The matter effects to neutrino oscillations $\nu_\mu \to \nu_e, \nu_\mu$ at very long baselines and the neutrino mixing angles θ_{13} and θ_{23}

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Abstract

We study the matter effects to neutrino oscillations $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_\mu$ at very long baselines. It has been observed that, for a very long baseline $L \simeq 10000$ km, the resonance peak for the $\nu_\mu \to \nu_e$ oscillation and the local minimum of the ν_μ survival probability occurs at similar energies. With a good knowledge on the absolute value of Δm^2_{31}, measurements of the above oscillation probabilities, $P_{\mu e}$ and $P_{\mu\mu}$, can be performed with the neutrino energy tuned to the resonance peak of the $\nu_\mu \to \nu_e$ oscillations. We show that the variations of CP violating phase and the solar neutrino mixing parameters have negligible effects on $P_{\mu e}$ and $P_{\mu\mu}$ for such a long baseline. Hence $P_{\mu e}$ and $P_{\mu\mu}$ together determine the mixing angles θ_{13} and θ_{23}. Around the resonance peak for $\nu_\mu \to \nu_e$ oscillations, the dependencies of $P_{\mu e}$ and $P_{\mu\mu}$ on the above mixing angles are worked out in detail, and the implications of such dependencies are discussed.

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The understanding of neutrino masses and mixing matrix is crucial to unveil the mystery of lepton flavor structure. The updated SKK analysis of the atmospheric neutrino data gives [1]

$$1.9 \cdot 10^{-3} \text{ eV}^2 < |\Delta m^2_{31}| < 3.0 \cdot 10^{-3} \text{ eV}^2, \ \sin^2 2\theta_{23} > 0.9. \tag{1}$$

This is a 90% C.L. range with the best fit values given by $\sin^2 2\theta_{23} = 1$ and $\Delta m^2_{31} = 2.4 \cdot 10^{-3}$ eV^2 respectively. The scenario of $\nu_\mu \to \nu_\tau$ oscillation for atmospheric neutrinos has been confirmed by the K2K experiment [2]. Furthermore the results in the solar neutrino oscillation measurements are also confirmed by KamLAND reactor measurements [3, 4]. Combining these measurements, the LMA solution of the solar neutrino problem is established and the updated 3σ parameter ranges are given by [5]

$$7.4 \cdot 10^{-5} \text{ eV}^2 < \Delta m^2_{21} < 9.2 \cdot 10^{-5} \text{ eV}^2, \ 0.28 < \tan^2\theta_{12} < 0.58, \tag{2}$$

with the best fit values $\Delta m^2_{21} = 8.2 \cdot 10^{-5}$ eV^2 and $\tan^2\theta_{12} = 0.39$.

Despite the achievements so far in measuring the neutrino mixing parameters, the sign of Δm^2_{31}, the mixing angle θ_{13} and the CP violating parameter $\delta_{\rm CP}$ in the mixing matrix remain to be determined. The sign of Δm^2_{31} can be determined through matter effects to $\nu_\mu \to \nu_e$ ($\bar\nu_\mu \to \bar\nu_e$) and $\nu_e \to \nu_\mu$ ($\bar\nu_e \to \bar\nu_\mu$) oscillations in the very long baseline experiments [6, 7]. However, such oscillations are sensitive to the mixing angle θ_{13}, which has been constrained by the reactor experiments [8, 9]. The CHOOZ experiment [8] gives a more stringent constraint on θ_{13} with $\sin^2 2\theta_{13} < 0.1$ for a large Δm^2_{31} (90% C.L.). A recent global fit gives $\sin^2 2\theta_{13} < 0.09$ (0.18) at 90% C.L. (3σ) [10].

It is known that the oscillation $\nu_\mu \to \nu_e$, for example, is enhanced (suppressed) by the matter effect for $\Delta m^2_{31} > 0$ ($\Delta m^2_{31} < 0$). On the contrary, the $\bar\nu_\mu \to \bar\nu_e$ oscillation is enhanced (suppressed) by the matter effect for $\Delta m^2_{31} < 0$ ($\Delta m^2_{31} > 0$). In the following discussions we shall assume $\Delta m^2_{31} > 0$ unless explicitly specified. It is noteworthy that, although the matter enhanced angle θ^m_{13} reaches to $\pi/4$ at the resonance energy, the oscillation probability, $P(\nu_\mu \to \nu_e) \equiv P_{\mu e}$, remains to be small unless for a very long baseline. Indeed, it is demonstrated that [11], in the constant density approximation for the Earth density profile, the condition for a maximal $P_{\mu e}$ reads:

$$\rho L^{\rm max} \simeq \frac{(2n+1)\pi}{\tan 2\theta_{13}} \times 5.18 \cdot 10^3 \text{ km} \cdot \text{g/cm}^3, \tag{3}$$

where ρ is the averaged mass density of the Earth, $L^{\rm max}$ is the required oscillation length, and n is an integer. For $n = 0$ and $\sin^2 2\theta_{13} = 0.1$, $L^{\rm max}$ is found to be 10200 km [12]. A smaller $\sin^2 2\theta_{13}$ requires an even larger oscillation length. Clearly, to take a full advantage of the matter enhancement, one is led to consider experiments with a baseline of the order of the Earth diameter [13].

It has been demonstrated that the matter effects to $P(\nu_\mu \to \nu_\tau) \equiv P_{\mu\tau}$ and $P(\nu_\mu \to \nu_\mu) \equiv P_{\mu\mu}$ can also be significant in the vicinity of $P_{\mu e}$'s resonance peak [12, 14]. In fact, the criterion for the maximal matter effect to $P_{\mu\tau}$ is identified to be $E_{\rm res} \simeq E^{\rm vac}_{\rm peak}$ where the former is the resonance energy for the oscillation $P_{\mu e}$ while the latter is the energy that the vacuum approximation of $P_{\mu\tau}$ attains its peak value. Such a criterion amounts to [12, 14]

$$\rho L^{\rm max} \simeq (2n+1)\pi(\cos 2\theta_{13}) \times 5.18 \cdot 10^3 \text{ km} \cdot \text{g/cm}^3. \tag{4}$$

For $n = 0$ and $\sin^2 2\theta_{13} = 0.1$, $L^{\rm max} \simeq 4400$ km, while, for $n = 1$ with the same $\sin^2 2\theta_{13}$ value, $L^{\rm max} \approx 9700$ km. In the latter case, $P_{\mu\tau}$ is decreased from its vacuum oscillation value by as much as 70%. This implies that the muon neutrino survival probability $P_{\mu\mu}$ also receives significant matter effects in the same energy range.

It is interesting to note that the matter effect to $P_{\mu e}$ and that to $P_{\mu\mu}$ are both significant in the same energy range for baselines of the order 10^4 km. Hence simultaneous measurements of $P_{\mu e}$ and $P_{\mu\mu}$ in such a baseline provide a stringent test on the matter effect, which is the focus of this work. To discuss these oscillations, we begin with the relation connecting flavor and mass eigenstates of neutrinos, $\nu_\alpha = \sum_i U_{\alpha i}\nu_i$, with U the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix given by

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix}, \tag{5}$$

where s_{ij} and c_{ij} denote $\sin\theta_{ij}$ and $\cos\theta_{ij}$, respectively. For the Dirac type CP-phase $\delta_{\rm CP}$, we allow its value ranging from 0 to 2π. The evolutions of neutrino flavor eigenstates are governed by the equation

$$i\frac{d}{dt}|\nu(t)\rangle = \left\{ \frac{1}{2E_\nu} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} V & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right\} |\nu(t)\rangle\,, \tag{6}$$

where $|\nu(t)\rangle = (\nu_e(t), \nu_\mu(t), \nu_\tau(t))^T$, $\Delta m^2_{ij} \equiv m^2_i - m^2_j$ is the mass-squared difference between the i-th and j-th mass eigenstates, and $V \equiv \sqrt{2}G_F N_e$ is the effective potential arising

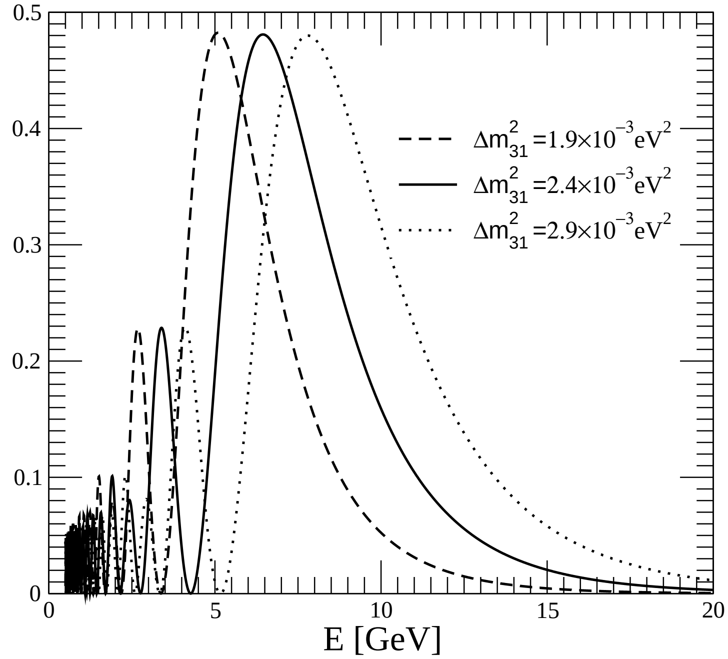

FIG. 1: The Δm^2_{31} dependence of the transition probability $P_{\mu e}$ for $\sin^2 2\theta_{13} = 0.1$, $\sin^2 2\theta_{23} = 1$, $\delta_{\rm CP} = 0$, and the baseline length $L = 9300$ km.

from the charged current interaction between ν_e and electrons in the medium with N_e the electron number density. Numerically $V = 7.56 \times 10^{-14}$ $(\rho/[\mathrm{g/cm^3}])Y_e[\mathrm{eV}]$ with Y_e denoting the number of electrons per nucleon. For the Earth matter, $Y_e \sim 0.5$. One solves Eq. (6) by diagonalizing the Hamiltonian on its right hand side. This amounts to writing the right hand side of Eq. (6) as $U'H'U'^{\dagger}|\nu(t)\rangle$ with U' the neutrino mixing matrix in the matter and $H' \equiv \mathrm{diag}(E_1, E_2, E_3)$ the Hamiltonian after diagonalization. To obtain various oscillation probabilities described later, we have used the parametrization in [15] for the Earth density profile [16]. For illustration, we shall take $L = 9300$ km, which is a distance between Fermilab and Kamioka [13]. This distance is in fact very ideal for studying $P_{\mu e}$ and $P_{\mu\mu}$ since both oscillations receive significant matter effects around such a distance.

To measure $P_{\mu e}$ and $P_{\mu\mu}$ near the resonance peak of the $\nu_\mu \to \nu_e$ oscillation, it is crucial to know the precise value of Δm^2_{31}. The current range for this parameter, Eq. (1), results in a corresponding range for the peak energy of $P_{\mu e}$ as shown in Fig. 1. The energy value at the peak of $P_{\mu e}$ is proportional to Δm^2_{31} as dictated by the resonance condition $2E_\nu V = \Delta m^2_{31} \cos\theta_{13}$ while the maximal magnitude of $P_{\mu e}$ is independent of Δm^2_{31}. This is easily understood by inspecting the expression for $P_{\mu e}$ in the constant density approximation [17]:

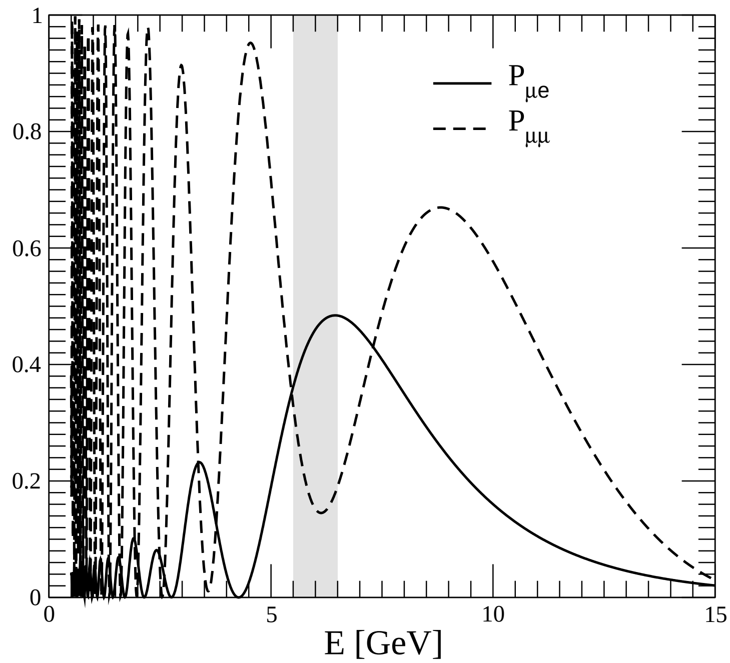

FIG. 2: The transition probabilities $P_{\mu e}$ and $P_{\mu\mu}$ for a baseline length $L = 9300$ km with $\sin^2 2\theta_{13} = 0.1$, $\sin^2 2\theta_{23} = 1$ and $\delta_{\rm cp} = 0$. The gray zone represents the region with the neutrino energy E_ν ranging from 5.5 GeV to 6.5 GeV, where the maximum of $P_{\mu e}$ and the local minimum of $P_{\mu\mu}$ are located.

$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\theta_{13}^{\rm m}\sin^2\left(1.27\Delta_{31}^{\rm m}L/E_\nu\right), \tag{7}$$

with L in km, E_ν in eV,

$$\Delta_{31}^{\rm m} = \sqrt{\left(\Delta m_{31}^2\cos 2\theta_{13} - 2E_\nu V\right)^2 + \left(\Delta m_{31}^2\sin 2\theta_{13}\right)^2}, \tag{8}$$

and

$$\sin 2\theta_{13}^{\rm m} = \frac{\sin 2\theta_{13}\cdot\Delta m_{31}^2}{\Delta_{31}^{\rm m}}. \tag{9}$$

Clearly, at the resonance energy, the last factor on the right hand side of Eq. (7) does not depend on Δm_{31}^2.

As pointed out earlier, the matter effects to $P_{\mu e}$ and $P_{\mu\mu}$ are both significant in the same energy range for our interested baseline length. This is indicated by the gray zone of Fig. 2 where the energy dependencies of $P_{\mu e}$ and $P_{\mu\mu}$ are depicted. The probability $P_{\mu\mu}$ shows an intriguing feature in this energy zone. Due to the large matter effect, $P_{\mu\mu}$ does not drop to zero while reaching to its local minimum. For $\Delta m_{31}^2 = 2.4\times 10^{-3}$ eV^2, $\sin^2 2\theta_{13} = 0.1$ and

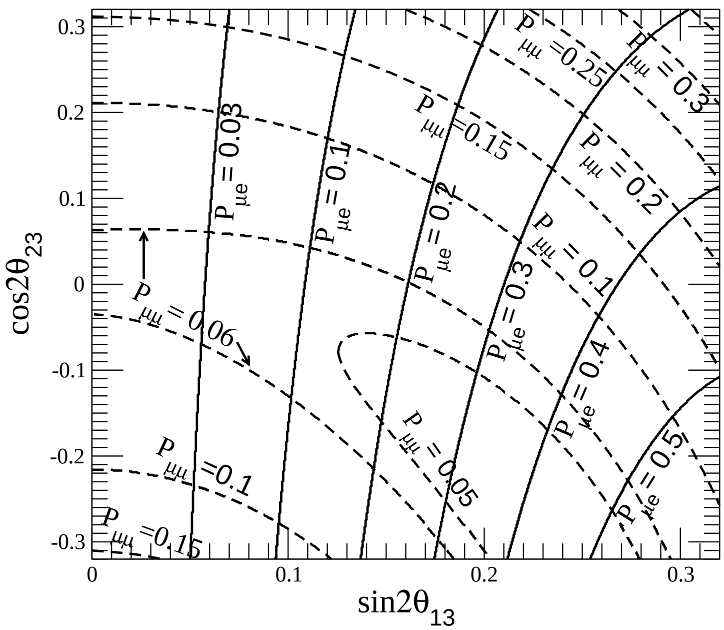

FIG. 3: The contour graph of the transition probabilities $P_{\mu e}$(solid line) and $P_{\mu\mu}$(dashed line) in the $\sin 2\theta_{13}$-$\cos 2\theta_{23}$ plane for $\delta_{\rm CP} = 0$ and the baseline length $L = 9300$ km. The plotted $P_{\mu e}$ and $P_{\mu\mu}$ are the averaged probabilities for neutrino energies ranging from 5.5 GeV to 6.5 GeV

$\sin^2 2\theta_{23} = 1$, the probability $P_{\mu e}$ peaks at 6.4 GeV while the local minimum for $P_{\mu\mu}$ is at 6.1 GeV. We have varied the values for θ_{13} and θ_{23}. The energy value for the maximal $P_{\mu e}$ and that for the local minimum of $P_{\mu\mu}$ are shifted. However, the shifts are within 300 MeV for both cases.

The probabilities $P_{\mu\mu}$ and $P_{\mu e}$ as functions of θ_{23} and θ_{13} are shown in Fig. 3. The range for $\sin 2\theta_{13}$ is taken from 0 to 0.32 in accordance with the reactor bound $\sin^2 2\theta_{13} < 0.1$ [8], while that for $\cos 2\theta_{23}$ is from -0.32 to 0.32 which is equivalent to $\sin^2 2\theta_{23} > 0.9$ [1]. The oscillation probabilities plotted here are the averaged ones for neutrino energies ranging from 5.5 GeV to 6.5 GeV. For $\Delta m^2_{31} = 2.4 \times 10^{-3}$ eV^2, both $P_{\mu e}$ and $P_{\mu\mu}$ receive large matter effects. We argue that these probabilities do reflect the relative event rates. In fact, to obtain neutrino event rates, one has to convolve the oscillation probabilities with the initial muon neutrino flux and the conversion rates from neutrinos to charged leptons. However, since our interested energy range is narrow enough, the neutrino-nucleon scattering cross section can be treated as a constant. The same is true for the ν_μ flux. We note that $P_{\mu e}$ is essentially only sensitive to $\sin 2\theta_{13}$ for $P_{\mu e} \leq 0.1$. However $P_{\mu e}$ is also sensitive to $\cos 2\theta_{23}$

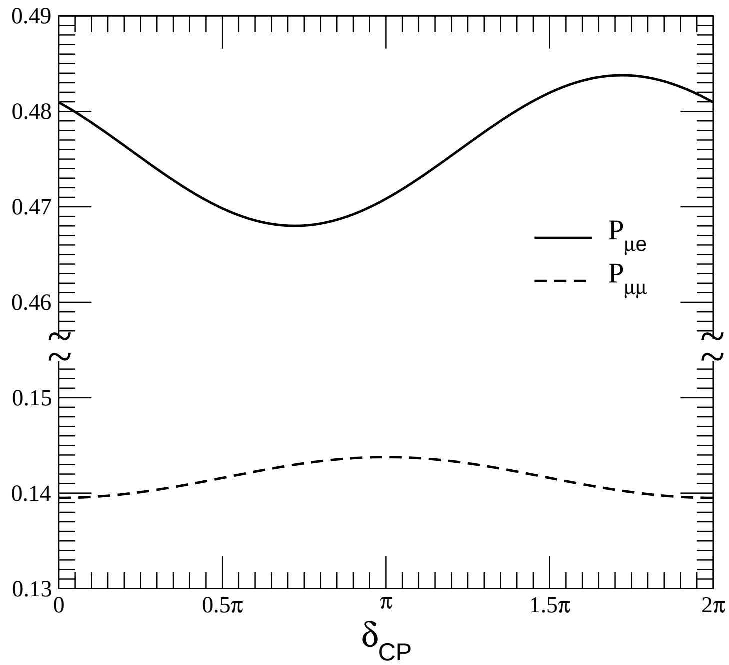

FIG. 4: The CP-phase dependencies of the transition probabilities $P_{\mu e}$ and $P_{\mu\mu}$ for $\sin^2 2\theta_{13} = 0.1$, $\sin^2 2\theta_{23} = 1$, and the baseline length $L = 9300$ km.

as it gets larger. The contours for constant $P_{\mu\mu}$'s show more complicated structures. We observe that, for a fixed θ_{13}, a given $P_{\mu\mu}$ yields two solutions for θ_{23} for $P_{\mu\mu} \leq 0.15$. The measurement of $P_{\mu e}$ might help to remove this degeneracy if $P_{\mu e}$ is large and consequently sensitive to $\cos 2\theta_{23}$. We also note that the above degeneracy no longer exists for $P_{\mu\mu} > 0.15$.

We point out that simultaneous measurements of $P_{\mu e}$ and $P_{\mu\mu}$ determine θ_{13} and θ_{23}. It is found that the variations of Δm^2_{21} and θ_{12} give negligible effects on the above oscillation probabilities. The effects of CP violating phase $\delta_{\rm CP}$ on $P_{\mu e}$ and $P_{\mu\mu}$ are also studied. With $\Delta m^2_{21} = 8.2 \cdot 10^{-5}$ eV^2, $\sin^2 2\theta_{23} = 1$, $\sin^2 2\theta_{13} = 0.1$, the dependencies of the peak value of $P_{\mu e}$ and the corresponding minimal value of $P_{\mu\mu}$ on $\delta_{\rm CP}$ are shown in Fig. 4. It is seen that the peak value of $P_{\mu e}$ differs by no more than 4% for the entire range of $\delta_{\rm CP}$. Similarly, the corresponding minimal value of $P_{\mu\mu}$ differs by no more than 3%.

The matter effect to $P_{\mu e}$ can be identified by the resonance enhancement to the oscillation $\nu_\mu \to \nu_e$, given our assumption that $\Delta m^2_{31} > 0$. On the other hand, the matter effect to $P_{\mu\mu}$ is seen by comparing Fig. 3 with a corresponding plot, Fig. 5, where matter effects to $P_{\mu\mu}$ are neglected. In Fig. 5, one can see that $P_{\mu\mu}$ is not sensitive to $\sin 2\theta_{13}$ and the contours for

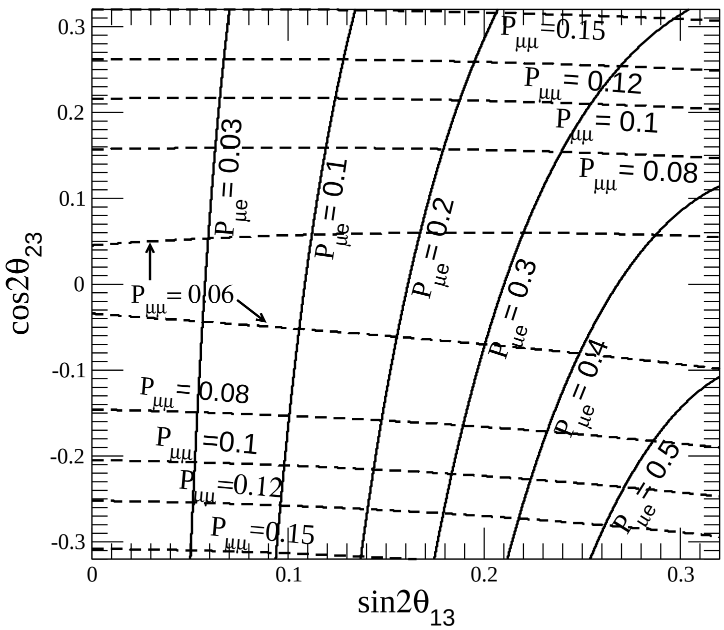

FIG. 5: The contour graph of the transition probabilities $P_{\mu e}$(solid line) and $P_{\mu\mu}$(dashed line) for $\delta_{\rm CP} = 0$ and the baseline length $L = 9300$ km. Vacuum oscillation is assumed for $P_{\mu\mu}$.

constant $P_{\mu\mu}$'s behave very differently from those in Fig. 3, particularly for a larger $\sin 2\theta_{13}$. We observe that, in the vacuum oscillation case, $P_{\mu\mu} \leq 0.15$ for almost all the parameter space in the $\sin 2\theta_{13}$-$\cos 2\theta_{23}$ plane. Hence an observation of $P_{\mu\mu}$ significantly greater than 0.15 is a signature of the matter effect. The parameter space for this to occur is situated in the upper right corner of $\sin 2\theta_{13}$-$\cos 2\theta_{23}$ plane. This is a region with $\theta_{23} < \pi/4$.

The matter effect to $P_{\mu\mu}$ can also be tested in other parameter regions. This is true when values of θ_{13} and θ_{23} are accurately measured in other future experiments. There are extensive discussions on measuring θ_{13} in reactor neutrino experiments [18, 19]. The issue of determining θ_{23} in the three-flavor oscillation framework has also been discussed in [20, 21, 22]. It is expected that $\sin^2 2\theta_{23}$ can be measured to an 1% accuracy in the J-PARC→ Super-Kamiokande (JPARC-SK) experiment [23]. This translates into a 10% accuracy for the determination of $\sin^2\theta_{23}$ for θ_{23} around $\pi/4$ at 90% C.L., provided $\sin^2 2\theta_{13}$ is accurately measured by the reactor experiment [22], say around 2% at 90% C.L. according to the analysis in [24]. Given the future accuracies for determining $\sin^2\theta_{23}$ and $\sin^2 2\theta_{13}$, it is possible to identify the matter effect in the muon survival probability $P_{\mu\mu}$. For illustration, let us assume, for example, $P_{\mu e}$ and $P_{\mu\mu}$ are measured to be 0.4 and 0.1 respectively in the very long baseline experiment, with some uncertainties understood for each measurement. Taking into account the matter effect to $P_{\mu\mu}$, these oscillation probabilities correspond to $(\sin 2\theta_{13}, \cos 2\theta_{23}) = (0.26, -0.03)$ according to Fig. 3. However, by assuming vacuum oscillations for $P_{\mu\mu}$, one obtains $(\sin 2\theta_{13}, \cos 2\theta_{23}) = (0.22, -0.22)$ according to Fig. 5. Therefore the extracted central values of $\sin 2\theta_{13}$ differ by 16% with and without the matter effects to $P_{\mu\mu}$ taken into account. This discrepancy is considerably larger than the expected uncertainty in the determination of $\sin 2\theta_{13}$ by the reactor experiment, which is approximately 4% with the above-mentioned uncertainty $\delta(\sin^2 2\theta_{13}) \simeq 2\%$ and the central value $\sin 2\theta_{13} = 0.26$. Hence the matter effect to $P_{\mu\mu}$ can in principle be identified by the extracted value of $\sin 2\theta_{13}$, provided there are sufficient accuracies in the measurements of $P_{\mu e}$ and $P_{\mu\mu}$. Furthermore, we note that $\sin^2\theta_{23} = 0.52$ with the matter effect to $P_{\mu\mu}$ taken into account while $\sin^2\theta_{23} = 0.61$ without the matter effect. The difference on the extracted value of $\sin^2\theta_{23}$ is about 17%, which is also larger than the expected 10% uncertainty in determining $\sin^2\theta_{23}$ by the JPARC-SK experiment. Therefore, if one can accurately measure $P_{\mu e}$ and $P_{\mu\mu}$, the matter effects to $P_{\mu\mu}$ can also be identified through the extracted value of $\sin^2\theta_{23}$.

In summary, we have discussed the matter effects to $\nu_\mu \to \nu_e, \nu_\mu$ oscillations at very long baselines. We illustrate these effects with a baseline $L = 9300$ km, the distance between Fermilab and Kamioka. Around this baseline length, the matter effects to $P_{\mu e}$ and $P_{\mu\mu}$ are both significant near the resonance energy for $\nu_\mu \to \nu_e$ oscillations. We have worked out in detail the dependencies of these probabilities on the mixing angles θ_{13} and θ_{23} with Δm^2_{31} taken to be 2.4×10^{-3} eV^2. We have also shown that these probabilities are not sensitive to the variations of other neutrino oscillation parameters. We have identified the parameter space in the $\sin 2\theta_{13}$-$\cos 2\theta_{23}$ plane in which the matter effect to $P_{\mu\mu}$ can be most easily identified. We also argued that it is possible to identify the same matter effect in the remaining parameter space provided θ_{13} and θ_{23} are accurately measured in other future neutrino experiments.

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