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Phase Structure of Color Superconductivity and Chiral Restoration

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We investigate color superconductivity and chiral symmetry restoration at finite temperature and baryon density in the frame of standard two flavor Nambu–Jona-Lasinio model. We derive the diquark mass in RPA, discuss its constraint on the coupling constant in the diquark channel, and find a strong competition between the two phase transitions when the coupling constant is large enough.

1. Introduction

In the ideal case at asymptotically high baryon density, the color superconductivity with two massless flavors and the color-flavor-locking (CFL) phase with three degenerated massless quarks have been widely discussed from first principle QCD calculations[1]. For physical applications we are more interested in the moderate baryon density region which may be related to the neutron stars and, in very optimistic cases, even to heavy-ion collisions. Usually, four-fermion interaction models such as the instanton, as well as the Nambu–Jona-Lasinio (NJL) model, are used in the study of color superconductivity phase transition. It was found that the color superconductivity gap can be of the order of 100 MeV[2], which is two orders larger than early perturbative QCD estimation[3]. The electric and color charge neutrality condition[4,5] leads to a new phase of color superconductivity, the gapless color superconductivity[6] or the breached pairing phase[7]. The most probable temperature for this new phase is finite but not zero[8].

It is generally accepted that the NJL model[9] applied to quarks[10] offers a simple but effective scheme to study spontaneous chiral symmetry breaking in the vacuum, chiral restoration at finite temperature and density, and spontaneous color symmetry breaking at high density[5,6,8,11]. From the study of chiral phase transition without considering color superconductivity, it is well-known that[10] the mean field approximation to quarks and random phase approximation (RPA) to mesons can describe well the thermodynamics of the system, especially the massless Goldstone mode in the spontaneous symmetry breaking phase. In this letter I will determine

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the diquark mass in RPA, discuss its constraint on the coupling constant in the diquark channel, and investigate the phase structure of color superconductivity and chiral restoration and its dependence on the coupling constant.

2. Quarks in Mean Field Approximation

The NJL Lagrange density is defined as

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m + \mu\gamma_0\right)\psi \tag{1}$$
$$+ G_S\left(\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\tau\gamma_5\psi\right)^2\right) + G_D\left(\bar{\psi}_{i\alpha}^C\epsilon^{ij}\epsilon^{\alpha\beta\gamma}i\gamma^5\psi_{j\beta}\right)\left(\bar{\psi}_{i\alpha}\epsilon^{ij}\epsilon^{\alpha\beta\gamma}i\gamma^5\psi_{j\beta}^C\right) \ ,$$

where G_S and G_D are coupling constants in color signet channel and color anti-triplet channel, respectively, and m is the current quark mass.

The quark-antiquark and diquark condensates which are order parameters of chiral and color superconductivity phase transitions are defined as

$$\sigma = \langle\bar{\psi}\psi\rangle \ , \quad \Delta = \Delta^3 = \langle\bar{\psi}_{i\alpha}^C\epsilon^{ij}\epsilon^{\alpha\beta3}i\gamma^5\psi_{j\beta}\rangle = \langle\bar{\psi}_{i\alpha}\epsilon^{ij}\epsilon^{\alpha\beta3}i\gamma^5\psi_{j\beta}^C\rangle \ , \tag{2}$$

where it has been regarded that only the first two colors participate in the condensate, while the third one does not. The condensates are assumed real but are otherwise as yet unspecified.

A usual way to treat systematically the spontaneous color and chiral symmetry breaking at finite temperature and density is to introduce the Nambu-Gorkov space[12] with color and flavor degrees of freedom. After performing the Fourier transformation, the quark propagator in momentum space at mean field level is diagonal in the 12 dimensional Nambu-Gorkov space,

$$S(k) = \begin{pmatrix} S_A(k) & & & & & \\ & S_B(k) & & & & \\ & & S_C(k) & & & \\ & & & S_D(k) & & \\ & & & & S_E(k) & \\ & & & & & S_F(k) \end{pmatrix} \ , \tag{3}$$

where $S_I(k)$ with $I = A, B, C, D, E, F$ are 2×2 matrices,

$$S_I = \begin{pmatrix} G_I^+ & \Xi_I^- \\ \Xi_I^+ & G_I^- \end{pmatrix} \ . \tag{4}$$

In general case, the quark chemical potential $\mu_{i\alpha}$ depends on color and flavor degrees of freedom, the elements of the 6 matrices S_I are totally different. However, if we consider only the baryon chemical potential μ_B, one has $\mu = \mu_B/3$, and some of the elements are identical ,

$$G_A^+ = G_B^+ = G_C^+ = G_D^+ \ , \quad G_A^- = G_B^- = G_C^- = G_D^- \ , \tag{5}$$
$$\Xi_A^+ = \Xi_B^+ = -\Xi_C^+ = -\Xi_D^+ \ , \quad \Xi_A^- = \Xi_B^- = -\Xi_C^- = -\Xi_D^- \ ,$$
$$G_E^+ = G_F^+ \ , \quad G_E^- = G_F^- \ , \quad \Xi_E^+ = \Xi_F^+ = \Xi_E^- = \Xi_F^- = 0 \ ,$$

$$G_A^+(k) = \frac{k_0 + E_k - \mu}{(k_0 - E_k^-)(k_0 + E_k^-)}\Lambda_+\gamma_0 + \frac{k_0 - E_k - \mu}{(k_0 - E_k^+)(k_0 + E_k^+)}\Lambda_-\gamma_0 \ ,$$

$$G_A^-(k) = \frac{k_0 - E_k + \mu}{(k_0 - E_k^-)(k_0 + E_k^-)}\Lambda_-\gamma_0 + \frac{k_0 + E_k + \mu}{(k_0 - E_k^+)(k_0 + E_k^+)}\Lambda_+\gamma_0 \ ,$$

$$\Xi_A^+(k) = \frac{2iG_D\Delta}{(k_0 - E_k^-)(k_0 + E_k^-)}\Lambda_-\gamma_5 + \frac{2iG_D\Delta}{(k_0 - E_k^+)(k_0 + E_k^+)}\Lambda_+\gamma_5 \ ,$$

$$\Xi_A^-(k) = \frac{2iG_D\Delta}{(k_0 - E_k^-)(k_0 + E_k^-)}\Lambda_+\gamma_5 + \frac{2iG_D\Delta}{(k_0 - E_k^+)(k_0 + E_k^+)}\Lambda_-\gamma_5 \ ,$$

$$G_E^+(k) = \frac{1}{k_0 - E_k + \mu}\Lambda_+\gamma_0 + \frac{1}{k_0 + E_k + \mu}\Lambda_-\gamma_0 \ ,$$

$$G_E^-(k) = \frac{1}{k_0 - E_k - \mu}\Lambda_+\gamma_0 + \frac{1}{k_0 + E_k - \mu}\Lambda_-\gamma_0 \ ,$$

with the quark energies $E_k^\pm = \sqrt{(E_k \pm \mu)^2 + (2G_D\Delta)^2}$, $E_k = \sqrt{|\mathbf{k}|^2 + M_Q^2}$, effective quark mass $M_Q = m - 2G_S\sigma$, and the energy projectors $\Lambda_\pm = \frac{1}{2}\left(1 \pm \frac{\gamma_0(\gamma\cdot\mathbf{k}+M_Q)}{E_k}\right)$.

While the first two colors are involved in the quark-antiquark and diquark condensates, and their dispersion relation $E_k^\pm$ are related to σ and Δ, the third color participates in the chiral condensate only, the non-diagonal elements of the last two matrices S_E and S_F are zero, and its energy E_k is associated with σ only.

From the definition of the two condensates (2), it is easy to obtain their relation with the elements of the quark propagator (3),

$$\sigma = 4M_Q \int \frac{d^3\mathbf{k}}{(\mathbf{2}\pi)^\mathbf{3}} \frac{1}{E_k}\Big[\frac{E_k - \mu}{E_k^-}\left(2f(E_k^-) - 1\right) \tag{6}$$

$$+\frac{E_k + \mu}{E_k^+}\left(2f(E_k^+) - 1\right) + f(E_k - \mu) + f(E_k + \mu) - 1\Big] \ ,$$

$$\Delta\left[1 + 8G_D \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left(\frac{1}{E_k^-}\left(2f(E_k^-) - 1\right) + \frac{1}{E_k^+}\left(2f(E_k^+) - 1\right)\right)\right] = 0 \ ,$$

with the Fermi-Dirac distribution function $f(x) = 1/(e^{x/T} + 1)$.

3. Coupling Constant G_D in Diquark Channel

The diquark polarization function can be represented in terms of the matrix elements of the quark propagator,

$$-i\Pi_D(k) = -4 \int \frac{d^4p}{(2\pi)^4}Tr\left[i\gamma_5 iG_A^+(p - k)i\gamma_5 iG_A^-(p)\right] \ . \tag{7}$$

In the mean field approximation to quarks and RPA to mesons and diquarks, the temperature and baryon chemical potential dependence of the diquark mass $M_D(T,\mu)$ is controlled by the pole equation

$$1 - 2G_D\Pi_D(k_0^2 = M_D^2, \mathbf{0}) = 0 \ . \tag{8}$$

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We consider now the physical constraints in the vacuum on the coupling constant G_D in the diquark channel. The diquark mass $M_D(0,0)$ in the vacuum is determined by

$$1 - 8G_D \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(\frac{1}{E_p + M_D(0,0)/2} + \frac{1}{E_p - M_D(0,0)/2} \right) = 0 \ . \tag{9}$$

In general, temperature effect disorders a system, and any condensate will be suppressed in hot medium. Therefore, the critical chemical potential of color superconductivity goes up monotonously with increasing temperature, and the minimum one μ_Δ at zero temperature satisfies the gap equation

$$1 - 8G_D \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(\frac{1}{E_p + \mu_\Delta} + \frac{1}{E_p - \mu_\Delta} \right) = 0 \ . \tag{10}$$

From the comparison of the above two equations, we obtain the relation between the diquark mass in the vacuum and the critical chemical potential for diquark condensate at zero temperature, $\mu_\Delta = M_D(0,0)/2$.

When the diquark is massless in the vacuum, the color superconductivity happens even at $\mu_\Delta = 0$, and the vacuum becomes instable. On the other hand, when the diquark mass in the vacuum is larger than two times the quark mass, the color superconductivity is far from the vacuum, and a diquark can not be in the bound state, but decay into two quarks. Therefore, the diquark mass $M_D(0,0)$ should satisfy the constraint $0 < M_D(0,0) < 2M_Q(0,0)$. From the comparison of the diquark mass equation (9) and the quark mass equation in the vacuum,

$$1 - 24G_S \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\sqrt{M_Q^2(0,0) + \mathbf{p}^2}} = 0 \ , \tag{11}$$

the constraint $0 < M_D(0,0) < 2M_Q(0,0)$ on the diquark mass resulus in the relation between the two coupling constants G_D and G_S,

$$G_D^{min} < G_D < G_D^{max} \ ,$$

$$G_D^{min} = \frac{\pi^2}{4 \left(\Lambda\sqrt{M_Q^2(0,0) + \Lambda^2} + M_Q^2(0,0) \ln \frac{\Lambda + \sqrt{M_Q^2(0,0) + \Lambda^2}}{M_Q(0,0)} \right)} \ ,$$

$$G_D^{max} = \frac{3}{2} G_S \frac{M_Q(0,0)}{M_Q(0,0) - m} \ . \tag{12}$$

With the known values of the parameters Λ, m and $M_Q(0,0)$ in real world, the minimum and maximum coupling constant take the values $G_D^{min} = 0.82 G_S$ and $G_D^{max} = 1.55 G_S$.

4. Phase Diagram

Before making numerical calculations, we first discuss the parameters in the model. There are four independent parameters in the current NJL model with both diquark

and meson channels, the three-momentum cutoff Λ, the current quark mass m, and the coupling constants G_S and G_D. The first three can be fixed by fitting the pion mass $m_\pi = 0.134$ GeV, the pion decay constant $f_\pi = 0.093$ GeV, and the quark condensate density $\sigma = (-0.25 \text{ GeV})^3$ in the vacuum. It leads to $\Lambda = 0.65$ GeV and $G_S = 5.01 \text{ GeV}^{-2}$ in chiral limit with zero current quark mass $m = 0$, and $\Lambda = 0.653$ GeV, $G_S = 4.93 \text{ GeV}^{-2}$ and $m = 0.005$ GeV in real world.

In chiral limit the chiral phase transition is well defined, and the phase transition line determined by $\sigma = 0$ is a solution of the gap equations (6). The phase diagram in the $T - \mu$ plane is shown in Fig.1 for different coupling constant G_D in the diquark channel.

At the minimum coupling constant G_D^{min}, the whole $T - \mu$ plane is separated into three regions, the region with chiral symmetry breaking but color symmetry, $\sigma \neq 0$ and $\Delta = 0$ at low density, the region with chiral symmetry but color symmetry breaking, $\sigma = 0$ and $\Delta \neq 0$ at low temperature, and the region with both chiral and color symmetries, $\sigma = \Delta = 0$ at high temperature and/or high density. The critical temperature $T_\chi = 185$ MeV for chiral phase transition at $\mu = 0$ is in good agreement with the lattice calculation[13,14], and the chiral symmetry breaking phase ends at μ_χ determined by

$$1 - 24G_S \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{p} \theta(k - \mu_\chi) = 0 . \qquad (13)$$

From the comparison with the quark mass equation (11) in the vacuum, one has $\mu_\chi = M_Q(0,0)$. Therefore, from the definition of the minimum coupling constant G_D^{min} at which the minimum chemical potential μ_Δ of color superconductivity is equal to the constituent quark mass in the vacuum, the end point of chiral phase transition and the starting point of color superconductivity coincide at $\mu = M_Q(0,0)$.

The order of a phase transition is determined by the behavior of the order parameter across the phase transition line. It is well known[10] that the chiral phase transition is of second order at high temperature and of first order at high density. The critical chemical potential at which the chiral transition goes from first to second order is 285 MeV in the NJL model[15]. One can also determine the critical exponent for the color superconductivity. Since the square bracket of the second gap equation of (6) depends quadratically on the order parameter Δ, one has a simple zero for Δ^2 as a function of T and therefore in the neighborhood of the critical temperature T_Δ of color superconductivity the order parameter behaviors as $\Delta(T) \sim |T - T_\Delta|^{1/2}$. This means that the transition between the phase with $\sigma = 0, \Delta \neq 0$ and the phase with $\sigma = \Delta = 0$ is of second order.

With increasing coupling constant G_D, the critical temperature T_Δ for melting the diquark condensate increases and the critical chemical potential μ_Δ for forming the color superconductivity decreases. Especially, a new phase with both chiral and color symmetry breaking, $\sigma \neq 0$ and $\Delta \neq 0$, is produced in between the phase with only chiral symmetry breaking and the phase with only color symmetry breaking,

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as shown in Fig.**??** for $G_D = 1.4G_S$. The left and right borders of this mixed phase are determined by $\sigma \neq 0, \Delta = 0$ and $\sigma = 0, \Delta \neq 0$, respectively. With increasing G_D the chiral breaking phase is swallowed up gradually by the mixed phase. At the maximum coupling constant G_D^{max} the chiral breaking phase disappears and the mixed phase reaches the maximum.
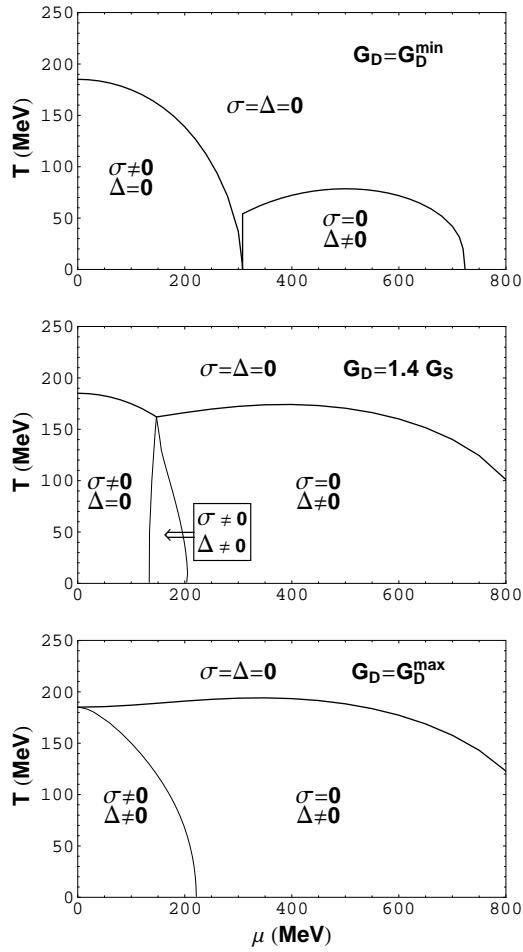

Fig. 1.   *The Phase diagram in the $T - \mu$ plane for different coupling constant G_D in the diquark channel.*

5. Summary

We have analytically derived the simple relation between the diquark mass in the vacuum and the minimum quark chemical potential for diquark condensate,

$\mu_\Delta = M_D(0,0)/2$. Under the conditions to keep the diquark bound state in the vacuum and to have a stable physical vacuum without diquark condensate in it, we obtained the constraint on the coupling constant in the diquark channel, $G_D^{min} < G_D < G_D^{max}$, with the relations to the coupling constant in the meson channel $G_D^{min} \simeq 0.82 G_S$ and $G_D^{max} \simeq 3/2 G_S$. There is a strong competition between the color superconductivity and chiral symmetry restoration: With increasing G_D the phase with chiral symmetry breaking but color symmetry is gradually taken over by the phase with both chiral and color symmetry breaking.

Acknowledgments

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