Mass spectra of excited meson states consisting of $u-$, $d-$quarks and antiquarks

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Abstract

Mass spectra of excited meson states consisting of $u-$, $d-$quarks and antiquarks with $I = 1$ are considered. The comparison between the existing experimental data and the mass values evaluated with phenomenological formulae is carried out. Problems of identification of vector and scalar meson excited states are discussed.

1. Introduction

QCD results concerning perturbative calculations of characteristics for different processes at high energies are widely used, however, considerable difficulties exist in the QCD framework, for example, in hadron spectroscopy for mass evaluations of hadrons consisting of light quarks. Mainly these difficulties are originated from the unsolved confinement and spontaneous chiral symmetry breaking problems. Nevertheless, a number of relations among hadron masses have been obtained using symmetry or phenomenological considerations. For instance, let us remind the mass relations [1] for Regge trajectories [2] or the Gell-Mann-Okubo relation [3, 4]. According to the Regge trajectories approach a hadron with its spin J and mass M within some errors belong to a straight trajectory on the $(J, M^2)-$plane with a slope α' and a intercept α_0

$$J = \alpha_0 + \alpha' M^2 \qquad (1)$$

Some hadrons belong to trajectories, so called, daughter trajectories, which are roughly parallel to the main trajectory and are distinguished with different values of a radial quantum number n^r or n, $n = n^r + 1$ [5, 6, 7].

Now there is a growing interest in improving the accuracy of existing mass relations and obtaining these relations in the framework of the QCD or QCD inspired models. For instance, on the base of QCD finite sum rules [8, 9] the following mass formulae for radially excited $\rho-$ and $\pi-$mesons have been obtained

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$$M^2(\rho^n) = M^2(\rho)(2n^r + 1), \tag{2}$$

$$M^2(\pi^n) = M^2(\pi')n^r, \tag{3}$$

where $M^2(\rho)$ is the mass squared of the $\rho-$meson, $M^2(\pi')$ is the mass squared of the first radial excitation of $\pi-$meson. In the work [10] more precise formulae for radially excited hadrons have been proposed. These formulae in general case can be written as

$$M_n^2 = M_0^2 + \mu^2(n - 1) \tag{4}$$

and describe trajectories on the $(n, M^2)-$plane with different M_0 values and approximately the same μ for each trajectory similarly Chew-Frautschi plots (1) on the $(J, M^2)-$plane.

In the present paper we evaluate mass spectra of excited meson states consisting of $u-$, $d-$quarks and antiquarks with mass formulae, which have the structure peculiar for mass formulae of the independent quark model [11]. These formulae have been proposed for the first time in the papers [12] and represent meson mass values in the $(L, M, n)-$space. We improve the accuracy of calculations of masses taking more precise values for formulae parameters, compare with existing data and make predictions for mass values of meson states, which have no experimental evidence. Besides that, we consider a possible compound structures of some mesons, which are under debates at present, such as scalar mesons and vector exited mesons.

2. Phenomenological mass formulae for superlight $\overline{q}q'-$mesons

In the standard quark model the ground state and exited mesons are formed from a quark and an antiquark and can be characterized with values of internal radial and orbital quantum numbers n^r and L. The total spin S of the pair $\overline{q}q'$ may be 1 or 0. The space and charge parities of mesons can be evaluated with formulae $P = (-1)^{L+1}$, $C = (-1)^{L+S}$ and any $\overline{q}q'-$meson with quantum numbers J^{PC} may be classified as $n^{2S+1}L_J-$state, where J is a meson spin value. Only superlight mesons, which are made from $u-$, $d-$quarks and antiquarks, are considered below. Isotopical spin values for mesons of such kind are 1 or 0. We restrict ourselves to those mesons, which have isotopical spin values equal to 1, in order to bypass problems concerning unknown in some cases mixing parameters for different mesons with the zero isotopical spin. Further on we neglect mass splittings within isotopic multiplets, so systematical errors of the order of $10\ MeV$ for the phenomenological scheme considered are without doubt admissible. Moreover, take into account the existing experimental errors for meson masses, we assume $30 \div 40 MeV$ as the typical absolute errors of the mass values evaluations.

In the framework of relativistic independent quark model a mass formula for $\overline{q}q'-$mesons has a structure, which is different from structures of mass formulae in other types of potential models. For instance, in Ref. [12] the following mass formula has been proposed

$$M(n^{2S+1}L_J) = E_1(n_1^r, j_1, c, \varkappa) + E_2(n_2^r, j_2, c, \varkappa), \tag{5}$$

where the mass terms (or energy spectral functions) $E_i(n_i, j_i, c, \varkappa), i = 1, 2$, for a quark and an antiquark are defined as

$$E_i(n_i, j_i, c, \varkappa) = \begin{cases} c + \varkappa\sqrt{2n^r + L + j - 1/2}, & L + j - 1/2 = 2k, \quad k = 0, 1, ... \\ \varkappa\sqrt{2n_r + L + j - 1/2}, & L + j - 1/2 = 2k + 1, \quad k = 0, 1, ... \end{cases} \quad (6)$$

Functions $E_i(n_i^r, j_i, c, \varkappa)$, $i = 1, 2$, give the relativistic effective energies of the quark and antiquark moving in the mean field inside the meson, and include also an energy of the mean field and possible nonpotential corrections, which cannot be evaluated within the mean field approximation. Possibly, a part of these corections can be taken into account with the help of constant c. Note that nonpotential corrections are the most important for mass spectra evaluations of light mesons [13, 14]. n_i^r and j_i are a radial quantum number and a quantum number of an angular moment of the $i-$th constituent, correspondingly, and $n_1^r = n_2^r = n - 1$ for the $n^{2S+1}L_J$ - meson state, $\varkappa$ is some constant. In general case the root part of mass term $E_i(n_i^r, j_i, c, \varkappa)$, which represent the energy of the $i-$th constituent in the mean field, should be determined from the solution of the Dirac equation [15]. However, for the superlight mesons the phenomenological energy spectral functions $E_i(n_i, j_i, c, \varkappa)$ in the form (5) are suitable with a rather good accuracy.

In order to exclude superfluous meson states, the following selection rules for $\overline{q}q'-$mesons with given J^{PC} values, quark masses m_1 and m_2 and quantum numbers j_1 and j_2 are used

$$\begin{aligned} j_1 = j_2 = J + 1/2, \quad if \quad J = L + S, \\ j_1 = j_2 + 1 = J + 3/2, \quad if \quad J \neq L + S, \quad m_1 \leq m_2 \end{aligned} \quad (7)$$

One can obtain with the help of formulae (5), (6) and (7) a few mass relations, which are fulfilled within the systematical errors of the phenomenological scheme considered. For instance, in the case of orbitally exitated vector mesons formulae (5) and (6) give the $\rho-$trajectory, for radially exitated vector mesons they bring to the formula (2). However, in general case in the framework of this approach instead of the meson trajectories in $(J, M^2)-$ and $(n, M^2)-$planes we have different series or trajectories in $(L, M, n)-$space for $n^{2S+1}L_J-$ meson states.

There is a $(\pi\rho)-$ series for $n^3L_{L-1}-$meson states with $P = (-1)^{L+1}$, $C = (-1)^{L+1}$. Mass values of the members of the $(\pi\rho)-$ series are determined by the formula:

$$M_{n,L}^{\pi\rho} = c + \varkappa\sqrt{2(n^r + L) - 1} + \varkappa\sqrt{2(n^r + L)} \quad (8)$$

There is a $\pi-$ series for n^1L_L-meson states with $P = (-1)^{L+1}$, $C = (-1)^L$. Mass values of the members of the $\pi-$ series are determined by the formula:

$$M_{n,L}^{\pi} = 2c + 2\varkappa\sqrt{2(n^r + L)} \quad (9)$$

There is a $(\rho\pi)-$ series for n^3L_L-meson states with $P = (-1)^{L+1}$, $C = (-1)^{L+1}$. Mass values of the members of the $(\rho\pi)-$ series are determined by the formula:

$$M_{n,L}^{\rho\pi} = c + \varkappa\sqrt{2(n^r + L)} + \varkappa\sqrt{2(n^r + L) + 1} \quad (10)$$

There is a $\rho-$ series for $n^3L_{L+1}-$meson states with $P = (-1)^{L+1}$, $C = (-1)^{L+1}$. Mass values of the members of $\rho-$ series are determined by the formula:

$$M_{n,L}^{\rho} = 2\varkappa\sqrt{2(n^r + L) + 1} \quad (11)$$

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With the help of these formulae for the series presented a few mass relations, which are not dependent on intrinsic quantum numbers L and n easily can be obtained. For instance, mass values of members of $\pi-$ series, $(\rho\pi)-$ series and $\rho-$ series with the same L obey the following mass relation:

$$2M_{n,L}^{\rho\pi}(n^3L_L) = M_{n,L}^{\pi}(n^1L_L) + M_{n,L}^{\rho}(n^3L_{L+1}) \tag{12}$$

It is interesting, that a quantity τ for members of P-wave meson multiplets, which played an important role in a determination of properties of spin-dependent forces between heavy quarks and antiquarks [14, 16], have the following value in the case considered (remind that we neglect the difference between masses of $u-$ and $d-$quarks).

$$\tau = \frac{M_{n,L}^{\rho}(n^3L_{L+1}) - M_{n,L}^{\rho\pi}(n^3L_L)}{M_{n,L}^{\rho\pi}(n^3L_L) - M_{n,L}^{\pi}(n^3L_{L-1})} = \frac{\varkappa\sqrt{2(n^r+L)+1} - c - \varkappa\sqrt{2(n^r+L)}}{\varkappa\sqrt{2(n^r+L)+1} - \varkappa\sqrt{2(n^r+L)-1}} \tag{13}$$

So for members of $P-$wave superlight meson multiplets with $n^r = 0$ τ is equal to 0.19. For the $c\bar{c}$ - states in $P-$wave the value τ is ~ 0.5, while for the $b\bar{b}$ - states it is ~ 0.65.

3. Evaluation of meson masses and comparison with experimental data

When evaluating masses of unknown excited meson states, we use the formulae written above together with the values of two parameters c and $\varkappa$, which have been obtained by fitting of mass values of experimentally detected meson states. The c and $\varkappa$ values obtained by this manner are $c = 69~MeV$, $\varkappa = 382 \pm 4~MeV$. Then a mass value of an ordinary superlight $\bar{q}q'-$meson, which consist of $u-$, $d-$quarks and antiquarks and have the isotopical spin equal to unity, can be evaluated with an absolute uncertainty less than $40 MeV$.

The results obtained with $c = 69~MeV$, $\varkappa = 385~MeV$ are shown in the Tables 1 and 2, where we present the mass values of orbital excitations of superlight meson states up to $L = 4$ and the radial excitations of superlight pseudoscalar, vector and scalar meson states up to $n^r = 4$. Although we restricts us with those meson states, which have isotopical spin values equal to unity, in order to exclude the unknown influence of mixing with superfluous states in $I = 0$ sector, nevertheless it is found that an account of mixing in $I = 1$ sector is also necessary. As it follows from the Tables 1 and 2, the masses evaluated for first radial and orbital excitations of the vector 1^{--} mesons have no reliable confirmations by existing data. May be it is due to the shortcoming of the mass formulae in this region. It seems rather unlikely because of other evaluated mass values especially for the orbital excitations coincide with the data [17] within the experimental errors. Thus we assume that mixing between the first radial and orbital excitations for the 1^{--} mesons take place. However, it is possible, that the more complicated situation should be considered, when in the mass region $\sim 1.5~GeV$ mixing between standard 1^{--} mesons and vector non $\bar{q}q'-$mesons take place as well. Below we consider only mixing between the first radial and orbital excited standard 1^{--} mesons. In this case the experimentally visible vector meson states V_R' and V_O' are connected with the pure ones through a rotation on some angle θ:

$$\begin{aligned} V_R' &= V_R \cos\theta - V_O \sin\theta, \\ V_O' &= V_R \sin\theta + V_O \cos\theta \end{aligned} \tag{14}$$

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Taking into account the existing experimental data the following cases are most preferable. In the first case the experimentally observed bump at $1465 MeV$ consists of one resonance V'_R, that is mainly the radial excitation of $\rho-$meson. In the second case the experimentally observed bump at $1465 MeV$ consist of two resonances V'_R and V'_O. Because of we know the masses of V_R and V_O (see Tables 1 and 2), mixing angles and mass values of V'_R and V'_O in these two cases easily can be evaluated. In the first case the V'_O resonanse lie below the V'_R and have mass $\sim 1350 \ MeV$, while in the second case the V'_R and V'_O have approximately equal masses $\sim 1450 \ MeV$. Take into account this consideration we expect that in the mass region between 1.3 GeV and 1.6 GeV two standard $\overline{q}q'-$meson resonances with $J^{PC} = 1^{--}$ and $I = 1$ exist, namely the first radial and orbital excitations mixing each another. The similar conclusion was presented in the paper [18]. In the general case, as mentioned above, the most complicated situation may arise, which is not disscussed here, when the mixing with cryptoexotic states can occur. This case demands further investigation.

Let us compare with the data [17] the mass relation (12), which must obey for the mesons with $L \neq 0$ and $S = 1$. At present this relation can be checked only for the mesons with $L = 1$. After the substitution the experimental values of the masses for the $a_1(1260)$, $b_1(1235)$ and $a_2(1320)$ mesons, we obtain, that this relation is fulfilled with account of experimental uncertanties. Moreover, the mass formulae presented above permit to explain degeneracy with respect to mass values for different J^{PC} mesons. For example, the mass formulae give $M(\rho'') = M(\rho_3)$. If one use the data, then $1720 \pm 20 MeV = 1688.8 \pm 2.1 MeV$. In the high mass value region the degeneracy of such kind will be increased, as it is seen in the $\sim 2315 MeV$ region, where the 0^{-+}, 1^{--}, 4^{-+}, 4^{--} and 5^{--} mesons are predicted.

In the framework of nonrelativistic potential models the following relation for the mass values of the members of P multiplets is well known

$$M(^1P_1) - 5/9M(^3P_2) - 3/9M(^3P_1) - 1/9M(^3P_0) = 0 \qquad (15)$$

Using the mass formulae for the superlight mesons with $n^r = 0$ written above, this relation goes into the form:

$$14(c + \varkappa\sqrt{2})MeV = \varkappa(13\sqrt{3} + 1)MeV \qquad (16)$$

When substituting the numerical c and $\varkappa$ values, we obtain $8589 MeV = 9054 MeV$. Thus, the relative error for the fulfillment of this relation for superlight mesons is $\sim 5\%$. This result has been presented for illustration only the numerical resemblance between the potential approach and the phenomenological one. Notwithstanding an approximated character of the relations (15) and (16) one can see, that these two approaches do not contradict each another. The comparison of the results presented in the Table 2 for the mass values of the radial excitations with the data, so as with the evaluations of the mass values of the radial excitations for the vector and pseudoscalar mesons with the help of Dirac equation with the potential $\sim -a/r + \gamma^0 kr$ [15, 19], also confirms the previous point. Moreover the obvious fact should be noted, that the accuracy of evaluations with the help of formulae (5), (6) and (7) for the radial excited mesons are worse that for orbital ones. For this reason we use in Table 2 the maximal systematical uncertainty for mass values.

4. Discussion and conclusions

It is useful to compare the mass values evaluated with the phenomenological mass formulae (5), (6) and (7) and shown in the Tables 1 and 2 with the values obtained in the framework of other approaches. As it was noted in the previous section the evaluated values are in a concordance with the values obtained in the relativistic model of quasi-independent quarks moving in the potential $\sim -a/r + \gamma^0 kr$ [15, 19]. However, the mass values of the first radial of ρ - meson and orbitally excited vector meson in the framework of the phenomenological approach considered show a discrepancy with data. We offer the mixing of these states in order to put mass values into accordance with data. It is possible, that at present the experimental data and their interpretation are not complete in this region and additional efforts are needed to clarify the situation. For instance, in the framework of the well-known potential model [20] the mass values of $2\ ^3S_1$, 1^3D_1 and $3\ ^3S_1$ states lie considerably higher (at 1.45,1.66, 2.00 GeV correspondingly) than the predictions of the method considered. Moreover, in Ref. [21] the mass value $1486 MeV$ has been evaluated the first radial excitation of ρ - meson.

Another importan problem is the interpretation of mesons discovered in scalar channel. Although, the mass values of the $a_0(980)$ and $f_0(980)$ mesons show the ideal mixing between them, the problems with the intensities of different decay modes exist (in particular the $K\overline{K}$ decay mode enhancement). The most widespread explanation of these facts now consist in the four-quark nature of the $a_0(980)$ and $f_0(980)$ mesons [22, 23, 24]. However, if one use typical mass values for scalar diquarks [25], estimations of mass values for lowest diquark-antidiquark scalar mesons give $\sim 1200 \div 1400 MeV$. The results of our evaluations (Table 1) also support the existence of the $P-$wave $\overline{q}q'-$mesons with masses $\sim 980\ MeV$. There is an accidental degeneracy of their mass values with the value of $\overline{K}K-$threshold, which complicates a mechanism of an extraction of $a_0(980)$ and $f_0(980)$ decays characteristics [26]. Moreover it is possible, that mixing of these mesons with a $\overline{K}K-$molecule arises [27]. The complementary reasoning in favour of the existence of the $P-$wave $\overline{q}q'-$mesons with masses $\sim 980\ MeV$ is the coincidence of the evaluated mass value for the first radial excitation of $a_0(980)$ meson (Table 2) with the mass value of the $a_0(1450)$ meson [17].

The problems discussed above are the problems of the phenomenological scheme for meson mass evaluations considered, as well as they represent the actual and unsolved during a long time problems of light meson spectroscopy.

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Table 1.
Evaluated masses in MeV for the ground states and the orbital excitations of the $\bar{q}q'-$ mesons in comparison with the data from Ref.[17].

Meson	J^{PC}	$M^{exp}(MeV)^{[17]}$	$M^{ev}(MeV)$	Meson	J^{PC}	$M^{exp}(MeV)^{[17]}$	$M^{ev}(MeV)$
π	0^{-+}	138 ± 3.1	138 ± 30	ρ_3	3^{--}	1688.8 ± 2.1	1722 ± 30
ρ	1^{--}	775.8 ± 0.5	770 ± 30	a_2^3	2^{++}	-	1873 ± 30
a_0	0^{++}	984.7 ± 1.2	998 ± 30	b_3^3	3^{+-}	-	2024 ± 30
b_1	1^{+-}	1229.5 ± 3.2	1227 ± 30	a_3^3	3^{++}	-	2031 ± 30
a_1	1^{++}	1230 ± 40	1280 ± 30	a_4	4^{++}	2040 ± 12	2037 ± 30
a_2	2^{++}	1318.3 ± 0.6	1334 ± 30	a_3^4	3^{--}	-	2177 ± 30
a_1^2	1^{--}	-	1506 ± 30	b_4^4	4^{-+}	-	2316 ± 30
π_2	2^{-+}	1672.4 ± 3.2	1678 ± 30	a_4^4	4^{--}	-	2313 ± 30
a_2^2	2^{--}	-	1700 ± 30	a_5^4	5^{--}	-	2310 ± 30

Table 2.
Evaluated masses in MeV for the radial excitations (from $n^r = 1$ to $n^r = 4$) of the pseudoscalar, vector and scalar $\bar{q}q'-$ mesons in comparison with the data from Ref.[17].

Meson	J^{PC}	$M^{exp}(MeV)^{[17]}$	$M^{ev}(MeV)$	Meson	J^{PC}	$M^{exp}(MeV)^{[17]}$	$M^{ev}(MeV)$
π'	0^{-+}	1300 ± 100	1227 ± 40	ρ'''	1^{--}	-	2037 ± 40
π''	0^{-+}	1812 ± 14	1678 ± 40	ρ^{IV}	1^{--}	-	2310 ± 40
π'''	0^{-+}	-	2024 ± 40	a_0'	0^{++}	1474 ± 12	1506 ± 40
π^{IV}	0^{-+}	-	2316 ± 40	a_0''	0^{++}	-	1873 ± 40
ρ'	1^{--}	1465 ± 25	1334 ± 40	a_0'''	0^{++}	-	2177 ± 40
ρ''	1^{--}	1720 ± 20	1722 ± 40	a_0^{IV}	0^{++}	-	2441 ± 40