

Relativistic Calculation of the Width of the $\Theta^+(1540)$ Pentaquark

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Abstract

We calculate the width of the $\Theta^+(1540)$ pentaquark in a relativistic model in which the pentaquark is considered to be composed of a scalar diquark and a spin 1/2 triquark. We consider both positive and negative parity for the pentaquark. There is a single parameter in our model which we vary and which describes the size of the pentaquark. If the pentaquark size is somewhat smaller than that of the nucleon, we find quite small widths for the pentaquark of about 1 MeV or less. Our model of confinement plays an important role in our analysis and makes it possible to use Feynman diagrams to describe the decay of the pentaquark.

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I. INTRODUCTION

There has been a great deal of interest in the recent observation of a quite narrow resonance which decays to a nucleon and a kaon [1, 2, 3, 4, 5, 6, 7, 8, 9]. These states have been interpreted as pentaquarks which are likely to have a $udud\bar{s}$ structure [10]. (The extremely narrow width of the state, the $\Theta^+(1540)$, has led some to question whether the state exists in nature.) In the present work we will describe the calculation of the width of the $\Theta^+(1540)$ in a relativistic quark model which includes a model of confinement that we have introduced previously in our study of meson and nucleon structure.

A number of reviews have appeared. In Ref. [11] the experimental evidence for the pentaquark is reviewed. Theoretical and experimental developments are reviewed in Refs. [12, 13, 14, 15]. A number of theoretical papers have also appeared [10, 16, 17, 18, 19, 20, 21]. We are particularly interested in the work of Ref. [21] in which a diquark-triquark model is introduced to describe the pentaquark. We will make use of a variant of that model in the present work, since that model lends itself to the analysis of the pentaquark decay using Feynman diagrams. The diagram we consider is presented in Fig. 1. There the pentaquark is represented by the heavy line with momentum P . The pentaquark is composed of a diquark of momentum $-k + P_N$ and a triquark of momentum $P + k - P_N$. The triquark emits a quark (u or d) which combines with the diquark to form a nucleon of momentum P_N . The final-state kaon of momentum $P - P_N$ is emitted along with the quark at the triquark vertex.

We have studied the structure of the nucleon in a quark-diquark model in Ref. [22]. In that work we considered both scalar and axialvector diquarks, however, in this work we will limit our considerations to a nucleon composed of a quark and a scalar diquark. The nucleon vertex is described in Ref. [22] for the case in which the diquark is placed on mass shell. (We remark that in Fig. 1 only the quark of momentum k and the triquark of momentum $P + k - P_N$ will be off mass shell in our analysis.) The mass of the scalar diquark was taken to be 400 MeV in Ref. [22]. We use that value here and also take the triquark to have a mass of 800 MeV. Therefore, it is clear that we need to introduce a model of confinement for the pentaquark which has a mass of 1540 MeV. Our (covariant) confinement model which we have used extensively in other works [23, 24, 25] will be discussed in the next section.

The organization of our work is as follows. In Section II we will describe our model of

confinement and in Section III we will calculate the width of the pentaquark, assuming it has negative parity. In Section IV we consider the width in the case the pentaquark has positive parity. Finally, Section V contains further discussion and conclusions.

II. A MODEL OF CONFINEMENT

As stated in the last section, our pentaquark is composed of a scalar diquark of mass 400 MeV and a triquark of mass 800 MeV. Since the mass of the pentaquark under consideration is 1540 MeV, the pentaquark would decay into its constituents in the absence of a model of confinement. Similarly, the nucleon of mass 939 MeV could decay into the scalar diquark of mass 400 MeV and a quark whose mass we take to be 350 MeV in this work.

In earlier work we have introduced a confining interaction which served to prevent the decay of mesons or nucleons into their constituents. Our covariant confinement model is described in a series of our papers [23, 24, 25]. In that model we solve a linear equation for a confining vertex function, Γ . This function has the following property. Consider the decay $A \rightarrow B + C$, in which the hadrons A and C are on mass shell. If we include the confining vertex we find the amplitude has a zero when particle B goes on mass shell, so that the amplitude for A to decay into two on-mass-shell particles (B and C) is zero.

For the decay $A \rightarrow B + C$ we may introduce a wave function that may be expressed in terms of the momentum of the off-shell particle B . If B is a scalar, we have

$$\Psi_B(k) = \frac{1}{k^2 - m_B^2} \Gamma_B(k). \quad (2.1)$$

Note that the ratio of $\Gamma_B(k)$ to $(k^2 - m_B^2)$ is an ordinary function which may often be well represented by a Gaussian function. (It is not necessary to include an $i\epsilon$ in the denominator of Eq. (2.1).)

In the case of the nucleon we may consider the decay into a quark and a diquark. In Ref. [22] we considered both scalar and axialvector diquarks, but for simplicity we will limit ourselves to only the scalar diquark. The relevant wave function in this case was given in Eq. (3.3) of Ref. [22]:

$$\Psi_S(P, k, s, t) = \tilde{\Psi}_{(1)}(P, k) \frac{2m_q \Lambda^{(+)}(\vec{k})}{\sqrt{2E(\vec{k}) (E(\vec{k}) + m_q)}} u_N(P, s) \chi_t \quad (2.2)$$

which we will simplify for the present work to read

$$\Psi_N(P_N, k, s, t) = \tilde{\Psi}_N(P_N, k) \Lambda^{(+)}(\vec{k}) u_N(P_N, s) \chi_t. \quad (2.3)$$

The function $\tilde{\Psi}_N(P, k)$ is represented in Fig. 5 of Ref. [22] by a dashed line. That function is well approximated by a Gaussian function which we will record at a latter point in our discussion. The factor of $\Lambda^{(+)}(\vec{k}) = (k_{on} + m_q)/2m_q$ arises from an approximation made in Ref. [22]. (Here $k_{on} = (E(\vec{k}), \vec{k})$ with $E(\vec{k}) = [\vec{k}^2 + m_q^2]^{1/2}$.) In Ref. [22] the quark propagator was written as

$$-iS(k) = \frac{m_q}{E_q(k)} \left[\frac{\Lambda^{(+)}(\vec{k})}{k^0 - E_q(\vec{k})} - \frac{\Lambda^{(-)}(-\vec{k})}{k^0 + E_q(\vec{k})} \right]. \quad (2.4)$$

The second term was neglected in our formalism when we studied the nucleon. (Thus we limited our analysis to positive-energy quark spinors.) Since we wish to make use of the nucleon wave function determined in Ref. [22], we will continue to include the projection $\Lambda^{(+)}(\vec{k})$ in our formalism. Here k is the quark momentum.

As stated earlier, the diquark of momentum $-k + P_N$ in Fig. 1 will be placed on mass shell, so that $k^0 = E_N(\vec{P}_N) - E_D(\vec{P}_N - \vec{k})$, where $E_D(\vec{P}_N - \vec{k}) = [(\vec{P}_N - \vec{k})^2 + m_D^2]^{1/2}$ and $E_N(\vec{P}_N) = [\vec{P}_N^2 + m_N^2]^{1/2}$. That approximation is achieved by writing

$$\frac{1}{(P_N - k)^2 - m_D^2 + i\epsilon} \longrightarrow -2\pi i \delta^{(+)}[(P_N - k)^2 - m_D^2], \quad (2.5)$$

as described in detail in Ref. [26]. Note that

$$\delta^{(+)}[(P_N - k)^2 - m_D^2] = \frac{1}{2E_D(\vec{P}_N - \vec{k})} \delta[P_N^0 - k^0 - E_D(\vec{P}_N - \vec{k})]. \quad (2.6)$$

The on-mass-shell specification used here arises when performing an integral in the complex k^0 plane [26].

III. CALCULATION OF THE WIDTH OF A NEGATIVE PARITY PENTAQUARK

We consider the diagram shown in Fig. 1. Recall that the heavy line of momentum P denotes the pentaquark. The line carrying momentum $-k + P_N$ is the on-mass-shell scalar diquark and the line with momentum $P + k - P_N$ is the triquark. The momentum k is that of an up or down quark, $P - P_N$ is the momentum of the nucleon. The pentaquark, nucleon

and kaon are on mass shell. In our analysis the diquark is also on mass shell, so that only the triquark and quark propagate off mass shell, as noted earlier. (As stated in the last section, the on-shell characterization of the diquark arises when we complete the k^0 integral in the complex k^0 plane.)

We now make use of the formula [27]

$$d\Gamma = |\mathfrak{M}|^2 \frac{d^3 k_1}{(2\pi)^3} \frac{m_N}{E_N(\vec{k}_1)} \frac{d^3 k_2}{(2\pi)^3} \frac{1}{2E_K(\vec{k}_2)} (2\pi)^4 \delta^4(P - k_1 + k_2) \quad (3.1)$$

where \vec{k}_1 and \vec{k}_2 are the momenta of the outgoing particles. We may put $\vec{k}_1 = \vec{P}_N$ and $\vec{k}_2 = \vec{P} - \vec{P}_N = -\vec{P}_N$ for a pentaquark at rest. (It is convenient to take \vec{P}_N along the z axis when calculating the width.)

In writing our expression for Γ we will represent the product of the quark propagator and nucleon vertex function by the nucleon wave function of Eq. (2.3). In a similar fashion we will represent the product of the triquark propagator and the pentaquark-triquark vertex function by a triquark wave function. (In the vertex, the diquark is on mass shell.) We then have to specify the vertex function of the triquark which describes the decay into the quark of momentum k and the kaon. That scalar part of the vertex is usefully written as

$$\Gamma_T(k) = \Psi_T(k)(k^2 - m_q^2). \quad (3.2)$$

Thus, the wave functions $\Psi_N(k)$, $\Psi_T(k)$ and $\Psi_\Theta(P_N - k)$ will appear in our expression for Γ_Θ . Note that

$$\vec{P}_N^2 = \left(\frac{m_\Theta^2 - m_N^2 + m_K^2}{2m_\Theta} \right) - m_K^2 \quad (3.3)$$

which yields $\vec{P}_N^2 = 0.0396 \text{ GeV}^2$, or $|\vec{P}_N| = 0.199 \text{ GeV}$.

We find that the width is given by

$$\begin{aligned} \Gamma_\Theta &= \frac{1}{2} \int \frac{d\vec{k}}{(2\pi)^3} \frac{1}{2E_D(\vec{k} - \vec{P}_N)} \int \frac{d\vec{k}'}{(2\pi)^3} \frac{1}{2E_D(\vec{k}' - \vec{P}_N)} \frac{1}{N_\Theta N_T N_N} \frac{1}{(2m_N)(2m_\Theta)} \quad (3.4) \\ &\times \Psi_N(\vec{k}) \Psi_N(\vec{k}') \Psi_\Theta(\vec{P}_N - \vec{k}) \Psi_\Theta(\vec{P}_N - \vec{k}') (k^2 - m_q^2) \Psi_T(\vec{k}) (k'^2 - m_q^2) \Psi_T(\vec{k}') \\ &\times \text{Tr} [(\not{k} - \not{P}_N + \not{P} + m_T) (\not{k}_{on} + m_q) (\not{P}_N + m_N) \gamma^0 \\ &\quad (\not{k}'_{on} + m_q) (\not{k}' - \not{P}_N + \not{P} + m_T) \gamma^0 (\not{P} + m_\Theta)] \rho, \end{aligned}$$

where ρ is the phase space factor. We find

$$\rho = \frac{1}{2\pi} \frac{m_N}{m_\Theta} |\vec{P}_N| \quad (3.5)$$

$$= 0.193 \text{ GeV}, \quad (3.6)$$

since $|\vec{P}_N| \simeq 0.199$ GeV.

The factor $\not{P}_N + m_N$ and $\not{P} + m_\Theta$ in the trace arise from the relations

$$\frac{\not{P}_N + m_N}{2m_N} = \sum_{s_N} u_N(\vec{P}_N, s_N) \bar{u}_N(\vec{P}_N, s_N) \quad (3.7)$$

and

$$\frac{\not{P} + m_\Theta}{2m_\Theta} = \sum_s u_\Theta(\vec{P}, s) \bar{u}_\Theta(\vec{P}, s). \quad (3.8)$$

The factors $(1/N_\Theta)^{1/2}$, $(1/N_T)^{1/2}$ and $(1/N_N)^{1/2}$ serve to normalize the wave functions. We determine that $N_N = 0.316$, $N_T = 0.0673$, and calculate N_Θ for each choice of the pentaquark wave function. We write, with $k = |\vec{k}|$,

$$\Psi_N(\vec{k}) = \frac{1}{\sqrt{N_N}} \left(y_0 + \frac{A}{w\sqrt{\frac{\pi}{2}}} e^{-\frac{2(k-k_c)^2}{w^2}} \right) \quad (3.9)$$

and

$$\Psi_T(\vec{k}) = \frac{1}{\sqrt{N_T}} \left(y_0 + \frac{A}{w\sqrt{\frac{\pi}{2}}} e^{-\frac{2(k-k_c)^2}{w^2}} \right). \quad (3.10)$$

We have determined y_0 , A , k_c and w from a fit to the wave function given in Fig. 5 of Ref. [22]. We find $y_0 = -3.66$, $k_c = -0.013$ GeV, $w = 0.660$ GeV and $A = 27.73$ GeV. For the pentaquark we write

$$\Psi_\Theta(\vec{k} - \vec{P}_N) = \frac{1}{\sqrt{N_\Theta}} \frac{A}{w_\Theta\sqrt{\frac{\pi}{2}}} e^{-\frac{2(\vec{k} - \vec{P}_N)^2}{w_\Theta^2}}, \quad (3.11)$$

where w_Θ is a variable in our analysis. (Note that N_Θ depends upon the choice of w_Θ .) Here $\vec{k} - \vec{P}_N$ is the relative momentum of the diquark and triquark when $\vec{P} = 0$.

In Fig. 2 we present the results of our calculation of Γ_Θ as a function of w_Θ . Quite small values are obtained in the region 0.7 GeV $< w_\Theta < 0.9$ GeV. The region 0.6 GeV to 1 GeV is shown in Fig. 3 where it may be seen that the width has a maximum of about 15 MeV. The minimum values are very small and it is quite difficult to make an accurate calculation of widths significantly less than 1 MeV. (We have calculated five-dimensional integrals with 40 points for each variable. Thus the number of points calculated is $(40)^5 \sim 10^8$.)

IV. CALCULATION OF THE WIDTH OF A POSITIVE PARITY PENTAQUARK

In the case of a positive parity pentaquark we assume that the pentaquark decays to a positive parity diquark and a positive parity triquark. As in Section III, the triquark and diquark have zero relative angular momentum. In this case we need to insert factors of $i\gamma_5$ at the triquark-kaon vertex where the quark of momentum k is emitted. In addition the calculation of the normalization factor, N_T , is modified. In Eq. (3.4), the trace becomes

$$\begin{aligned} \text{Trace} = \text{Tr} & \left[(k - \not{P}_N + \not{P} + m_T) i\gamma_5 (\not{k}_{on} + m_q) (\not{P}_N + m_N) \gamma^0 \right. \\ & \left. (\not{k}'_{on} + m_q) i\gamma_5 (\not{k}' - \not{P}_N + \not{P} + m_T) \gamma^0 (\not{P} + m_\Theta) \right]. \end{aligned} \quad (4.1)$$

We may define $\tilde{k} = \gamma^0 k \gamma^0$, etc. Thus

$$\begin{aligned} \text{Trace} = \text{Tr} & \left[(k - \not{P}_N + \not{P} + m_T) \gamma_5 (\not{k}_{on} + m_q) (\not{P}_N + m_N) (\tilde{k}'_{on} + m_q) \right. \\ & \left. \gamma_5 (\tilde{k}' - \tilde{\not{P}}_N + \tilde{\not{P}} + m_T) (\not{P} + m_\Theta) \right] \end{aligned} \quad (4.2)$$

$$\begin{aligned} = \text{Tr} & \left[(k - \not{P}_N + \not{P} + m_T) (-\not{k}_{on} + m_q) (-\not{P}_N + m_N) (-\tilde{k}'_{on} + m_q) \right. \\ & \left. (\tilde{k}' - \tilde{\not{P}}_N + \tilde{\not{P}} + m_T) (\not{P} + m_\Theta) \right]. \end{aligned} \quad (4.3)$$

In this case we find $N_N = 0.316$ and $N_T = 0.0201$, where only the second value has changed relative to the values given in Section III. In Fig. 4 we show the values obtained for the width of the positive parity pentaquark, and in Fig. 5 we show the results of our calculation using a different scale.

V. DISCUSSION

It is of interest to note that, despite extensive efforts, no signal of a 1540 MeV pentaquark has been observed in lattice QCD studies [28, 29]. (These work contain more comprehensive references to the literature than given here.) In Ref. [29] pion masses in the range 400-630 MeV are used. The authors state that they cannot rule out the existence of a pentaquark at the physical quark mass of $m_\pi = 135$ MeV. They also suggest the possibility is that the pentaquark has a more exotic wave function than that considered in Ref. [29].

One feature that has led some researches to question the existence of the pentaquark is its very small width [14]. We have shown in the present work that very small widths are

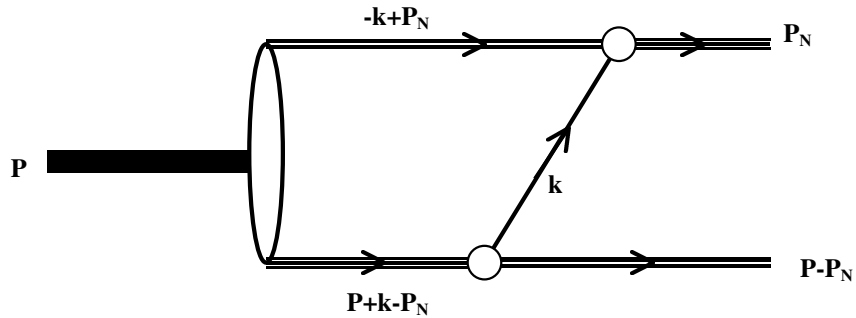


FIG. 1: In this figure the heavy line denotes the pentaquark and the line of momentum $-k + P_N$ denotes an on-mass-shell diquark. The line of momentum k represents the quark and $P + k - P_N$ is the momentum of the triquark. In the final state we have a nucleon of momentum P_N and a kaon of momentum $P - P_N$.

obtained in our model if the pentaquark wave function parameter, w_Θ , is somewhat larger than that we have found for the nucleon ($w_N = 0.66$ GeV) in Ref. [22]. [See Fig. 3.]

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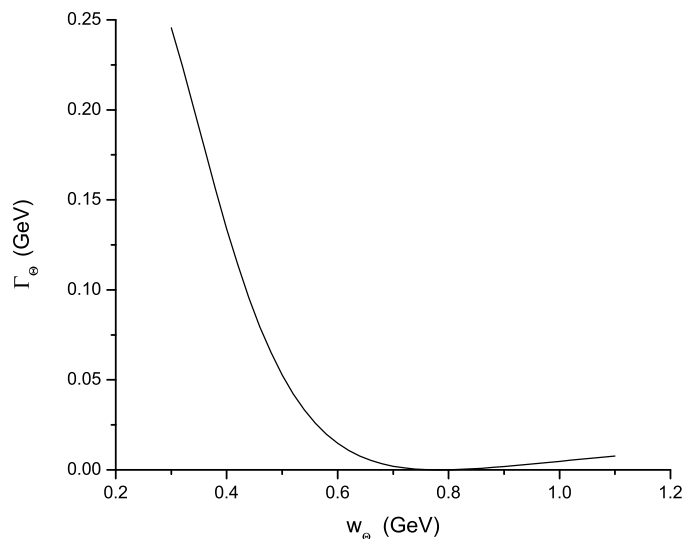


FIG. 2: The figure shows the width of the pentaquark as a function of the parameter, w_Θ , which governs the extent of the pentaquark wave function in momentum space. In the case of the nucleon we found $w = 0.66$ GeV for the wave function calculated in Ref. [22]. In this calculation the pentaquark had negative parity as did the triquark.

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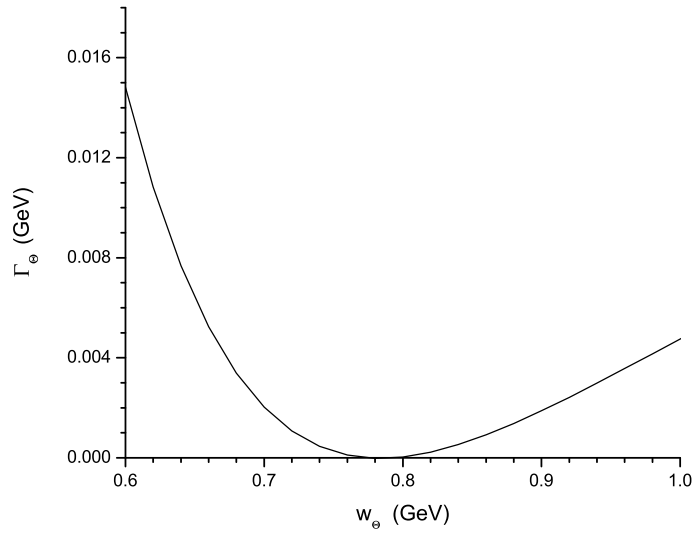


FIG. 3: The width of the negative parity pentaquark is shown as a function of the parameter w_Θ . (See Fig. 2.) (The quite small values in the vicinity of the minimum have rather large uncertainties because of the limitation of the number of points used in our five-dimensional integral which determines the width.)

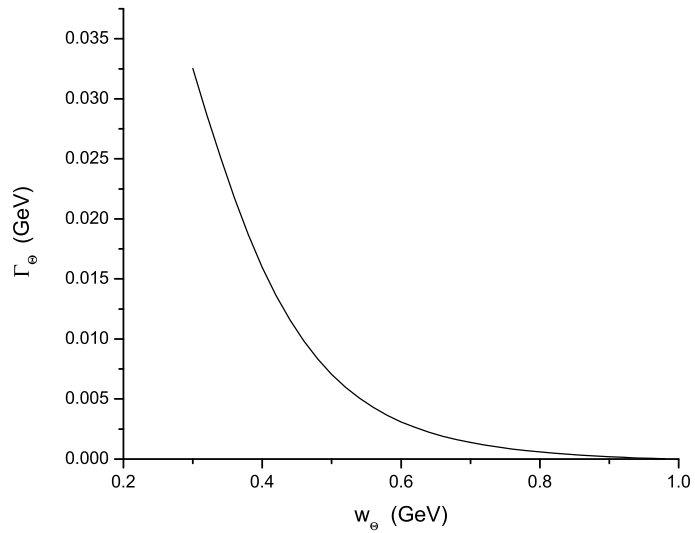


FIG. 4: The width of a pseudoscalar pentaquark calculated in our model is shown.

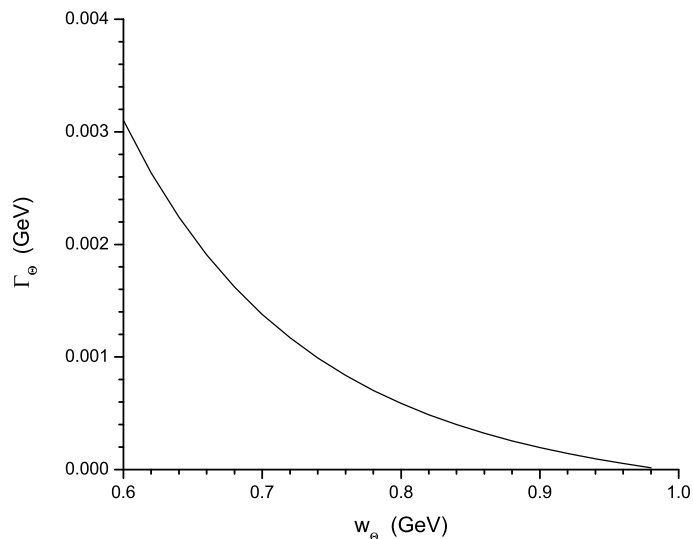


FIG. 5: The results of the calculation reported in Fig. 4 are shown using a different scale. Widths of the pentaquark less than 1 MeV are found for values of w_Θ greater than 0.72 GeV. (We recall that w for the nucleon was 0.66 GeV.)

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