

# Cahn and Sivers effects in the target fragmentation region of SIDIS

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## Abstract

LEPTO event generator is modified to describe the azimuthal modulations arising from Cahn and Sivers effects. The comparisons with some existing data in the current fragmentation region of SIDIS are presented. The predictions for Cahn and Sivers asymmetries in the target fragmentation region are presented for SIDIS of 12 GeV electrons off proton target.

## 1 Introduction

In [1] the role of parton intrinsic motion in semi-inclusive DIS (SIDIS) processes within QCD parton model has been considered at leading order; intrinsic  $\mathbf{k}_\perp$  is fully taken into account in quark distribution functions and in the elementary processes as well as the hadron transverse momentum,  $\mathbf{p}_\perp$ , with respect to fragmenting quark momentum, see Fig. 1<sup>1</sup>. The average values of  $k_\perp$  for quarks inside protons and  $p_\perp$  for final hadrons inside the fragmenting quark jet were fixed by a comparison with data on Cahn effect [2] – the dependence of the unpolarized cross section on the azimuthal angle between the leptonic and the hadronic planes. The single spin asymmetry (SSA)  $A_{UT}^{\sin(\phi_\pi - \phi_S)}$  recently observed by HERMES Collaboration [3] was successfully described by Sivers mechanism [4]. It was also shown that the Sivers distribution functions resulting from this analysis are compatible with the preliminary data from COMPASS collaboration [5].

The description of SIDIS within standard QCD parton model approach using the distribution and fragmentation functions is valid only in the current fragmentation region, CFR ( $x_F > 0$ ) and at high energies. A more general approach allowing to describe SIDIS in the whole kinematic region available for final hadrons is based on the LUND string

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<sup>1</sup>In the following the notations of [1] are used.

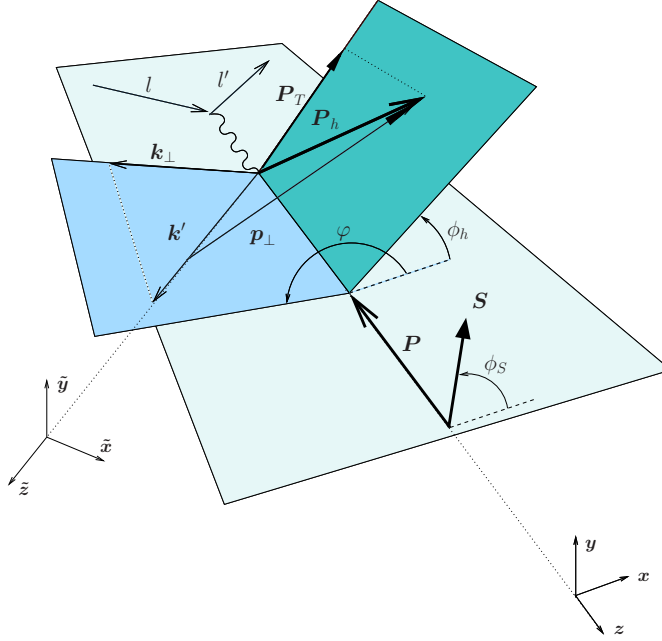


Figure 1: Three dimensional kinematics of the SIDIS process.

fragmentation model and is incorporated into LEPTO event generator [6]. In the simplest case, corresponding to LO approximation of parton model, event generation in LEPTO proceeds in several steps:

1. The active quark inside the nucleon is chosen according to the quark density function  $f_q(x, Q^2)$ ,
2. The hard scattering kinematics is generated,
3. The transverse momentum of the final quark is simulated with Gaussian  $k_\perp$  and flat  $\varphi$  distributions. Note that the transverse momentum of the final final quark is equal to that of initial quark for leading order hard subprocesses.
4. The string fragmentation machinery of JETSET program [7] is applied to form the final hadrons.

Within this approach the SIDIS cross section at LO can be expressed as

$$\frac{d^5 \sigma^{\ell p \rightarrow \ell h X}}{dx dQ^2 dx_F d^2 \mathbf{P}_T} = \sum_q \int d^2 \mathbf{k}_\perp f_q(x, k_\perp) \frac{d\hat{\sigma}^{\ell q \rightarrow \ell q}}{dQ^2} H_{q/N}^h(x, x_F, \mathbf{k}_\perp, \mathbf{P}_T), \quad (1)$$

where  $\frac{d\hat{\sigma}^{\ell q \rightarrow \ell q}}{dQ^2}$  is the lepton–quark hard scattering cross section and  $H_{q/N}^h(x, x_F, \mathbf{k}_\perp, \mathbf{P}_T)$  represents the hadronization function of the system formed by struck quark with transverse momentum  $\mathbf{k}_\perp$  and target remnant. In the standard version of LEPTO the quark distribution function and the LO lepton–quark cross section are independent of  $\varphi$ , and, thus the final quarks are uniformly distributed in azimuthal angle. For final hadrons this implies also a uniform azimuthal distribution. However, already in unpolarized SIDIS the observed azimuthal distribution of hadrons is not flat.

In this paper two types of azimuthal modulation at the quark level and their influence on the produced hadron azimuthal distribution will be considered:

- azimuthal modulation of the hard scattering cross section in unpolarized SIDIS (Cahn effect)
- azimuthal modulation of the initial quark transverse momentum in SIDIS of unpolarized leptons off the transversely polarized nucleon (Sivers effect).

It is possible to incorporate both effects in the LEPTO event generator and obtain predictions for azimuthal asymmetries in the whole kinematical region for the final hadrons. The way how the LEPTO code is modified to include Cahn and Sivers effects are described in Sec. 2 and Sec. 3, respectively. In Sec. 4 some discussion and conclusions are presented.

## 2 Including Cahn effect in LEPTO

The Cahn effect [2] is a kinematical effect arising due to the presence of nonzero intrinsic transverse momentum of quarks in the nucleon. The underlying physics is very simple. The lepton–quark scattering cross section is given by the QED expression

$$d\hat{\sigma}^{\ell q \rightarrow \ell q} \propto \hat{s}^2 + \hat{u}^2. \quad (2)$$

In the general case of non collinear kinematics Mandelstam variables depend on the quark transverse momentum and its azimuthal angle and at order  $\mathcal{O}(k_{\perp}/Q)$  one has

$$\begin{aligned} \hat{s}^2 &= \frac{Q^4}{y^2} \left( 1 - 4 \frac{k_{\perp}}{Q} \sqrt{1-y} \cos \varphi \right), \\ \hat{u}^2 &= \frac{Q^4}{y^2} (1-y)^2 \left( 1 - 4 \frac{k_{\perp}}{Q} \frac{\cos \varphi}{\sqrt{1-y}} \right). \end{aligned} \quad (3)$$

Then, the lepton–quark elastic scattering cross section is given by

$$d\hat{\sigma}^{\ell q \rightarrow \ell q} \propto 1 - \frac{(2+y)\sqrt{1-y}}{1+(1-y)^2} \frac{k_{\perp}}{Q} \cos \varphi. \quad (4)$$

Eq. (4) shows that the azimuthal angle of the final quark (and of the string’s end associated with the struck quark) is now modulated with amplitude depending on  $y, Q$  and  $k_{\perp}$ .

This effect can be introduced in the LEPTO event generator at the step 3) of the event generation, when the transverse momentum and azimuthal angle of the scattered quark are generated. To do this the generation of the quark transverse momentum,  $k_{\perp}$ , is left unchanged and then the azimuthal angle is generated according to Eq. (4). This leads to azimuthal modulation of the string axis (axis  $\tilde{z}$  on Fig. 1). The momentum conservation means that the transverse momentum of the quark is balanced by that of the target remnant, which in turn means that the azimuthal angle of the target remnant  $\varphi_{qq} = \varphi + \pi$ . Then, one expects that the azimuthal angle of the hadrons in the target fragmentation region (TFR),  $x_F < 0$ , will be modulated with shifted a phase by  $\pi$  with respect to that in CFR.

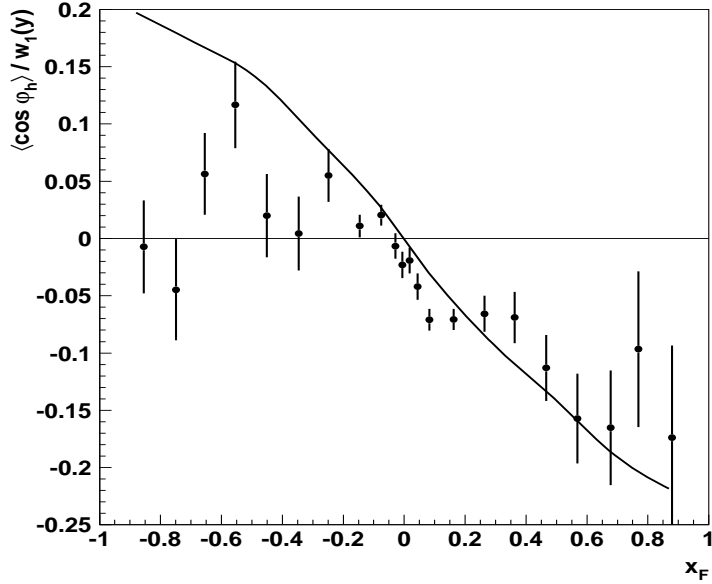


Figure 2: The  $x_F$  dependence of  $\langle \cos \phi_h \rangle / w_1(y)$  for charged hadrons compared with EMC data.

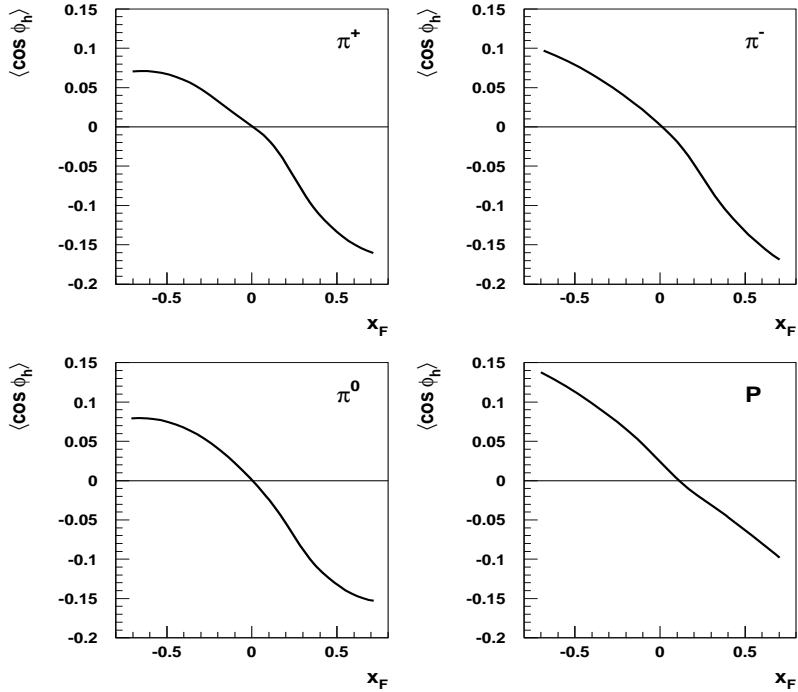


Figure 3: Predictions of modified LEPTO for  $x_F$  dependence of  $\langle \cos \phi_h \rangle$  for different hadrons produced in 12 GeV unpolarized SIDIS process.

Data on azimuthal dependencies of SIDIS covering a large  $x_F$  range have been obtained by the EMC Collaboration [8] for a beam energy of 280 GeV. The  $x_F$  dependence of  $\langle \cos \phi_h \rangle / w_1(y)$ , where  $w_1(y) = (2 - y)\sqrt{1 - y} / (1 + (1 - y)^2)$ , obtained by using modified LEPTO for EMC kinematics are presented in Fig. 2 together with data points from [8]. The simulations has been done with LO setting of LEPTO (LST(8)=0) and with values of the parameters describing intrinsic  $k_T$  (PARL(3)=0.5) and fragmentation  $p_T$  (PARL(21)=0.45) as adopted in [1].

The predictions of modified LEPTO for  $\langle \cos \phi_h \rangle$  of different hadron ( $\pi^+$ ,  $\pi^-$ ,  $\pi^0$  and  $p$ ) produced in SIDIS on a proton target at future CEBAF 12 GeV facility at JLab [9] are presented in Fig. 3. One can see from Fig. 2 and Fig. 3 that the predicted mean value of  $\cos \phi_h$  in the CFR is negative  $\langle \cos \phi_h \rangle_{CFR} < 0$ , while in the TFR is positive  $\langle \cos \phi_h \rangle_{TFR} > 0$ , as suggested by arguments based on transverse momentum conservation.

### 3 Including Sivers effect in LEPTO

The azimuthal modulation of the string transverse momentum in the previous section was due to Cahn effect – the dependence of the non planar lepton-quark scattering cross section on the quark azimuth. The quark distribution,  $f_q(x, k_\perp)$  itself is independent of quark azimuthal angle.

The situation is different when one considers SIDIS on a transversely polarized nucleon. Now a correlation between transverse momentum of quark and target transverse polarization is possible – the so called Sivers effect [4]. For quite some time it was believed that this correlation is forbidden because of T-invariance of the strong interactions. However the spectator model calculations [10] demonstrated that there exists a nonzero SSA in SIDIS when the final state interaction between struck quark and target remnant is taken into account. Then, the effective description of this SSA is possible within QCD factorized approach by introducing a new distribution function – the Sivers function (for further discussion see, for example, [11]).

The unpolarized quark (and gluon) distributions inside a transversely polarized proton (generically denoted by  $p^\uparrow$ , with  $p^\downarrow$  denoting the opposite polarization state) can be written as:

$$f_{q/p^\uparrow}(x, \mathbf{k}_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp), \quad (5)$$

where  $\mathbf{P}$  and  $\mathbf{S}_T$  are respectively the proton momentum and transverse polarization vector, and  $\mathbf{k}_\perp$  is the parton transverse momentum; transverse refers to the proton direction. Eq. (5) implies

$$\begin{aligned} f_{q/p^\uparrow}(x, \mathbf{k}_\perp) + f_{q/p^\downarrow}(x, \mathbf{k}_\perp) &= 2f_{q/p}(x, k_\perp), \\ f_{q/p^\uparrow}(x, \mathbf{k}_\perp) - f_{q/p^\downarrow}(x, \mathbf{k}_\perp) &= \Delta^N f_{q/p^\uparrow}(x, k_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp), \end{aligned} \quad (6)$$

where  $f_{q/p}(x, k_\perp)$  is the unpolarized parton density and  $\Delta^N f_{q/p^\uparrow}(x, k_\perp)$  is referred to as the Sivers function. Notice that, as requested by parity invariance, the scalar quantity  $\mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp)$  singles out the polarization component perpendicular to the  $\mathbf{P} - \mathbf{k}_\perp$  plane. For a proton moving along  $-z$  and a generic transverse polarization vector  $\mathbf{S}_T = |\mathbf{S}_T| (\cos \phi_S, \sin \phi_S, 0)$  (see Fig. 1) one has:

$$\mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp) = |\mathbf{S}_T| \sin(\varphi - \phi_S) \equiv |\mathbf{S}_T| \sin \phi_{Siv}, \quad (7)$$

where  $(\varphi - \phi_S) = \phi_{Siv}$  is the Sivers angle.

In [1] the Sivers function for each light quark flavor  $q = u, d$  are parameterized in the following factorized form:

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = 2\mathcal{N}_q(x) h(k_\perp) f_{q/p}(x, k_\perp), \quad (8)$$

where

$$\mathcal{N}_q(x) = N_q x^{a_q} (1-x)^{b_q} \frac{(a_q + b_q)^{(a_q + b_q)}}{a_q^{a_q} b_q^{b_q}}, \quad (9)$$

$$h(k_\perp) = \sqrt{2} e \frac{k_\perp}{M} e^{-k_\perp^2/M^2}, \quad (10)$$

where  $N_q$ ,  $a_q$ ,  $b_q$  and  $M$  (GeV/c) are parameters. Then Eq. (5) can be rewritten as

$$f_{q/p^\uparrow}(x, \mathbf{k}_\perp) = f_{q/p}[x, k_\perp] (1 + |\mathbf{S}_T| \mathcal{N}_q(x) h(k_\perp) \sin \phi_{Siv}). \quad (11)$$

Again, the Sivers effect is incorporated into LEPTO at the stage 3) of the event generation in the same way as for the Cahn effect but now the azimuthal angle is generated according to Eq. (11). For simulations the following set of parameters compatible with those obtained in [1] have been used:  $N_u = N_{\bar{u}} = 0.5$ ,  $N_d = N_{\bar{d}} = -0.2$ ,  $a_q = 0.3$ ,  $b_q = 2$  and  $M^2 = 0.36$  (GeV/c)<sup>2</sup>.

In Fig. 4 the results of simulation for HERMES experimental conditions are compared with observed Sivers asymmetries [3].

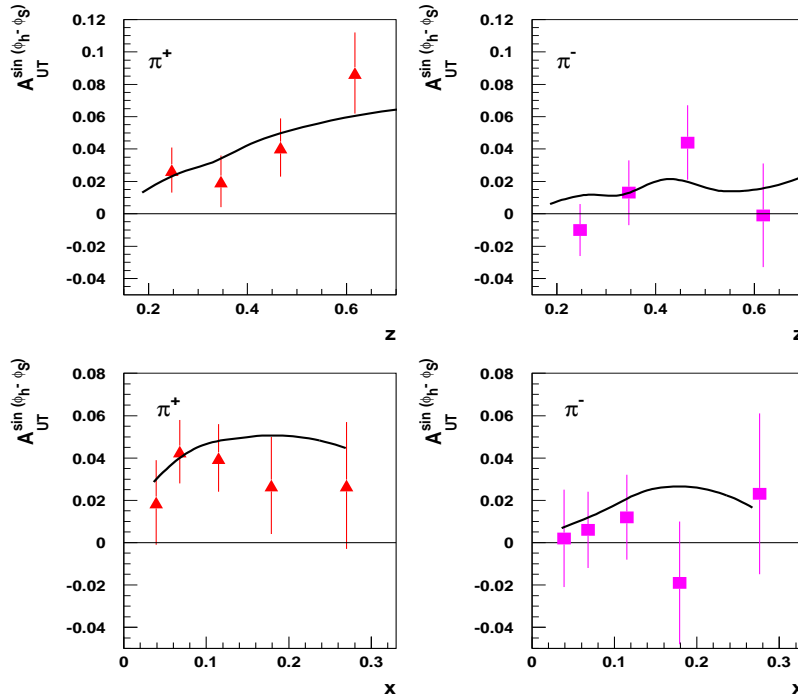


Figure 4: HERMES data on  $A_{UT}^{\sin(\phi_\pi - \phi_S)}$  [3] for scattering off a transversely polarized proton target. The curves are the results of simulations obtained with modified LEPTO.

Future facilities as Electron Ion Colliders or upgraded JLab will have larger kinematic coverage and will offer the possibility of studying the Sivers effect also with hadrons produced in the TFR. As an example, the simulations have been done for 12 GeV electron SIDIS of a proton target. The DIS cut  $Q^2 > 1(\text{GeV}/c)^2$  and  $W^2 > 4\text{GeV}^2$  and a cut on the produced hadron transverse momentum  $P_T > 0.05$  GeV/c was imposed. The predictions for  $x_F$ ,  $x$  and  $P_T$  dependencies for JLab kinematics are presented on the Fig. 5, Fig. 6 and Fig. 7, respectively.

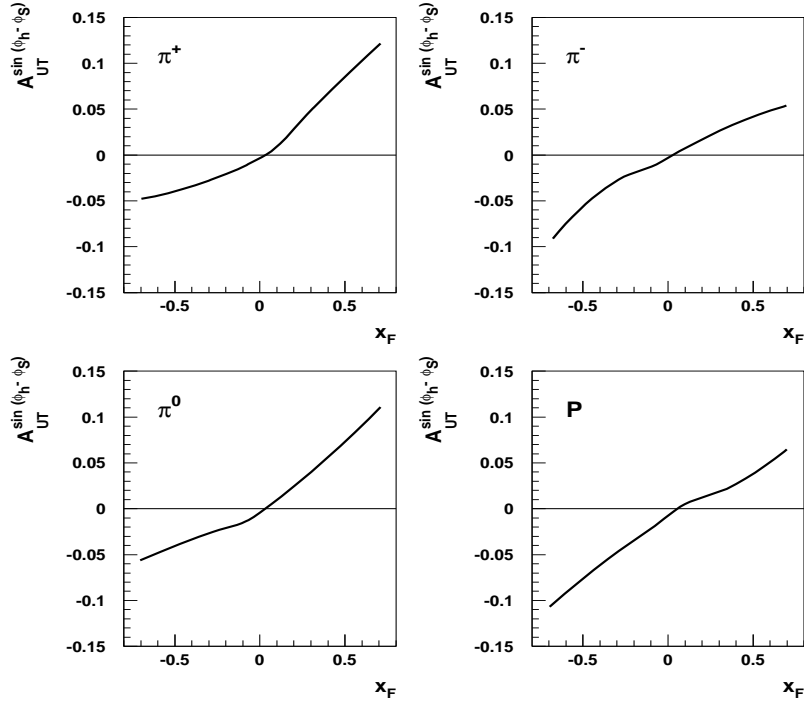


Figure 5: Predicted dependence of  $A_{UT}^{\sin(\varphi_h - \varphi_S)}$  on  $x_F$  for different hadrons produced in SIDIS of 12 GeV electrons off a transversely polarized proton target.

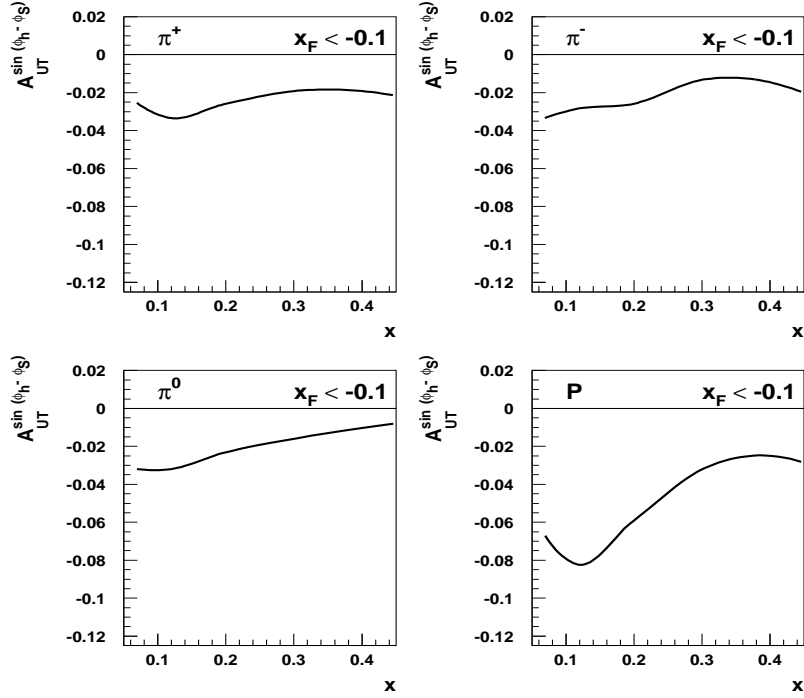


Figure 6: Predicted dependence of  $A_{UT}^{\sin(\varphi_h - \varphi_S)}$  on  $x$  for different hadrons produced in the TFR ( $x_F < -0.1$ ) of SIDIS of 12 GeV electrons off a transversely polarized proton target.

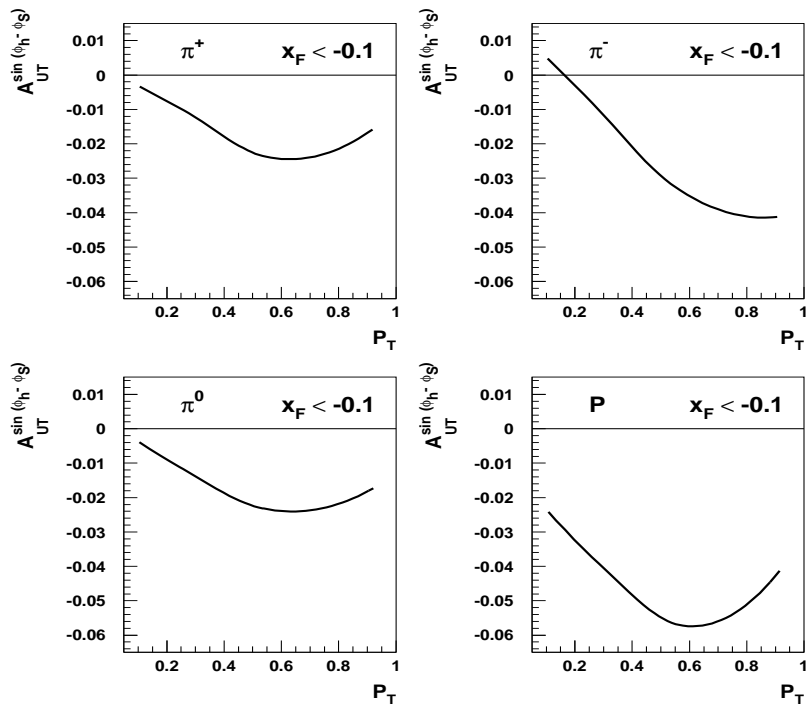


Figure 7: Predicted dependence of  $A_{UT}^{\sin(\phi_h - \phi_S)}$  on  $p_T$  for different hadrons produced in the TFR ( $x_F < -0.1$ ) of SIDIS of 12 GeV electrons off a transversely polarized proton target.

## 4 Discussion and Conclusions

In this article the way of modifying the standard LEPTO event generator in order to include the azimuthal asymmetries arising from Cahn and Sivers effects is described. Only LO effects have been taken into account. The azimuthal modulations for Cahn and Sivers effects have different origins. In the case of Cahn effect the initial quark transverse momentum is independent of azimuthal angle but the hard scattering cross section in a non planar kinematics depends on the final quark azimuthal angle. In the case of Sivers effect already the initial quark transverse momentum has an azimuthal modulation. The azimuthal asymmetries are introduced in both cases by changing the struck quark/string azimuthal distribution during event generation. The hadronization part of program (JETSET) is left unchanged. The possible influence of the higher twist distribution functions as well as possible modifications of hadronization in the case of polarized target [12] have been ignored.

The advantage of this MC based approach compared to standard QCD factorized approach is in the full coverage of produced hadron phase space. The modified generator will be useful for complete MC simulations of experiments including Cahn and Sivers effects both in the CFR and in the TFR and also for global analysis of these effects.

Figs. 2 and 4 demonstrate that the modified LEPTO event generator well describing the data in the CFR both for Cahn and Sivers asymmetries. The description of Cahn effect in the TFR looks unsatisfactory. This discrepancy can be explained either by some unaccounted contributions in the TFR or by insufficient precision of experimental data. One can notice from the experimental points in Fig. 2 that the integrated value of  $\langle \cos \phi_h \rangle$  for charged hadrons in the CFR is not compensated by that in TFR. It seems improbable that this imbalance can be compensated by larger values of  $\langle \cos \phi_h \rangle$  of neutral hadrons



at  $x_F \simeq -1$ .

The new high statistic measurements will allow to check the predictions of the approach presented here and better understand the effects of the quark intrinsic transverse momentum and hadronization mechanism in SIDIS.

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