds to the parametrization given by Eq. (2-a). On the other hand, in order to achieve the parametrization given by Eq. (2-b), one should take V_M to be identity matrix and the product of two matrix $V_d^T U_d$ in Eq. (10) should give bi-maximal mixing pattern. Since the left-handed rotation matrix U_d for down type quark can be almost diagonal to leading order, V_d^T should have almost bi-maximal mixing form, which can be achieved in the so-called lopsided form of Yukawa matrix. The case given by Eq. (2-c) can also be achieved by taking $V_d^T \simeq U_{23}^m$ and $V_M \simeq U_{12}^m$ in Eq. (10). In such ways, U_{PMNS} can be connected with U_{CKM} in the framework of the quark-lepton unification.

Although the minimal quark-lepton unification can lead to an elegant relation between U_{PMNS} and U_{CKM} as shown above, it indicates undesirable mass relations between quarks and leptons at the GUT scale such as $m_d^{diag} = m_l^{diag}$. Thus, we need to modify the simple relations between quark and lepton Yukawa matrices so as to achieve desirable mass relations. From the well known empirical relation

$$|V_{us}| \simeq \sqrt{\frac{m_d}{m_s}} \simeq 3\sqrt{\frac{m_e}{m_\mu}}, \tag{12}$$

it has been shown that the $U(\lambda)$ in Eq. (2) should have the CKM-like form but with the replacement of λ with $\lambda/3$ as shown in Refs. [7, 14], which can be obtained by introducing the Higgs sector transforming under the representation **45** of $SU(5)$ or **126** of $SO(10)$ [15].

Now, let us consider how the relative ratio of $Br(l_i \rightarrow l_j \gamma)$ can be connected with the structure of the neutrino Dirac Yukawa matrix, which is constructed from the grand unification scenario above. It is well known that the RG running induces off-diagonal terms in the slepton mass matrix even for the case of universal slepton masses at GUT scale [16]:

$$m_{\tilde{l}_{ij}}^2 \simeq -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)(Y_\nu' Y_\nu'^\dagger)_{ij} \log \frac{M_G}{M_X}, \tag{13}$$

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where m_0, A_0 are universal soft scalar mass and soft trilinear A parameter, and Y'_ν is defined in the basis where the charged lepton Yukawa matrix and the heavy Majorana mass matrix are real and diagonal. Here M_G and M_X denote the GUT scale and the characteristic scale of the right-handed neutrinos at which off-diagonal contributions are decoupled [16], respectively. Thus, one can expect that some specific form of Y'_ν is crucial to estimate the sizes of LFV processes which are caused by non-diagonal slepton mass matrix. First of all, let us consider the parametrization (a). It follows from Eqs. (2-a,11) that

$$Y'_\nu \equiv U^\dagger_{\rm CKM} Y^D_\nu V^T_0 V_R, \tag{14}$$

where Y^D_ν is diagonal neutrino Dirac Yukawa matrix and V_R is the rotation matrix of the heavy Majorana neutrino mass matrix M_R in Eq. (3). From Eq. (14), the term $(Y'_\nu Y'^\dagger_\nu)$ becomes

$$Y'_\nu Y'^\dagger_\nu = U^\dagger_{\rm CKM} (Y^D_\nu)^2 U_{\rm CKM}. \tag{15}$$

The induced off-diagonal terms in the slepton mass matrix can be a source of the lepton flavor violation in SSM and they can yield sizable contributions to LFV decays, $l_i \to l_j \gamma$. The contribution to the branching ratios of the LFV decays due to the slepton mass term is roughly given by

$$Br(l_i \to l_j \gamma) \simeq \frac{\alpha^3}{G^2_F} \tan^2 \beta \left| \frac{m^2_{\tilde{l}_{ij}}}{m^4_S} \right|^2, \tag{16}$$

where m_S is a supersymmetric mass scale. Let us define Y^D_ν in a hierarchical form expressed in terms of the power of λ :

$$Y^D_\nu \equiv Y_3 \begin{pmatrix} \lambda^{n_1} & & \\ & \lambda^{n_2} & \\ & & 1 \end{pmatrix}. \tag{17}$$

For the quark-lepton unification, $Y_3 = m_t/v_u$ and the powers of λ are given by $n_1 = 8, n_2 = 4$ so as to be the same hierarchy of up-type quark sector at high energy scale.

The term $(Y'_\nu Y'^\dagger_\nu)$ is roughly given to leading order as

$$Y'_\nu Y'^\dagger_\nu \sim \left(\frac{m_t}{v_u} \right)^2 \times \tag{18}$$

$$\begin{pmatrix} \lambda^{2n_1} + \lambda^{2n_2+2} + \lambda^6 & \lambda^{2n_1+1} - \lambda^{2n_2+1} - \lambda^5 & \lambda^3 \\ \lambda^{2n_1+1} - \lambda^{2n_2+1} - \lambda^5 & \lambda^{2n_1+2} + \lambda^{2n_2} + \lambda^4 & -\lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}.$$

Inserting this into Eq. (16), we can estimate how large $Br(l_i \to l_j\gamma)$ could be by fixing the parameters m_S and M_X. Instead of considering the values of $Br(l_i \to l_j\gamma)$, we focus on the ratio of $Br(l_i \to l_j\gamma)$. The ratio of $Br(l_i \to l_j\gamma)$ only depends on $Y'_\nu Y'^\dagger_\nu$, and thus from Eqs. (13,16,18), we can simply obtain the ratio:

$$Br(\mu \to e\gamma) : Br(\tau \to e\gamma) : Br(\tau \to \mu\gamma)$$
$$\simeq \quad (-\lambda^{2n_1-1} + \lambda^{2n_2-1} + \lambda^3)^2 : \lambda^2 : 1. \tag{19}$$

When Y^D_ν is the same as Y^D_u due to the quark-lepton unification, we predict that the ratio given in Eq. (19) should be $(\lambda^6 : \lambda^2 : 1)$. But, we note that this ratio may not necessarily indicate quark-lepton unification just considered because we can obtain the same ratio in the limit of large values of n_1, n_2. However, if the ratio of $Br(l_i \to l_j\gamma)$ is measured to be inconsistent with the prediction of the ratio given above, it may indicate $Y^D_\nu \neq Y^D_u$.

For the case of realistic quark-lepton unification satisfying Eq. (12), the terms $(Y'_\nu Y'^\dagger_\nu)_{i,j}$ for $(i,j) = (2,1), (3,1), (3,2)$ are roughly given as

$$(Y'_\nu Y'^\dagger_\nu)_{21} \simeq \frac{\lambda^5}{6} + \frac{\lambda^{1+2n_1}}{3} - \frac{\lambda^{1+2n_2}}{3}$$
$$(Y'_\nu Y'^\dagger_\nu)_{31} \simeq \frac{\lambda^3}{6} - \frac{\lambda^{3+2n_1}}{2} + \frac{\lambda^{3+2n_2}}{3}$$
$$(Y'_\nu Y'^\dagger_\nu)_{32} \simeq \lambda^2 - \frac{\lambda^{4+2n_1}}{6} - \lambda^{2+2n_2}. \tag{20}$$

Then, the ratio of $Br(l_i \to l_j\gamma)$ among the three different flavors is

$$Br(\mu \to e\gamma) : Br(\tau \to e\gamma) : Br(\tau \to \mu\gamma)$$
$$\simeq \quad (\lambda^{2n_1} - \lambda^{2n_2} + \lambda^4)^2 : \lambda^4 : 1, \tag{21}$$

in order of magnitude estimation. For $n_1 = 8, n_2 = 4$, the ratio becomes $\lambda^8 : \lambda^4 : 1$. Therefore, we may confirm the validity or breaking of the quark-lepton unification through the measurements of the ratios of $Br(l_i \to l_j\gamma)$. As can be seen from Eq. (18), the important elements in U_{CKM} which actually determine the hierarchy among $Br(l_i \to l_j\gamma)$ are $(U_{\mathrm{CKM}})_{13}$ and $(U_{\mathrm{CKM}})_{23}$. For more precise predictions of the relative branching ratios, it is urgently required to determine $(U_{\mathrm{CKM}})_{13}$ and $(U_{\mathrm{CKM}})_{23}$ experimentally with better accuracy. Note that the lepton mixing matrix given by Eq. (11) leads to a new QLC relation,

$$(U_{\mathrm{PMNS}})_{e3} = [-\lambda + (U_{\mathrm{CKM}})_{31}]/\sqrt{2}. \tag{22}$$

Therefore, we are led to further confirm or discard quark-lepton unification through the measurement of $(U_{\mathrm{CKM}})_{31}$ and $(U_{\mathrm{PMNS}})_{e3}$. Note that similar to the QLC relation between θ_{sol} and θ_C, we can get another QLC relation between the mixing angle θ_{atm} and $(\theta_{23})_{\mathrm{CKM}}$,

$$\theta_{atm} + (\theta_{23})_{\mathrm{CKM}} \simeq \pi/4. \tag{23}$$

Similar to the parametrization (2-a), we can easily estimate the relative ratios of $Br(l_i \rightarrow l_j\gamma)$ for the parameterizations in (2-b) and (2-c). In these cases, the term $Y'_\nu Y'^\dagger_\nu$ becomes

$$Y'_\nu Y'^\dagger_\nu = \begin{cases} U_{\mathrm{bimax}} U^\dagger_{\mathrm{CKM}} (Y^D_\nu)^2 U_{\mathrm{CKM}} U^\dagger_{\mathrm{bimax}} & \text{(2-b),} \\ U^m_{23} U^\dagger_{\mathrm{CKM}} (Y^D_\nu)^2 U_{\mathrm{CKM}} U^{m\dagger}_{23} & \text{(2-c).} \end{cases} \tag{24}$$

Imposing the hierarchy of Y^D_ν given by Eq. (17), the relative ratios of $Br(l_i \rightarrow l_j\gamma)$ become

$$Br(\mu \rightarrow e\gamma) : Br(\tau \rightarrow e\gamma) : Br(\tau \rightarrow \mu\gamma)$$
$$\simeq \quad \lambda^4 : \lambda^4 : 1 \ \ \text{(2-b),} \qquad \lambda^6 : \lambda^6 : 1 \ \ \text{(2-c).} \tag{25}$$

From the predictions (19,25), one can see that experimental determination of the relative ratios of $Br(l_i \rightarrow l_j\gamma)$ can differentiate the parameterizations of the quark-lepton unification if the empirical QLC relations indeed indicate the quark-lepton unification.

We note that the RG-induced off-diagonal terms in the slepton mass matrix is more precisely given by [17]

$$m^2_{\tilde{l}_{ij}} \simeq -\frac{1}{8\pi^2}(3m^2_0 + A^2_0) \left(Y'_{\nu ik} \log \frac{M_G}{M_{R_k}} Y'^\dagger_{\nu kj} \right). \tag{26}$$

In this expression, we see that the prediction of $Br(l_i \rightarrow l_j\gamma)$ depends on the hierarchy of the heavy Majorana neutrino mass eigenvalues M_{R_k}. However, we note that the hierarchy is not arbitrary but derived from seesaw formulae if we fix a light neutrino mass m_{ν_1} in our framework. According to our numerical estimation on the relative ratios of $Br(l_i \rightarrow l_j\gamma)$ based on Eq. (26), the hierarchical patterns given in Eqs. (19,21,25) are almost kept, as long as $m_{\nu_1} \geq 10^{-5}$ eV. This is due to the mild hierarchy of $\log \frac{M_G}{M_{R_k}}$.

In summary, interpreting the surprising empirical relation, $\theta_{sol} + \theta_C \simeq \frac{\pi}{4}$, as a support of the quark-lepton unification, we find that the PMNS mixing matrix can be parameterized by a CKM-like matrix and maximal mixing matrices in various ways. Each parametrization may imply very different fundamental flavor structure. We have shown that the various parameterizations of U_{PMNS} with regard to quark-lepton unification would give very different

and profound implication to the radiative leptonic decays, $l_i \to l_j\gamma$, in the context of SSM. Therefore, by measuring the relative size of the radiative decay branching ratios, we will be able to pin down the U_{PMNS} parametrization, assuming the quark-lepton unification. There have been proposed experiments [18] to measure these radiative decays. The proposal in this Letter can soon be tested for the quark-lepton unification.

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