

Binding Energies and Dissociations of J/ψ , Υ , and Toponium in Quark-Gluon Plasma

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Abstract

In this work, we calculate the binding energies of the J/ψ , the Υ , and the Toponium in quark-gluon plasma. We also calculate the temperature of the quark-gluon plasma at which these quarkonia dissociate. We approach the problem from a quantum mechanical point of view. We use the variational method and the screening Debye potential in order to obtain these binding energies.

I. Introduction

Since heavy quarks at hadron colliders can be produced, the quarkonia such as $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$ can also be produced. Experiments at LHC will eventually reach an energy that will lead to a large number of $t\bar{t}$ events. When we collide heavy ions to produce the quark-gluon plasma, we will produce the quarkonia in the process, which some of it will get embedded inside the quark-gluon plasma and dissociate due to the hot temperature. The binding energies of the quarkonia will get smaller when the quarkonia get inserted in the quark-gluon plasma. For example, the binding energy of free Υ is about 800 MeV, but it decreases to 10 MeV once we immerse the Υ in a 150 MeV QGP [1]. The quarkonia get pumped up, so to speak, to a lower binding energy due screening. We shouldn't just look at the effect of screening on the binding energy, we should also consider the effect of the absorption of gluon by the quark, and the effect of the collision $Q+g \rightarrow Q+g$. In our model we are looking at the effect of screening only, and we agree that another mechanism would involve absorption by gluons, or collision of a gluon and a quark. Other mechanism could be the subject of another investigation.

II Description

In the high temperature deconfined phase, the quark-antiquark free energy $V_{q\bar{q}}$, which is the Debye potential with inverse screening length m_{el} , is given by [2]:

$$V_{q\bar{q}} = -\frac{4}{3} \frac{\alpha_s}{r} e^{-m_{el}r} \quad (1)$$

Using the variational method the binding energy is

$$E = \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} \quad (2)$$

Since the Debye potential is spherically symmetric, and similar to Coulomb potential, we choose a hydrogen-like trial wave function

$$|\psi\rangle = N e^{-r/a_T}. \quad (3)$$

This trial wave function gives us a normalization term

$$\langle \psi | \psi \rangle = \pi N^2 a_T^3,$$

where N is just a constant and a_T is a variational parameter. Calculating the value

$$\langle \psi | H | \psi \rangle = 4\pi N^2 \left[\frac{a_T}{4m_Q} - \frac{4}{3} \alpha_s \frac{a_T^2}{(2 + m_{el}a_T)^2} \right],$$

we find the variational estimate of the binding energy of the two quarks in the meson to be:

$$E = \frac{1}{m_Q a_T^2} - \frac{16}{3} \alpha_s \frac{1}{a_T (2 + m_{el}a_T)^2}. \quad (4)$$

One can see that the binding energy in Eq.(4) is written in terms of the observables, m_Q, m_{el}, α_s , and the parameter a_T which we can eliminate through the variational method. Taking the derivative of the energy with respect to the parameter a_T and making it equal to zero, we get:

$$\frac{dE}{da_T} = \frac{32\alpha_s m_{el}}{3a_T(2 + a_T m_{el})^3} + \frac{16\alpha_s}{3a_T^2(2 + a_T m_{el})^2} - \frac{2}{a_T^3 m_Q} = 0.$$

Minimizing the energy, we can find the parameter a_T and then plug it back into the energy. But the binding energy is related to the inverse screening length, which is related to the temperature. There is a relationship between the inverse screening length, the number of flavors in the quark-gluon plasma, and the temperature of the quark-gluon plasma. If we increase the temperature of the plasma, the quarks and the gluons will become very active and some of them will eventually get in between the quark and the antiquark of the meson and cause more screening. So

the screening increases with the temperature, and the quark and the antiquark can barely “feel” each other. This relationship can be algebraically noted [2] in the following equation

$$m_{el}^2 = \frac{1}{3}g^2(N + \frac{N_f}{2})T^2, \quad (5)$$

where $N = 3$ from the $SU(N)$ group, g is the dimensional coupling constant of the field strength, $F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - gf_{abc}A_b^\mu A_c^\nu$, and $N_f = 3$ is the number of light flavors, which are the up, the down, and the strange quarks.

The uncertainties in using the leading perturbation of Eq.(5) relies on the second order, or the leading correction. There has been an interest in computing the leading correction to Eq.(5)[3,4]. It is known that this correction cannot be computed perturbatively in non-Abelian gauge[5]. The $O(g^2T)$ correction to the inverse screening length receives contributions from fundamentally non-perturbative physics associated with the interactions, at high temperature, of magnetic gluons with momenta of order g^2T [6].

We will also use the temperature-dependent running coupling constant of QCD, which is given by [3]

$$\frac{g^2}{4\pi} = \frac{12\pi}{(11N - 2N_f)\ln(T^2/\Lambda^2)}. \quad (6)$$

This equation explicitly displays asymptotic freedom: $g^2 \rightarrow 0$ as $T \rightarrow \infty$. We notice that there is no intrinsic coupling “constant” on the right hand side of this equation. The only free parameter of the theory on the right side of this equation is the QCD energy scale, Λ , whose numerical value is dependent on the gauge and on the renormalization scheme chosen. If we choose the QCD energy scale to be $\Lambda = 50$ MeV, and $N_f = 3$, and $N = 3$ we get

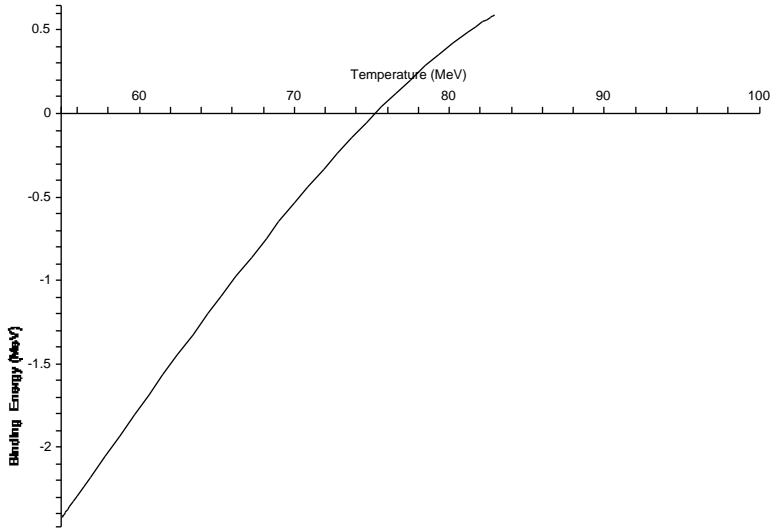
$$\frac{g^2}{4\pi} = \frac{6\pi}{27\ln(T/50\text{MeV})}. \quad (7)$$

Now combining Eq.(5) and Eq.(7) we get a simple relationship between the inverse screening length and the temperature.

$$m_{el} = \frac{2\pi T}{\sqrt{3\ln(T/50)}}. \quad (8)$$

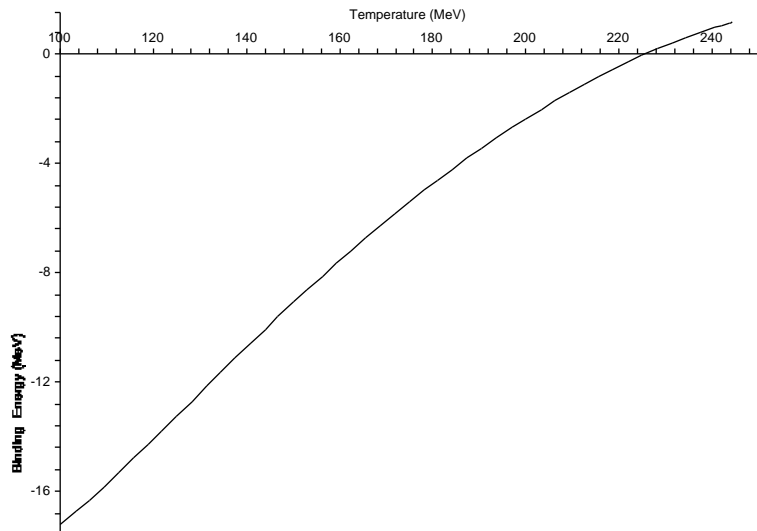
Since the inverse-screening length, m_{el} , in Eq.(8) is found to be written in terms of the temperature only, and the quarkonia binding energy of Eq.(4) is found to be written in terms of m_{el} , we can now, to certain extent, calculate the binding energy of any specific quarkonium we choose, knowing the temperature of the medium. We have other observables in the binding energy, of course, such as the mass of the quark and the coupling constant. But these observables are necessary and need to be there. For example, the mass of the quark in Eq.(4) is tied to the quarkonia we choose.

Using $\alpha_s(m_c) = 0.30 \pm 0.02$ [7] for the J/ψ and Plotting the binding energy in terms of the temperature, we get:



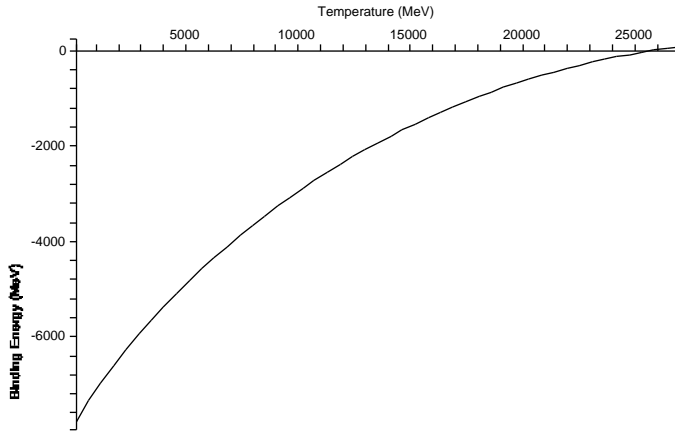
When we try to look for the binding energy of the J/ψ in quark-gluon plasma, we find that its curve doesn't cross the zero-line energy, but goes over it instead, indicating that the J/ψ is unbound already. But when we get too curious and we lower the Λ_{QCD} to 30 MeV—this was done just to get the curve to cross the zero-line energy—we find the J/ψ to be unbound at the 75 MeV temperature. This might indicate that the J/ψ got dissociated before even the quark-gluon plasma was formed. In the literature, the quark-gluon plasma materializes at a temperature of about 100 MeV or over, which makes us believe that the J/ψ has broken down before even thermalization. Our interest here is not to know exactly at what temperature the J/ψ dissociates, but to have an idea of where the range of that temperature might be. We tweaked the Λ_{QCD} just to have an idea about that temperature and not know exactly what it is.

Using $\alpha_s(m_b) = 0.2325 \pm 0.0044$ [8] for the Υ and Plotting the binding energy in terms of the temperature, we get:



In the graph just above, one can see that Υ dissociates at a temperature of about 225 MeV. This temperature is convenient and can be achieved through experiment. Since Υ states in a quark-gluon plasma are also sensitive to the color screening effect [9, 10, 11], the study of the upsilon meson suppression in high energy heavy ion collisions can be used as a signature for the quark-gluon plasma as well. Because the binding energy of Υ is larger than that of J/ψ , the critical energy density at which Υ is dissociated in the quark-gluon plasma is also higher [12]. One therefore expects to see the effects of the quark-gluon plasma on the production of Υ only in ultra-relativistic heavy ion collisions, such as at the RHIC and the LHC. As in the case of J/ψ , one needs to understand the effects of Υ absorption in hadronic matter in order to use its suppression as a signal for the quark-gluon plasma in heavy ion collisions.

Using $\alpha_s(m_t) = 0.32$ [13], and Plotting the binding energy in terms of the temperature, we get:



We can see in the graph above that the toponium dissociates at the high temperature of 25 GeV. As of now, this temperature is unachievable in the lab, for it is 25 times larger than the temperature LHC is trying to reach in the near future.

III Conclusion

We state that Υ dissociates at 225 MeV quark-gluon plasma and the toponium dissociates at 25 GeV. We are not satisfied with our result in regard to the J/ψ , because we changed Λ_{QCD} considerably to get the result of 75 MeV. We know that Λ_{QCD} is not arbitrary. Once we specify how Λ_{QCD} should be determined, it is fixed.

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