

Quark mass effects in the soft-collinear effective theory and $\bar{B} \rightarrow X_s \gamma$ in the endpoint region

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Abstract

We consider the effects of a light quark mass in the soft-collinear effective theory (SCET) and we apply them to $\bar{B} \rightarrow X_s \gamma$ in the endpoint region. We find that the reparameterization invariance can be extended by including the collinear quark mass in the SCET Lagrangian. This symmetry constrains the theory with the quark mass terms, and we present explicit results at one loop. It also relates the Wilson coefficients of some mass operators to those of the leading operators, which are useful in organizing the subleading effects due to the quark mass in $\bar{B} \rightarrow X_s \gamma$. We present strange quark mass corrections to $\bar{B} \rightarrow X_s \gamma$ in the endpoint region as an application. The forward scattering amplitude from the mass corrections is factorized, and it can be expressed as a convolution of the m_s^2/p_X^2 -suppressed jet function and the leading-order shape function of the B meson. This contribution should be added to the existing subleading contributions from the B meson shape functions to obtain complete subleading corrections.

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I. INTRODUCTION

The soft-collinear effective theory (SCET) [1, 2, 3] has been widely used to describe high-energy processes which include energetic light particles. It is obtained from QCD by integrating out the degrees of freedom which are larger than a typical energy scale, Q . The effective theory contains a rich class of symmetries, and these symmetries of SCET provide us with new insight into factorization theorems [3, 4, 5] and enable us to perform a systematic power counting in hadronic processes [6]. SCET has been applied successfully to many high energy processes such as exclusive B decays [7, 8, 9, 10, 11, 12], inclusive B decays [1, 13], quarkonium production and decay [14], deep inelastic scattering [15], and jet physics [16].

In SCET, the momentum of a light energetic particle has three distinct scales and can be written as

$$p^\mu = \bar{n} \cdot p \frac{n^\mu}{2} + p_\perp^\mu + n \cdot p \frac{\bar{n}^\mu}{2} = \mathcal{O}(\lambda^0) + \mathcal{O}(\lambda^1) + \mathcal{O}(\lambda^2). \quad (1)$$

Here n and \bar{n} are light-cone vectors satisfying $n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$ and λ is a small parameter. In many processes, λ is chosen as $\sqrt{\Lambda/Q}$ or Λ/Q , where Λ is a typical hadronic scale. The effective theory which has a small expansion parameter $\lambda \sim \sqrt{\Lambda/Q}$ is called SCET_I and the effective theory in which physical quantities are expanded in powers of $\lambda \sim \Lambda/Q$ is called SCET_{II}. If there are contributions at intermediate scales of order $\sqrt{Q\Lambda}$, we employ the two-step matching in which SCET_I is obtained from the full theory by integrating out the hard modes of order $p^2 \sim Q^2$, and SCET_{II} is obtained by successively integrating out hard-collinear modes of order $p^2 \sim Q\Lambda$ [3].

At leading order in SCET, the collinear quarks are regarded as massless. Because the mass of a light quark, m , is very small compared to the hard scale Q or the intermediate hard-collinear scale $\sqrt{Q\Lambda}$, the quark mass can be neglected at leading order in λ . However, the light-quark mass terms [12, 17, 18, 19], and in some situations the charm quark mass [20] can be included in the framework of SCET. In Ref. [18], the authors first considered the quark mass in the SCET Lagrangian. Any operators including the light quark mass are formally suppressed by Λ/Q or more compared to the leading contribution. However if there are no leading terms, the quark mass can appear at leading order. $SU(3)$ breaking effects can be of this type since the strange quark mass can be numerically regarded as of order Λ (it is not possible to treat isospin breaking effects in this way since the masses of the up and down quarks are too small to be regarded as of order Λ). Another remarkable point about the quark mass is that it can give an enhanced contribution to some hadronic processes in SCET_{II} due to the different power counting schemes in SCET_I and SCET_{II}. Although they do not appear at leading order in SCET_I, since the quark mass terms are suppressed by Λ/Q , light quark masses can give significant corrections to the matching process related to hard-collinear degrees of freedom. The contribution of the quark mass to the decay rate can be of order $m^2/(Q^2(1-x)) \sim \Lambda/Q$ near the endpoint $1-x \sim \Lambda/Q$. This is one of the main themes to be investigated in this paper.

SCET can be extended to include the light quark mass, which we regard as of order Λ . We can systematically implement the quark mass in SCET and consider its renormalization behavior. We find that the reparameterization invariance [7, 21] still holds for the transformations of type-I and type-III in spite of the presence of the quark mass. But the transformation of type-II does not hold in its original form. However the transformation of type-II can be modified (or extended) to include the quark mass so that the symmetry still exists. This extended reparameterization invariance relates the leading operators to some subleading operators that include the quark mass. In practical applications, the strange quark mass is the only light quark mass that is relevant and we consider the quark mass effects in $\bar{B} \rightarrow X_s \gamma$ near the endpoint as a concrete example. Naively, the mass terms give corrections of order m^2/m_b^2 compared to the leading order contribution. But contributions of order $m^2/[m_b^2(1-x_\gamma)]$ with $x_\gamma = 2E_\gamma/m_b$ can arise, which are of order Λ/m_b near the endpoint region.

In this paper we investigate the effects of the quark mass in SCET and consider the symmetries including a quark mass. We also consider the renormalization effects and the Wilson coefficients of the mass operators in SCET. We then apply these results to $\bar{B} \rightarrow X_s \gamma$ in the endpoint region and discuss the contribution of the quark mass terms. In section II, the SCET Lagrangian with the light quark mass is constructed. We find an extended reparameterization transformation under which the Lagrangian is invariant, and we divide the Lagrangian into two reparameterization-invariant combinations. In this procedure, we show that the original reparameterization invariance symmetry without a quark mass can be extended by modifying the transformation of the collinear quark. We describe the consequence of the extended reparameterization invariance on the renormalization behavior of the mass operators. In section III, the Wilson coefficients of the effective operators including the quark mass are obtained to first order in α_s from the matching between full QCD and SCET. Also their renormalization behavior is presented with the effective theory quark mass renormalization at one loop. In section IV, the corrections due to the strange quark mass in $\bar{B} \rightarrow X_s \gamma$ near the endpoint region are considered. They can give corrections of order Λ/m_b , contrary to naive expectations. From the matching of the heavy-to-light current between the full theory and SCET_I, we obtain the subleading current operators including the quark mass. We then consider the time-ordered products of the currents and mass operators contributing to the decay rate in SCET. We show that the forward scattering amplitude with the mass corrections factorizes, similar to the leading-order result, and the jet function can be expanded in powers of the quark mass. Finally the results are summarized and the conclusions are presented in the final section.

II. MASS OPERATORS AND THE REPARAMETERIZATION INVARIANCE

In SCET, the collinear quark in the full theory is decomposed into

$$\begin{aligned}\psi(x) &= \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x} q_{n,p}(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x} \left(\frac{\not{n}\not{\tilde{p}}}{4} + \frac{\not{\tilde{p}}\not{n}}{4} \right) q_{n,p}(x) \\ &= \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x} \left(\xi_{n,p}(x) + \xi_{\bar{n},p}(x) \right),\end{aligned}\quad (2)$$

where \tilde{p}^μ is a label momentum, and $\frac{\not{n}\not{\tilde{p}}}{4} q_{n,p} = \xi_{n,p}$, $\frac{\not{\tilde{p}}\not{n}}{4} q_{n,p} = \xi_{\bar{n},p}$ are the projected spinors. After integrating out the off-shell field $\xi_{\bar{n},p}$ [18], the SCET Lagrangian with a quark mass is written as

$$\begin{aligned}\mathcal{L}_{\text{SCET}} &= \bar{\xi}_{n,p'} n \cdot i\mathcal{D} \frac{\not{\tilde{p}}}{2} \xi_{n,p} + \bar{\xi}_{n,p'} i\mathcal{D}_\perp \frac{1}{\bar{n} \cdot i\mathcal{D}} i\mathcal{D}_\perp \frac{\not{\tilde{p}}}{2} \xi_{n,p} \\ &\quad + m \bar{\xi}_{n,p'} [i\mathcal{D}_\perp, \frac{1}{\bar{n} \cdot i\mathcal{D}}] \frac{\not{\tilde{p}}}{2} \xi_{n,p} - m^2 \bar{\xi}_{n,p'} \frac{1}{\bar{n} \cdot i\mathcal{D}} \frac{\not{\tilde{p}}}{2} \xi_{n,p},\end{aligned}\quad (3)$$

where a summation over the label momenta is implied, and the covariant derivative \mathcal{D}^μ is given by [2]

$$\begin{aligned}\mathcal{D}^\mu &= D_c^\mu + D_{us}^\mu, \\ iD_c^\mu &= \mathcal{P}^\mu + gA_{n,q}^\mu, \quad iD_{us}^\mu = i\partial^\mu + gA_{us}^\mu.\end{aligned}\quad (4)$$

Let us consider first SCET_I with the expansion parameter $\lambda \sim \sqrt{\Lambda/Q}$, in which the ultrasoft (usoft) fields can interact with the collinear fields. The usoft momentum is of order Λ , and $p_\perp \sim \sqrt{Q\Lambda}$. For a collinear strange quark, if we treat the sizes of the quark mass m and iD_{us} to be of the same order $\mathcal{O}(\lambda^2)$, the term proportional to m in Eq. (3) is of order $\mathcal{O}(\lambda)$ and the term proportional to m^2 starts from $\mathcal{O}(\lambda^2)$. In this case, the mass terms in SCET are suppressed at least by order λ compared to the leading Lagrangian, and the spin of the collinear quark is preserved at leading order in SCET.

Integrating out the hard-collinear degrees of freedom with $p_{hc}^2 \sim Q\Lambda$ to obtain SCET_{II}, the usoft fields are decoupled from the collinear fields [3], and the Lagrangian of the collinear quark sector in SCET_{II} can be written as

$$\begin{aligned}\mathcal{L}_c^{\text{II}} &= \bar{\xi}_{n,p'} n \cdot iD_c \frac{\not{\tilde{p}}}{2} \xi_{n,p} + \bar{\xi}_{n,p'} i\mathcal{D}_c^\perp \frac{1}{\bar{n} \cdot iD_c} i\mathcal{D}_c^\perp \frac{\not{\tilde{p}}}{2} \xi_{n,p} \\ &\quad + m \bar{\xi}_{n,p'} [i\mathcal{D}_c^\perp, \frac{1}{\bar{n} \cdot iD_c}] \frac{\not{\tilde{p}}}{2} \xi_{n,p} - m^2 \bar{\xi}_{n,p'} \frac{1}{\bar{n} \cdot iD_c} \frac{\not{\tilde{p}}}{2} \xi_{n,p},\end{aligned}\quad (5)$$

where

$$\begin{aligned}iD_c^\mu &= (\bar{n} \cdot \mathcal{P} + g\bar{n} \cdot A_{n,q}) \frac{n^\mu}{2} + (\mathcal{P}_\perp^\mu + gA_{n,q,\perp}^\mu) + (n \cdot \mathcal{P} + gn \cdot A_{n,q}) \frac{\bar{n}^\mu}{2} \\ &= \mathcal{O}(\lambda^0) + \mathcal{O}(\lambda^1) + \mathcal{O}(\lambda^2).\end{aligned}\quad (6)$$

Here the expansion parameter λ is of order Λ/Q , and the collinear fields have momenta $p_c^2 \sim \Lambda^2$. Contrary to SCET_I, the mass terms in Eq. (5) belong to the leading-order Lagrangian. Therefore the effects of the mass terms can be important at leading order.

Before we investigate the effects of the radiative corrections for the new operators with a quark mass in Eq. (5), it is useful to consider the symmetries of SCET with the quark mass. In Refs. [7, 21], it has been shown that the SCET Lagrangian without the quark mass has a reparameterization invariance. One of the consequences is that the kinetic energy in SCET is not renormalized to all orders in α_s . And when we consider current operators in SCET, there are subleading operators which form a reparameterization-invariant combination with the leading operators. In this case, the Wilson coefficients of these subleading operators are the same as those of the leading operators to all orders in α_s . When the mass terms are included in SCET, the situation is slightly different. In this case, we can find an extended reparameterization transformation under which the Lagrangian is still invariant, and the Lagrangian consists of two independent sets of the operators which are separately reparameterization invariant. A similar example exists in the heavy quark effective theory (HQET) [22, 23, 24], in which the chromomagnetic operator belongs to a different reparameterization invariant combination from the kinetic term in HQET, and has a nontrivial Wilson coefficient.

Let us consider the effect of the mass term on the reparameterization invariance and how we can extend the reparameterization symmetry with the quark mass. The Lagrangian before integrating out $\xi_{\bar{n},p}$ is given by

$$\mathcal{L} = \sum_{\bar{p}, \tilde{p}'} e^{i(\tilde{p}' - \bar{p}) \cdot x} \left(\bar{q}_{n,p'} i \not{D} q_{n,p} - m \bar{q}_{n,p'} q_{n,p} \right), \quad (7)$$

where the quark field in SCET is given by $q_{n,p} = \xi_{n,p} + \xi_{\bar{n},p}$, and the covariant derivative is $\mathcal{D}^\mu = D_c^\mu + D_{us}^\mu$. Here the covariant derivative \mathcal{D}^μ is invariant under the reparameterization transformation since it is a four-vector, which does not change under a different basis of n^μ and \bar{n}^μ . Furthermore, the quantity $\sum_{\bar{p}} e^{-i\bar{p} \cdot x} q_{n,p}$ is the quark field in the full theory, which also does not change under the reparameterization transformation. Therefore the two terms in Eq. (7) are separately reparameterization invariant. Thus, there are two independent reparameterization-invariant combinations in Eq. (3), (5), and (7) if we can still find the appropriate reparameterization invariance.

In fact, there is a reparameterization invariance which can be extended to the case with the mass term. The original reparameterization invariance combined with the gauge invariance requires that the covariant derivative \mathcal{D}_μ not change under the transformations of type-I, II and III in Ref. [21]. We can find the same types of the reparameterization transformations under which the Lagrangian with the quark mass is invariant. In this case, we only need to check if the quark field $\sum_{\bar{p}} e^{-i\bar{p} \cdot x} q_{n,p}$ in the full theory remains invariant under these three types of the transformation. Using the equation of motion, we can write the

quark field in the full theory in terms of $\xi_{n,p}$ as [18]

$$\psi(x) = \sum_{\vec{p}} e^{-i\vec{p}\cdot x} q_{n,p} = \sum_{\vec{p}} e^{-i\vec{p}\cdot x} \left[1 + \frac{1}{\bar{n}\cdot i\mathcal{D}} (i\mathcal{D}_\perp + m) \frac{\vec{\eta}}{2} \right] \xi_{n,p}. \quad (8)$$

Without the mass term, the quark field ψ has the original reparameterization invariance.

With the mass term, ψ is not invariant under all of the original reparameterization transformations. ψ is still invariant under the reparameterization transformations of type-I and III, but not the transformation of type-II. In order to see this, it is enough to look at the term proportional to the quark mass in Eq. (8) under the type-II transformation, in which the light-cone vector \bar{n}^μ changes to $\bar{n}^\mu + \varepsilon_\perp^\mu$ with infinitesimal ε_\perp^μ . The transformation yields

$$\begin{aligned} \frac{m}{\bar{n}\cdot i\mathcal{D}} \frac{\vec{\eta}}{2} \xi_n &\rightarrow \frac{m}{\bar{n}\cdot i\mathcal{D} + \varepsilon_\perp\cdot i\mathcal{D}_\perp} \left(\frac{\vec{\eta}}{2} + \frac{\not{\varepsilon}_\perp}{2} \right) \left(1 + \frac{\not{\varepsilon}_\perp}{2} \frac{1}{\bar{n}\cdot i\mathcal{D}} i\mathcal{D}_\perp \right) \xi_n \\ &= m \left(\frac{1}{\bar{n}\cdot i\mathcal{D}} \frac{\vec{\eta}}{2} \xi_n - \frac{1}{\bar{n}\cdot i\mathcal{D}} \varepsilon_\perp\cdot i\mathcal{D}_\perp \frac{1}{\bar{n}\cdot i\mathcal{D}} \frac{\vec{\eta}}{2} \xi_n + \frac{1}{\bar{n}\cdot i\mathcal{D}} \frac{\not{\varepsilon}_\perp}{2} \xi_n \right. \\ &\quad \left. + \frac{1}{\bar{n}\cdot i\mathcal{D}} \frac{\not{\varepsilon}_\perp}{2} \frac{1}{\bar{n}\cdot i\mathcal{D}} i\mathcal{D}_\perp \frac{\vec{\eta}}{2} \xi_n \right) \neq \frac{m}{\bar{n}\cdot i\mathcal{D}} \frac{\vec{\eta}}{2} \xi_n, \end{aligned} \quad (9)$$

which clearly shows that the mass term is not invariant under the transformation of type-II. However, we can find an extended transformation of the spinor under the type-II transformation such that ψ remains invariant, and in the limit of the zero quark mass, the transformation reduces to the original transformation of type-II.

Suppose that the spinor ξ_n changes as $\xi_n \rightarrow \xi_n + \delta\xi_n$ under the transformation of type-II. Then ψ transforms as

$$\begin{aligned} \left[1 + \frac{1}{\bar{n}\cdot i\mathcal{D}} (i\mathcal{D}_\perp + m) \frac{\vec{\eta}}{2} \right] \xi_n &\rightarrow \left[1 + \frac{1}{\bar{n}\cdot i\mathcal{D} + \varepsilon_\perp\cdot i\mathcal{D}_\perp} (i\mathcal{D}_\perp - \frac{\not{\varepsilon}_\perp}{2} n\cdot i\mathcal{D} - \frac{\not{\eta}}{2} \varepsilon_\perp\cdot i\mathcal{D}_\perp + m) \right] \\ &\quad \times \left(\frac{\vec{\eta}}{2} + \frac{\not{\varepsilon}_\perp}{2} \right) (\xi_n + \delta\xi_n). \end{aligned} \quad (10)$$

Requiring that it be invariant under the transformation, the solution for $\delta\xi_n$ is given by

$$\delta\xi_n = \frac{\not{\varepsilon}_\perp}{2} \frac{1}{\bar{n}\cdot i\mathcal{D}} (i\mathcal{D}_\perp - m) \xi_n, \quad (11)$$

which reduces to the original reparameterization transformation of type-II without the quark mass. If we plug this solution into Eq. (10), we obtain

$$\begin{aligned} \delta_\Pi \left[1 + \frac{1}{\bar{n}\cdot i\mathcal{D}} (i\mathcal{D}_\perp + m) \frac{\vec{\eta}}{2} \right] \xi_n &= -\frac{1}{\bar{n}\cdot i\mathcal{D}} \frac{\not{\varepsilon}_\perp}{2} \\ &\times \left[n\cdot i\mathcal{D} + i\mathcal{D}_\perp \frac{1}{\bar{n}\cdot i\mathcal{D}} i\mathcal{D}_\perp + m (i\mathcal{D}_\perp \frac{1}{\bar{n}\cdot i\mathcal{D}} - \frac{1}{\bar{n}\cdot i\mathcal{D}} i\mathcal{D}_\perp) - m^2 \frac{1}{\bar{n}\cdot i\mathcal{D}} \right] \frac{\vec{\eta}}{2} \xi_n = 0, \end{aligned} \quad (12)$$

using the equation of motion, Eq. (3). So the extended transformation of type-II on the spinor with the quark mass can be written as

$$\xi_n \xrightarrow{\Pi} \left[1 + \frac{\not{\varepsilon}_\perp}{2} \frac{1}{\bar{n}\cdot i\mathcal{D}} (i\mathcal{D}_\perp - m) \right] \xi_n. \quad (13)$$

Therefore the reparameterization symmetries in the presence of the light quark mass in SCET still exist with the only modification of the spinor under the transformation of type-II, while the other transformations remain intact.

As mentioned above, there are two independent reparameterization-invariant combinations in Eq. (7). Putting Eq. (8) into Eq. (7), each combination can be written as

$$\begin{aligned}\bar{q}_{n,p'} i\mathcal{D} q_{n,p} &= \bar{\xi}_{n,p'} \left[n \cdot i\mathcal{D} + i\mathcal{D}_\perp \frac{1}{\bar{n} \cdot i\mathcal{D}} i\mathcal{D}_\perp \right] \frac{\bar{\eta}}{2} \xi_{n,p} + m^2 \bar{\xi}_{n,p'} \frac{1}{\bar{n} \cdot i\mathcal{D}} \frac{\bar{\eta}}{2} \xi_{n,p} \\ &\equiv \mathcal{K} - \mathcal{O}_m^{(2)},\end{aligned}\tag{14}$$

$$\begin{aligned}-m \bar{q}_{n,p'} q_{n,p} &= m \bar{\xi}_{n,p'} \left[i\mathcal{D}_\perp, \frac{1}{\bar{n} \cdot i\mathcal{D}} \right] \frac{\bar{\eta}}{2} \xi_{n,p} - 2m^2 \bar{\xi}_{n,p'} \frac{1}{\bar{n} \cdot i\mathcal{D}} \frac{\bar{\eta}}{2} \xi_{n,p} \\ &\equiv \mathcal{O}_m^{(1)} + 2\mathcal{O}_m^{(2)},\end{aligned}\tag{15}$$

where \mathcal{K} is the kinetic term of SCET and the mass operators $\mathcal{O}_m^{(i)}$ are suppressed by λ^i compared to \mathcal{K} in SCET_I. Because the kinetic term in the effective theory is not renormalized to all orders in α_s , it is also true for the reparameterization-invariant combination $\mathcal{K} - \mathcal{O}_m^{(2)}$. But the other combination in Eq. (15) does not have such a constraint, and in general it can have a nontrivial Wilson coefficient at higher orders. Putting these together, to all orders in α_s , the SCET Lagrangian can be written as

$$\begin{aligned}\mathcal{L}_{\text{SCET}} &= \mathcal{K} - \mathcal{O}_m^{(2)} + C(\mu)(\mathcal{O}_m^{(1)} + 2\mathcal{O}_m^{(2)}) \\ &= \mathcal{K} + C(\mu)\mathcal{O}_m^{(1)} + (-1 + 2C(\mu))\mathcal{O}_m^{(2)}.\end{aligned}\tag{16}$$

The Wilson coefficient $C(\mu)$ can be obtained from matching the full QCD Lagrangian onto SCET by treating the mass term as a perturbation. As will be explicitly shown in the next section, when dimensional regularization is used both for the ultraviolet and the infrared divergences, all the radiative corrections at order α_s are zero since the ultraviolet divergences cancel the infrared divergences. Therefore there is no finite contribution in matching, and the Wilson coefficient remains as 1. The SCET Lagrangian, at least to first order in α_s , can be written as

$$\mathcal{L}_{\text{SCET}} = \mathcal{K} + \mathcal{O}_m^{(1)} + \mathcal{O}_m^{(2)}.\tag{17}$$

If the radiative corrections remain zero at higher orders, the Wilson coefficient is equal to 1 to all order in α_s . An argument to the non-renormalization to all orders was presented in the first reference of [8], and in Ref. [25] including the quark mass.

The scaling behavior of the quark mass can be considered by extracting the ultraviolet divergent part in the radiative corrections of the operators $\mathcal{O}_m^{(1,2)}$ since these operators involve the quark mass. It can be obtained by computing the radiative corrections for the quark mass with the wavefunction renormalization of the spinor ξ_n . Physically, the scaling behavior of the quark mass should be the same as that in the full theory since there are no degrees of freedom integrated out, which contribute to the evolution of the quark mass of order Λ . For example, the self energy for ξ_n is the same as that for the spinor ψ in the full theory. This

is in contrast to HQET, where the magnetic operator has a nontrivial Wilson coefficient because the hard calculation of the full theory has a dependence on the heavy quark mass. All these aspects will be verified explicitly to order α_s in the next section.

III. MATCHING AND RENORMALIZATION OF THE MASS OPERATORS

The matching between full QCD and SCET can be performed by considering the quark propagator. The quark propagator in the full theory can be written to all orders in α_s as

$$\frac{i}{\not{p} - m} \longrightarrow \frac{i}{\not{p} - m - \Sigma(\not{p}, m)}, \quad (18)$$

where $\Sigma(\not{p}, m)$ is the self energy of the quark, and the higher-order corrections of the full QCD Lagrangian can be obtained by replacing the Lagrangian in momentum space as

$$\bar{\psi}(\not{p} - m)\psi \longrightarrow \bar{\psi}[\not{p} - m - \Sigma(\not{p}, m)]\psi. \quad (19)$$

When we match SCET₁ onto the full theory at the scale $\mu \sim Q$ where Q is the large momentum of the collinear quark, the self energy can be written as

$$\Sigma(\not{p}, m) = A(p^2, \mu)\not{p} + B(p^2, \mu)m, \quad (20)$$

where the virtuality of the collinear quark p^2 is treated as $\mu^2 \gg p^2 \gg m^2$. At first order in α_s , the coefficients are given as

$$\begin{aligned} A(p^2, \mu) &= -\frac{\alpha_s C_F}{4\pi} \left(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2} + 1 \right), \\ B(p^2, \mu) &= \frac{\alpha_s C_F}{4\pi} \left(\frac{4}{\varepsilon} + 4 \ln \frac{\mu^2}{-p^2} + 6 \right), \end{aligned} \quad (21)$$

where $D = 4 - 2\varepsilon$ and $1/\varepsilon$ represents the ultraviolet divergence and the infrared divergences are regulated by the logarithmic terms. This method is useful in extracting the ultraviolet divergences. For example, the counterterms for the wavefunction renormalization Z_ψ and the mass renormalization Z_m are given by

$$Z_\psi = 1 - \frac{\alpha_s C_F}{4\pi} \frac{1}{\varepsilon}, \quad Z_m = 1 - \frac{\alpha_s C_F}{4\pi} \frac{3}{\varepsilon}. \quad (22)$$

A more convenient method is to use pure dimensional regularization with all the external particles on their mass shell. This greatly simplifies the computation both in the full theory and in the effective theory. In both theories the on-shell graphs have no finite parts since there are scaleless integrals, which vanish in pure dimensional regularization. Furthermore the matching results are gauge independent and renormalization-scheme independent only

when we put the external particles on their mass shell. Eq. (21) can be written in pure dimensional regularization as

$$A(\mu) = -\frac{\alpha_s C_F}{4\pi} \left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon_{\text{IR}}} \right), \quad B(\mu) = \frac{\alpha_s C_F}{4\pi} \left(\frac{4}{\varepsilon} - \frac{4}{\varepsilon_{\text{IR}}} \right), \quad (23)$$

where the infrared poles in ε_{IR} can be explicitly computed or can be inferred from the ultraviolet divergence with the fact that the radiative corrections are zero.

At one loop after the ultraviolet divergence is removed, the radiative correction of the full QCD Lagrangian is given by

$$\left(1 - \frac{\alpha_s C_F}{4\pi} \frac{1}{\varepsilon_{\text{IR}}} \right) \bar{\psi} \not{p} \psi + \left(1 - \frac{\alpha_s C_F}{4\pi} \frac{4}{\varepsilon_{\text{IR}}} \right) (-m \bar{\psi} \psi). \quad (24)$$

To match this result onto SCET_I, we convert \not{p} to $i\not{D}$ and apply Eqs. (8), (14), and (15) to Eq. (24). Then we obtain

$$\begin{aligned} \bar{\psi} [\not{p} - m - \sum(\not{p}, m)] \psi &\longrightarrow \left(1 - \frac{\alpha_s C_F}{4\pi} \frac{1}{\varepsilon_{\text{IR}}} \right) \mathcal{K} \\ &+ \left(1 - \frac{\alpha_s C_F}{4\pi} \frac{4}{\varepsilon_{\text{IR}}} \right) \mathcal{O}_m^{(1)} + \left(1 - \frac{\alpha_s C_F}{4\pi} \frac{7}{\varepsilon_{\text{IR}}} \right) \mathcal{O}_m^{(2)}, \end{aligned} \quad (25)$$

where the operators \mathcal{K} , $\mathcal{O}_m^{(1)}$, and $\mathcal{O}_m^{(2)}$ are defined in Eqs. (14) and (15), and we use the on-shell renormalization scheme in which the infrared divergences are regulated by the poles in ε_{IR} .

In order to examine if the effective theory reproduces the infrared divergences of the full theory and to extract the Wilson coefficients of $\mathcal{O}_m^{(1)}$ and $\mathcal{O}_m^{(2)}$, we need to calculate the one-loop corrections of $\mathcal{O}_m^{(1)}$ and $\mathcal{O}_m^{(2)}$ in SCET_I. For the strange collinear quark (in the case of up or down quarks the mass operators are more suppressed), both mass operators are subleading because the operators start at order λ or λ^2 since they are given as

$$\begin{aligned} \mathcal{O}_m^{(1)} &= m \bar{\xi}_{n,p'} \left[i\not{D}_c^\perp, W \frac{1}{\overline{\mathcal{P}}} W^\dagger \right] \frac{\overline{\not{n}}}{2} \xi_{n,p} + \dots = \mathcal{O}(\lambda) + \dots, \\ \mathcal{O}_m^{(2)} &= -m^2 \bar{\xi}_{n,p'} W \frac{1}{\overline{\mathcal{P}}} W^\dagger \frac{\overline{\not{n}}}{2} \xi_{n,p} + \dots = \mathcal{O}(\lambda^2) + \dots. \end{aligned} \quad (26)$$

Here $\overline{\mathcal{P}} = \overline{n} \cdot \mathcal{P}$ and W is the collinear Wilson line,

$$W(x) = \left[\sum_{\text{perms}} \exp\left(-g \frac{1}{\overline{\mathcal{P}}} \overline{n} \cdot A_{n,q}(x)\right) \right]. \quad (27)$$

Since the subleading terms in the right side of Eq. (26) are connected to the leading terms by the reparameterization invariance and the gauge symmetries, it is sufficient to consider the loop corrections of the leading operators which we will denote as $\mathcal{O}_m^{(1)}$ and $\mathcal{O}_m^{(2)}$.

First let us consider the one loop corrections of $\mathcal{O}_m^{(2)}$. The relevant interaction vertices and their Feynman rules are shown in Fig. 1. The Feynman diagrams for the radiative

The figure shows three Feynman diagrams and their corresponding mathematical expressions. The first diagram shows a fermion line with momentum \$p\$ entering from the left and \$p'\$ exiting to the right. A gluon with momentum \$\vec{\eta}\$ and indices \$\mu, a\$ is attached to the line at a vertex labeled \$O_m^{(2)}\$. The expression is \$= igT^a \bar{n}^\mu \frac{m^2}{\bar{n} \cdot p \bar{n} \cdot p'} \frac{\vec{\eta}}{2}\$. The second diagram is similar but the vertex is labeled \$O_m^{(1)}\$. The expression is \$= igT^a m \left[\left(\frac{1}{\bar{n} \cdot p} - \frac{1}{\bar{n} \cdot p'} \right) \gamma_\perp^\mu + \frac{\not{p}_\perp - \not{p}'_\perp}{\bar{n} \cdot p \bar{n} \cdot p'} \bar{n}^\mu \right] \frac{\vec{\eta}}{2}\$. The third diagram shows a fermion line with momentum \$p\$ entering from the left and \$p'\$ exiting to the right. Two gluons with momenta \$q_1, q_2\$ and indices \$\mu, a\$ and \$\nu, b\$ are attached to the line at a vertex labeled \$O_m^{(1)}\$. The expression is \$= igT^a T^b m \left[\frac{1}{\bar{n} \cdot (p + q_2)} \left(\frac{\bar{n}^\mu \gamma_\perp^\nu}{\bar{n} \cdot p} - \frac{\gamma_\perp^\mu \bar{n}^\nu}{\bar{n} \cdot p} \right) + \frac{\not{p}'_\perp - \not{p}_\perp}{\bar{n} \cdot p \bar{n} \cdot (p + q_2)} \bar{n}^\mu \bar{n}^\nu \right] \frac{\vec{\eta}}{2} + (\mu \leftrightarrow \nu, a \leftrightarrow b, q_1 \leftrightarrow q_2)\$.

FIG. 1: Feynman rules for the operators $O_m^{(1)}$ and $O_m^{(2)}$ with one or two collinear gluons.

corrections at one loop are shown in Fig. 2. The radiative corrections for $O_m^{(2)}$ with collinear gluons will be the same as those in Fig. 2 due to the gauge invariance. If we put the external particle off the mass shell, the radiative corrections in Fig. 2 are given as

$$\begin{aligned}
M_a^{(2)} &= -\frac{\alpha_s C_F}{4\pi} \frac{m^2}{\bar{n} \cdot p} \frac{\vec{\eta}}{2} \left[-2 - 2 \left(\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon} - \frac{\pi^2}{12} \right) (-p^2)^{-\varepsilon} \right], \\
M_b^{(2)} &= M_c^{(2)} = -\frac{\alpha_s C_F}{4\pi} \frac{m^2}{\bar{n} \cdot p} \frac{\vec{\eta}}{2} \left(\frac{2}{\varepsilon} + 4 \right) (-p^2)^{-\varepsilon}, \\
M_d^{(2)} &= -\frac{\alpha_s C_F}{4\pi} \frac{m^2}{\bar{n} \cdot p} \frac{\vec{\eta}}{2} \left(\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon} + 3 - \frac{\pi^2}{6} \right) (-p^2)^{-\varepsilon}.
\end{aligned} \tag{28}$$

Adding all these, we obtain

$$M_a^{(2)} + M_b^{(2)} + M_c^{(2)} + M_d^{(2)} = -\frac{\alpha_s C_F}{4\pi} \frac{m^2}{\bar{n} \cdot p} \frac{\vec{\eta}}{2} \left(\frac{7}{\varepsilon} + 7 \ln \frac{\mu^2}{-p^2} + 9 \right). \tag{29}$$

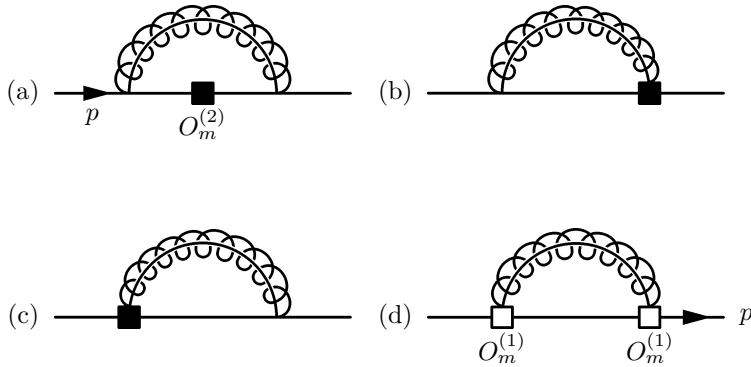


FIG. 2: Feynman diagrams for the radiative corrections to $O_m^{(2)}$ at one loop.

This result will be used in computing the jet function for $\overline{B} \rightarrow X_s \gamma$ at order α_s . [See Fig. 6 (a), (b) and (c).] In pure dimensional regularization, all these diagrams vanish since the integrals are dimensionless. However, it is transparent to write the result in pure dimensional regularization from Eq. (29) as

$$M_a^{(2)} + M_b^{(2)} + M_c^{(2)} + M_d^{(2)} = \frac{\alpha_s C_F - m^2 \overline{\not{n}}}{4\pi \overline{n} \cdot p} \frac{1}{2} \left(\frac{7}{\varepsilon} - \frac{7}{\varepsilon_{\text{IR}}} \right), \quad (30)$$

since the ultraviolet divergent piece is known.

For the one-loop corrections of the operator $O_m^{(1)}$, which has at least one collinear gluon, it is convenient to use the background gauge field method [26]. Since the product of g and the background field A_n is not renormalized in the background field gauge, the number of Feynman diagrams to compute is fairly reduced, and they are shown in Fig. 3. The computation of the diagrams is straightforward using the on-shell dimensional regularization scheme with the external quark momenta $p^2 = p'^2 = 0$. The results without and with a triple gluon vertex are given by

$$\begin{aligned} & M_a^{(1)} + M_b^{(1)} + M_c^{(1)} + M_d^{(1)} + M_e^{(1)} + M_f^{(1)} + M_g^{(1)} \\ &= 3 \frac{\alpha_s C_F}{4\pi} \left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon_{\text{IR}}} \right) mg \overline{\xi}_{n,p'} \left[\left(\frac{1}{\overline{n} \cdot p} - \frac{1}{\overline{n} \cdot p'} \right) A_n^+ + \frac{\not{p}_\perp - \not{p}'_\perp}{\overline{n} \cdot p \overline{n} \cdot p'} \overline{n} \cdot A_n \right] \frac{\overline{\not{n}}}{2} \xi_{n,p} \\ &+ \frac{\alpha_s}{4\pi} \left(-\frac{1}{2N} \right) \left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon_{\text{IR}}} \right) mg \overline{\xi}_{n,p'} \left[\left(\frac{1}{\overline{n} \cdot p} - \frac{1}{\overline{n} \cdot p'} \right) A_n^+ + \frac{\not{p}_\perp - \not{p}'_\perp}{\overline{n} \cdot p \overline{n} \cdot p'} \overline{n} \cdot A_n \right] \frac{\overline{\not{n}}}{2} \xi_{n,p}, \quad (31) \end{aligned}$$

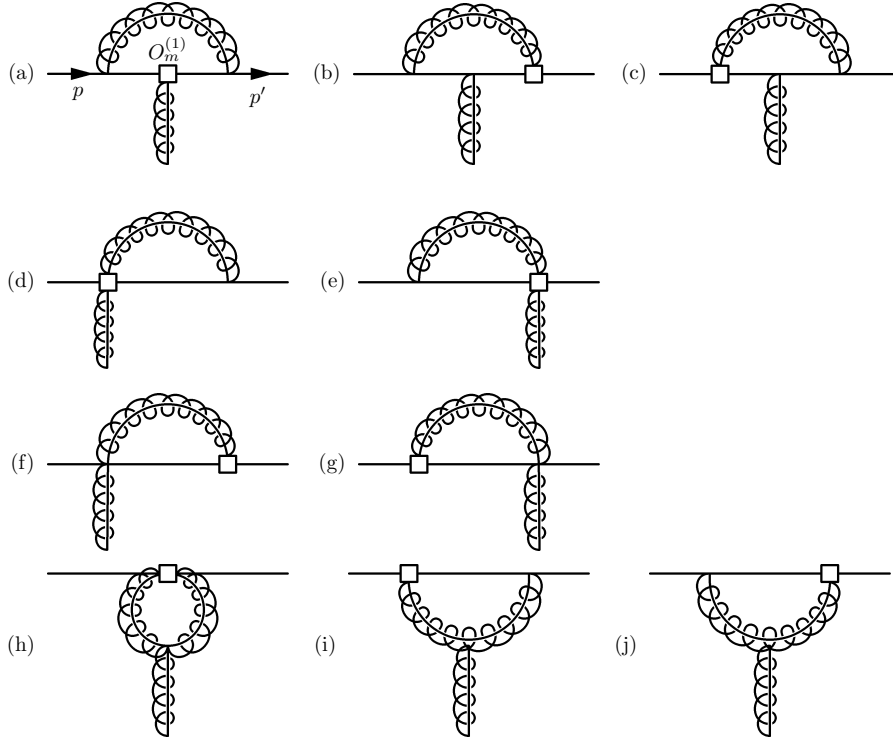


FIG. 3: Feynman diagrams for one-loop corrections to $O_m^{(1)}$ in the background field gauge.

$$\begin{aligned}
& M_h^{(1)} + M_i^{(1)} + M_j^{(1)} \\
&= \frac{\alpha_s N}{4\pi} \frac{1}{2} \left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon_{\text{IR}}} \right) m g \bar{\xi}_{n,p'} \left[\left(\frac{1}{\bar{n} \cdot p} - \frac{1}{\bar{n} \cdot p'} \right) A_n^\perp + \frac{\not{p}_\perp - \not{p}'_\perp}{\bar{n} \cdot p \bar{n} \cdot p'} \bar{n} \cdot A_n \right] \frac{\not{\eta}}{2} \xi_{n,p}, \tag{32}
\end{aligned}$$

where $M_i^{(1)}$ represents the i th diagram in Fig. 3, and N is the number of colors. Summing these two results with $C_F = (N^2 - 1)/(2N)$, the radiative correction of the operator $O_m^{(1)}$ at one loop is given as

$$\begin{aligned}
M^{(1)} &= \frac{\alpha_s C_F}{4\pi} \left(\frac{4}{\varepsilon} - \frac{4}{\varepsilon_{\text{IR}}} \right) m g \bar{\xi}_{n,p'} \left[\left(\frac{1}{\bar{n} \cdot p} - \frac{1}{\bar{n} \cdot p'} \right) A_n^\perp + \frac{\not{p}_\perp - \not{p}'_\perp}{\bar{n} \cdot p \bar{n} \cdot p'} \bar{n} \cdot A_n \right] \frac{\not{\eta}}{2} \xi_{n,p} \\
&= \frac{\alpha_s C_F}{4\pi} \left(\frac{4}{\varepsilon} - \frac{4}{\varepsilon_{\text{IR}}} \right) O_m^{(1)}. \tag{33}
\end{aligned}$$

From Eqs. (30) and (33), we can see that the radiative corrections of the operators $O_m^{(2)}$ and $O_m^{(1)}$ in SCET_I reproduce the infrared divergences in the full theory. And since the radiative corrections are the same in both theories, the Wilson coefficients of both operators are 1 with no contribution at one loop. We can also extract the counterterm for the quark mass in SCET_I. The counterterm Z_ξ for the wavefunction renormalization of a collinear quark is given by

$$Z_\xi = 1 - \frac{\alpha_s C_F}{4\pi} \frac{1}{\varepsilon}, \tag{34}$$

which is the same as the counterterm in the full theory for the quark field. Therefore we obtain the counterterm for the quark mass from $O_m^{(1)}$ and $O_m^{(2)}$ as

$$Z_m^{\text{SCET}_I} = 1 - \frac{\alpha_s C_F}{4\pi} \frac{3}{\varepsilon}, \tag{35}$$

which is the same as the full theory mass renormalization to first order in α_s . This is to be expected since we do not integrate out any degrees of freedom relevant to the collinear quark mass from the matching. In summary, it has been shown that the counterterms for the wavefunction and the quark mass are the same as those in the full theory, and there are no contributions to the coefficients of the operators at one loop; that is, the operators are not renormalized to order α_s .

The matching between SCET_I and SCET_{II} is trivial because there is no hard-collinear degrees of freedom ($p_{hc}^2 \sim Q\Lambda$) to be integrated out in the SCET_I Lagrangian. Note that the situation is different for heavy-to-light currents with the spectator interactions in B decays and for soft-collinear currents [27, 28], in which there arise nontrivial Wilson coefficients (or jet functions) from the matching between SCET_I and SCET_{II}. A more concrete analysis on the hard-collinear modes is discussed in Refs. [25, 28]. However the operators $O_m^{(1)}$ and $O_m^{(2)}$ in SCET_{II} remain as they are in SCET_I since the collinear momentum $p_c^2 = m^2$ is still very small compared to matching scale $\mu \sim \sqrt{Q\Lambda}$. Therefore in SCET_{II}, the mass operators are regarded as the leading operators for the strange collinear quark, and the operators have the same Wilson coefficients and the same renormalization behavior as in SCET_I with the same mass renormalization given by Eq. (35).

IV. QUARK MASS CORRECTIONS TO $\bar{B} \rightarrow X_s \gamma$ DECAYS

Inclusive B decays based on HQET [29] have been widely studied to extract Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and to search for possible new physics. When an emitted photon is energetic in the region of the phase space with $p_X^2 \sim m_B \Lambda$, SCET along with HQET is applicable and has been successfully applied [1, 13]. In this case, the differential decay rate can be given by a factorized form as

$$\frac{d\Gamma}{dE_\gamma} \propto H J \otimes f, \quad (36)$$

where \otimes means the appropriate convolution. Here H is a hard factor obtained from the matching between the full theory and SCET_I, J is a jet function obtained by integrating out hard-collinear objects, and f represents the shape function of a B meson, which consists of only soft interactions and is purely nonperturbative.

Recently the corrections of order Λ/m_b to $\bar{B} \rightarrow X_s \gamma$ and $\bar{B} \rightarrow X_u l \bar{\nu}$ decays in the endpoint region have been investigated using SCET [30, 31, 32]. Here the factorization formula Eq. (36) still holds, and the subleading shape functions are studied to clarify the uncertainty from the theoretical analysis. When the effect of the strange quark mass of order Λ is included in $\bar{B} \rightarrow X_s \gamma$ or $\bar{B} \rightarrow X_s \bar{l} l$, the mass corrections can also give a nonnegligible contribution of order Λ/m_b . In this section we focus on this fact and analyze the mass corrections to the decay $\bar{B} \rightarrow X_s \gamma$ in the endpoint region. The result is also applicable to the $\bar{B} \rightarrow X_s \bar{l} l$, but it is not considered here.

The effective weak Hamiltonian for $\bar{B} \rightarrow X_s \gamma$ is given by [33]

$$\mathcal{H}_{\text{eff}} = -4 \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^8 C_i^{\text{full}}(\mu) Q_i(\mu), \quad (37)$$

where the main contribution comes from the operator

$$Q_7 = \frac{e}{16\pi^2} \bar{s} \sigma_{\mu\nu} F^{\mu\nu} (m_b P_R + m_s P_L) b. \quad (38)$$

Here $P_{R,L} = (1 \pm \gamma_5)/2$ and $F^{\mu\nu}$ is the electromagnetic field strength tensor. We choose the frame in which the photon momentum q^μ is in the \bar{n}^μ direction, $q^\mu = n \cdot q \bar{n}^\mu / 2 = E_\gamma \bar{n}^\mu$, where the photon energy E_γ near the endpoint satisfies $m_B - 2E_\gamma \lesssim \Lambda$. The strange quark can be taken as a collinear quark in the n^μ direction in the rest frame of a B meson.

Let us define the forward scattering amplitude $T_{\mu\nu}$ as

$$T_{\mu\nu} = \frac{1}{2m_B} \langle \bar{B} | \hat{T}_{\mu\nu} | \bar{B} \rangle, \quad (39)$$

where $\hat{T}_{\mu\nu}$ is given by

$$\hat{T}_{\mu\nu} = -i \int d^4 z e^{-iq \cdot z} T [J_\mu^\dagger(z) J_\nu(0)] \quad (40)$$

with the current

$$J^\mu = i \bar{s} \sigma_{\mu\nu} q^\nu P_R b + \frac{m_s}{m_b} i \bar{s} \sigma_{\mu\nu} q^\nu P_L b. \quad (41)$$

The inclusive photon energy spectrum can be written as

$$\frac{1}{\Gamma_0} \frac{d\Gamma}{dE_\gamma} = \frac{8E_\gamma}{m_B^3} \frac{1}{\pi} \text{Im} T_\mu^\mu(E_\gamma), \quad (42)$$

where

$$\Gamma_0 = \frac{G_F^2 m_B^3 m_b^2}{32\pi^4} \alpha |V_{tb} V_{ts}^*|^2 |C_7^{\text{full}}(m_b)|^2. \quad (43)$$

The forward scattering amplitude $T_{\mu\nu}(E_\gamma)$ in SCET near the endpoint region is given by the factorized form in Eq. (36) and the power counting can be performed systematically. The hard part can be computed from the matching between the full theory and SCET_I, and the heavy-to-light current can be expanded in terms of the currents in SCET_I in powers of $\lambda \sim \sqrt{\Lambda/m_b}$. Then the time-ordered product of the effective currents can be expressed as a convolution of the jet function and the shape function of the B meson by matching onto SCET_{II}. As a result, the forward scattering amplitude is given by the convolution of the hard part, the jet function, and the shape functions.

We investigate the strange quark mass corrections to the inclusive decay rate to first order in Λ/m_b and α_s . We show that these corrections can also be written in a factorized form and the mass corrections reside only in the jet functions. This mass correction should be included in the subleading contribution along with other subleading corrections from the shape function to order Λ/m_b , which was extensively discussed in Refs. [30, 31, 32].

A. Matching a heavy-to-light current with a quark mass

Let us consider matching the heavy-to-light tensor current $J_{\mu\nu} = \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b$ at $\mu \sim m_b \sim \bar{n} \cdot p$ where $\bar{n} \cdot p$ is the large momentum component of the collinear strange quark. The full-theory current can be matched onto the currents in SCET_I, in which the hard degrees of freedom such as m_b and the large off-shellness $p_{\text{hard}}^2 \sim m_b^2 \sim m_b \bar{n} \cdot p$ are integrated out. By choosing the heavy quark velocity as $v_\perp = 0$, $n \cdot v = \bar{n} \cdot v = 1$, the heavy-to-light current can be expanded in SCET_I as

$$J_{\mu\nu} = e^{i(\bar{p}\cdot z - m_b v\cdot z)} \left\{ \sum_i \int d\omega C_i(\omega) j_{i\mu\nu}^{(0)}(\omega) + \sum_i \int d\omega B_i(\omega) j_{i\mu\nu}^{(1)}(\omega) + \sum_i \int d\omega A'_i(\omega) j_{i\mu\nu}^{(2)}(\omega) + \sum_i \int d\omega A_i(\omega) j_{i\mu\nu}^{(m)}(\omega) + \dots \right\}, \quad (44)$$

where the superscripts k ($k = 0, 1, 2$) denote the order in λ , and another superscript m indicates the operators with the strange quark mass. The currents $j_{i\mu\nu}^{(m)}$ are of the same order as $j_{i\mu\nu}^{(2)}$ as long as the mass is regarded as $m \sim \Lambda$. From now on, we suppress the exponential factors with the understanding that the label momenta are conserved. Since

we focus on the mass corrections of the heavy-to-light currents and their relations to the leading or the subleading currents in λ , we will not consider the currents $j_{i\mu\nu}^{(2)}$ any more. The detailed analysis on these currents can be found in Ref. [30].

At tree level, the current operator in the full theory can be expressed in terms of the currents in SCET_I as

$$\begin{aligned} J_{\mu\nu} &= \bar{s}\sigma_{\mu\nu}(1 + \gamma_5)b \\ &\longrightarrow \bar{\xi}_n W \sigma_{\mu\nu}(1 + \gamma_5)h_v + \bar{\xi}_n \frac{\not{n}}{2} i \overleftarrow{D}_c^\perp W \frac{1}{\bar{n} \cdot \mathcal{P}^\dagger} \sigma_{\mu\nu}(1 + \gamma_5)h_v \\ &\quad + \frac{1}{m_b} \bar{\xi}_n \sigma_{\mu\nu}(1 + \gamma_5) i \overleftarrow{D}_c^\perp W \frac{\not{n}}{2} h_v + m \bar{\xi}_n W \frac{\not{n}}{2} \frac{1}{\bar{n} \cdot \mathcal{P}^\dagger} \sigma_{\mu\nu}(1 + \gamma_5)h_v \end{aligned} \quad (45)$$

$$\begin{aligned} &= j_{1\mu\nu}^{(0)} + j_{1\mu\nu}^{(1)} + j_{2\mu\nu}^{(1)} + j_{1\mu\nu}^{(m)}, \\ \tilde{J}_{\mu\nu} &= \frac{m}{m_b} \bar{s}\sigma_{\mu\nu}(1 - \gamma_5)b \longrightarrow \frac{m}{m_b} \bar{\xi}_n \sigma_{\mu\nu}(1 - \gamma_5)h_v = j_{3\mu\nu}^{(m)}, \end{aligned} \quad (46)$$

where ξ_n is a collinear strange quark field and h_v is a heavy quark field.

At order α_s , we employ the modified minimal subtraction ($\overline{\text{MS}}$) scheme using on-shell dimensional regularization. In the full theory, the matrix element of the tensor current $J_{\mu\nu}$ at one loop is given as

$$\begin{aligned} \langle J_{\mu\nu} \rangle^{(1)} &= \frac{\alpha_s C_F}{4\pi} \left\{ \left[-\frac{1}{\varepsilon^2} - \frac{2}{\varepsilon} + \frac{2}{\varepsilon} \ln \frac{\bar{n} \cdot p}{\mu} - 4 \ln \frac{\mu}{m_b} + \frac{2(1-2x)}{1-x} \ln x \right. \right. \\ &\quad \left. \left. - 2 \ln^2 \frac{\bar{n} \cdot p}{\mu} - \frac{\pi^2}{12} - 2Li_2(1-x) - 4 \right] \langle \bar{s}\sigma_{\mu\nu}(1 + \gamma_5)b \rangle \right. \\ &\quad + \left[\frac{4}{1-x} \ln x \right] \frac{1}{m_b} \langle i\bar{s}(\gamma_\mu p_\nu - \gamma_\nu p_\mu)(1 - \gamma_5)b \rangle \\ &\quad + \left[\frac{2}{x} \left(\frac{1}{\varepsilon} - 2 \ln \frac{\bar{n} \cdot p}{\mu} + 2 \right) \right] \frac{m}{m_b} \langle \bar{s}\sigma_{\mu\nu}(1 - \gamma_5)b \rangle \\ &\quad - \left[\frac{4}{x} \left(\frac{1}{\varepsilon} - 2 \ln \frac{\bar{n} \cdot p}{\mu} + 2 - \frac{x}{1-x} \ln x \right) \right] \frac{m}{m_b^2} \langle i\bar{s}(\gamma_\mu p_{b\nu} - \gamma_\nu p_{b\mu})(1 + \gamma_5)b \rangle \\ &\quad \left. - \frac{2m}{xm_b^2} \langle i\bar{s}(\gamma_\mu p_\nu - \gamma_\nu p_\mu)(1 + \gamma_5)b \rangle \right\}, \end{aligned} \quad (47)$$

where $x = \bar{n} \cdot p/m_b$, and all the poles in $1/\varepsilon$ represent the IR divergences. Here we use the equations of motion $\not{p}_b b = m_b b$ and $\bar{s} \not{p} = m \bar{s}$ putting each quark on shell with $p_b^2 = m_b^2$, $p^2 = m^2 \rightarrow 0$, keeping the terms to first order in the strange quark mass m . For $\tilde{J}_{\mu\nu}$ the matrix element at one loop is given by

$$\begin{aligned} \langle \tilde{J}_{\mu\nu} \rangle^{(1)} &= \frac{\alpha_s C_F}{4\pi} \left\{ \left[-\frac{1}{\varepsilon^2} - \frac{2}{\varepsilon} + \frac{2}{\varepsilon} \ln \frac{\bar{n} \cdot p}{\mu} - 4 \ln \frac{\mu}{m_b} + \frac{2(1-2x)}{1-x} \ln x \right. \right. \\ &\quad \left. \left. - 2 \ln^2 \frac{\bar{n} \cdot p}{\mu} - \frac{\pi^2}{12} - 2Li_2(1-x) - 4 \right] \frac{m}{m_b} \langle \bar{s}\sigma_{\mu\nu}(1 - \gamma_5)b \rangle \right. \\ &\quad \left. + \frac{4m}{m_b^2} \frac{\ln x}{1-x} \langle i\bar{s}(\gamma_\mu p_\nu - \gamma_\nu p_\mu)(1 + \gamma_5)b \rangle \right\}. \end{aligned} \quad (48)$$

Now we expand the current operators in Eq. (47) in powers of λ using the momentum decomposition $p^\mu = \bar{n} \cdot pn^\mu/2 + p_\perp^\mu + n \cdot p\bar{n}^\mu/2$. These operators can be written in terms of the gauge-invariant effective currents as

$$\begin{aligned}
\bar{s}\sigma_{\mu\nu}(1 + \gamma_5)b &\longrightarrow C_1\bar{\xi}_n W\sigma_{\mu\nu}(1 + \gamma_5)h_v + B_1\bar{\xi}_n \frac{\overleftarrow{\not{n}}}{2} i\overleftarrow{D}_c^\perp W \frac{1}{\bar{n} \cdot \mathcal{P}^\dagger} \sigma_{\mu\nu}(1 + \gamma_5)h_v \\
&+ B_2 \frac{1}{m_b} \bar{\xi}_n \sigma_{\mu\nu}(1 + \gamma_5) i\overleftarrow{D}_c^\perp W \frac{\not{n}}{2} h_v + A_1 m \bar{\xi}_n W \frac{\overleftarrow{\not{n}}}{2} \frac{1}{\bar{n} \cdot \mathcal{P}^\dagger} \sigma_{\mu\nu}(1 + \gamma_5)h_v + \dots \\
&= C_1 j_{1\mu\nu}^{(0)} + B_1 j_{1\mu\nu}^{(1)} + B_2 j_{2\mu\nu}^{(1)} + A_1 j_{1\mu\nu}^{(m)} + \dots, \\
\frac{2}{xm_b} i\bar{s}(\gamma_\mu p_\nu - \gamma_\nu p_\mu)(1 - \gamma_5)b &\longrightarrow C_2 i\bar{\xi}_n W(\gamma_\mu n_\nu - \gamma_\nu n_\mu)(1 - \gamma_5)h_v \\
&+ B_3 i\bar{\xi}_n \frac{\overleftarrow{\not{n}}}{2} i\overleftarrow{D}_c^\perp W \frac{1}{\bar{n} \cdot \mathcal{P}^\dagger} (\gamma_\mu n_\nu - \gamma_\nu n_\mu)(1 - \gamma_5)h_v \\
&+ 2B_4 i\bar{\xi}_n (\gamma_\mu i\overleftarrow{D}_{c\nu}^\perp - i\overleftarrow{D}_{c\mu}^\perp \gamma_\nu) \frac{1}{\bar{n} \cdot \mathcal{P}^\dagger} (1 - \gamma_5)h_v \\
&+ A_2 i m \bar{\xi}_n \frac{\overleftarrow{\not{n}}}{2} W \frac{1}{\bar{n} \cdot \mathcal{P}^\dagger} (\gamma_\mu n_\nu - \gamma_\nu n_\mu)(1 - \gamma_5)h_v + \dots \\
&= C_2 j_{2\mu\nu}^{(0)} + B_3 j_{3\mu\nu}^{(1)} + 2B_4 j_{4\mu\nu}^{(1)} + A_2 j_{2\mu\nu}^{(m)} + \dots, \tag{49} \\
\frac{m}{m_b} \bar{s}\sigma_{\mu\nu}(1 - \gamma_5)b &\longrightarrow (A_3 + \tilde{A}_3) \frac{m}{m_b} \bar{\xi}_n W\sigma_{\mu\nu}(1 - \gamma_5)h_v + \dots = (A_3 + \tilde{A}_3) j_{3\mu\nu}^{(m)} + \dots, \\
\frac{m}{m_b^2} i\bar{s}(\gamma_\mu p_{b\nu} - \gamma_\nu p_{b\mu})(1 + \gamma_5)b &\longrightarrow A_4 \frac{m}{m_b} i\bar{\xi}_n W(\gamma_\mu v_\nu - \gamma_\nu v_\mu)(1 + \gamma_5)h_v = A_4 j_{4\mu\nu}^{(m)} + \dots, \\
\frac{2m}{xm_b^2} i\bar{s}(\gamma_\mu p_\nu - \gamma_\nu p_\mu)(1 + \gamma_5)b &\longrightarrow (A_5 + \tilde{A}_5) \frac{m}{m_b} i\bar{\xi}_n W(\gamma_\mu n_\nu - \gamma_\nu n_\mu)(1 + \gamma_5)h_v \\
&= (A_5 + \tilde{A}_5) j_{5\mu\nu}^{(m)} + \dots,
\end{aligned}$$

where we keep the effective currents to $\mathcal{O}(\lambda^2)$. Here we use the fact that

$$\tilde{J}_{\mu\nu} = \frac{m}{m_b} \bar{s}\sigma_{\mu\nu}(1 - \gamma_5)b \longrightarrow \tilde{A}_3 j_{3\mu\nu}^{(m)} + \tilde{A}_5 j_{5\mu\nu}^{(m)}. \tag{50}$$

All the Wilson coefficients at tree level are 0 except

$$C_1 = B_1 = A_1 = B_2 = \tilde{A}_3 = 1. \tag{51}$$

Due to the reparameterization invariance, some of the Wilson coefficients at subleading order are related to the leading coefficients. They are given by

$$C_1 = B_1 = A_1, \quad C_2 = B_3 = 2B_4 = A_2 \tag{52}$$

to all orders in α_s since those subleading operators with the corresponding Wilson coefficients are obtained by expanding the collinear field in a reparameterization-invariant way. However the subleading operator with B_2 is an independent operator. This operator can be obtained when we consider the heavy-to-light current in which a collinear gluon is emitted from the heavy quark and we integrate out the intermediate-state heavy particle. Therefore

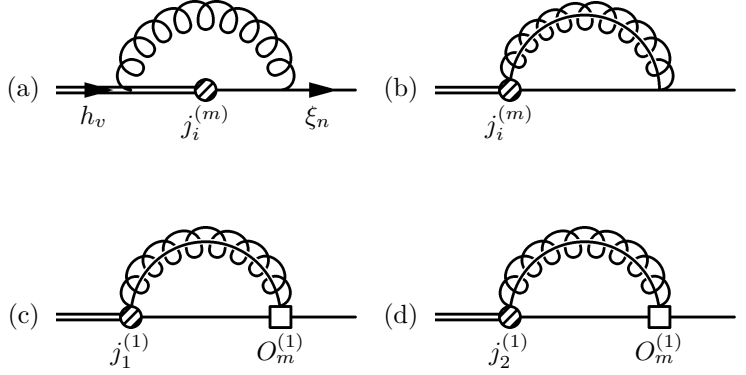


FIG. 4: Feynman diagrams for the mass correction to the heavy-to-light current with $j_i^{(m)}$ ($i = 1, 2, \dots, 5$) and $O_m^{(1)}$.

the coefficient B_2 is not related to other Wilson coefficients and should be computed independently. The radiative correction for the operator with B_2 was considered at one loop in Refs. [34, 35]. And \tilde{A}_3 and \tilde{A}_5 come from the operator proportional to $m_s = m$ in Q_7 , while A_3 and A_5 come from the subleading contribution of the leading operator in Q_7 at higher orders in α_s .

In order to match the full theory onto SCET_I, we compute the radiative corrections in SCET_I. The relevant Feynman diagrams in SCET_I at one loop are shown in Fig. 4. We have verified that the infrared divergences of the full theory in Eq. (47) are fully reproduced in the effective theory from the explicit calculations of the diagrams with the self energy of the external quarks, and they cancel out in matching. In computing the Wilson coefficients, the point is that all the radiative corrections in the effective theory are simply zero when the on-shell dimensional regularization scheme is employed, and the Wilson coefficients can be easily obtained.

The difference of the residues in the wave function renormalization between the full theory and the effective theory for the heavy quark at one loop is given by

$$\frac{1}{2}(R_b^{(1)} - R_h^{(1)}) = -\frac{\alpha_s C_F}{4\pi} \left(3 \ln \frac{\mu}{m_b} + 2 \right), \quad (53)$$

and we find the Wilson coefficients C_i for $j_{i\mu\nu}^{(0)}$, B_i for $j_{i\mu\nu}^{(1)}$, and A_i, \tilde{A}_i for $j_{i\mu\nu}^{(m)}$ as

$$\begin{aligned} C_1 = B_1 = A_1 &= 1 + \frac{\alpha_s C_F}{4\pi} \left[-6 - 7 \ln \frac{\mu}{m_b} + \frac{2(1-2x)}{1-x} \ln x - 2 \ln^2 \frac{\bar{n} \cdot p}{\mu} \right. \\ &\quad \left. - 2Li_2(1-x) - \frac{\pi^2}{12} \right], \\ C_2 = B_3 = 2B_4 = A_2 &= \frac{\alpha_s C_F}{4\pi} \left(\frac{2x}{1-x} \ln x \right), \\ A_3 &= \frac{\alpha_s C_F}{4\pi} \left(-\frac{4}{x} \ln \frac{\bar{n} \cdot p}{\mu} + \frac{4}{x} \right), \end{aligned}$$

$$\begin{aligned}
A_4 &= \frac{\alpha_s C_F}{4\pi} \left(\frac{8}{x} \ln \frac{\bar{n} \cdot p}{\mu} - \frac{8}{x} + \frac{4}{1-x} \ln x \right), & A_5 &= -\frac{\alpha_s C_F}{4\pi}, \\
\tilde{A}_3 &= 1 + \frac{\alpha_s C_F}{4\pi} \left[-6 - 7 \ln \frac{\mu}{m_b} + \frac{2(1-2x)}{1-x} \ln x - 2 \ln^2 \frac{\bar{n} \cdot p}{\mu} \right. \\
&\quad \left. - 2Li_2(1-x) - \frac{\pi^2}{12} \right], \\
\tilde{A}_5 &= \frac{\alpha_s C_F}{4\pi} \left(\frac{2x}{1-x} \ln x \right).
\end{aligned} \tag{54}$$

The Wilson coefficients $C_1(\mu)$ and $C_2(\mu)$ are basically identical to those obtained in Ref. [2] although the operator basis is different. The Wilson coefficients A_i , \tilde{A}_i and B_i except B_2 are new and first calculated here.

Note that all the operators in the basis $\{j_{1\mu\nu}^{(m)}, \dots, j_{5\mu\nu}^{(m)}\}$ are not independent. Because $\frac{\bar{n}}{2} = \not{p} - \frac{\not{p}}{2}$ in the B meson rest frame with the choice of $v_\perp^\mu = 0$, the operator $j_{1\mu\nu}^{(m)}$ can be written as

$$\begin{aligned}
j_{1\mu\nu}^{(m)} &= m \bar{\xi}_n W \left(\not{p} - \frac{\not{p}}{2} \right) \frac{1}{\bar{n} \cdot \mathcal{P}^\dagger} \sigma_{\mu\nu} (1 + \gamma_5) h_\nu \\
&= m \bar{\xi}_n W \frac{1}{\bar{n} \cdot \mathcal{P}^\dagger} \sigma_{\mu\nu} (1 - \gamma_5) h_\nu - 2m i \bar{\xi}_n W (\gamma_\mu v_\nu - \gamma_\nu v_\mu) \frac{1}{\bar{n} \cdot \mathcal{P}^\dagger} \sigma_{\mu\nu} (1 + \gamma_5) h_\nu \\
&= \frac{1}{x} \left(j_{3\mu\nu}^{(m)} - 2j_{4\mu\nu}^{(m)} \right).
\end{aligned} \tag{55}$$

Therefore the number of the independent operators in the basis is four. But it is useful to use this basis because the reparameterization invariance is shown transparently, as shown in Eq. (54).

B. Jet functions and factorization in SCET_{II}

Let us consider the contribution of the quark mass to the forward scattering amplitude $T_{\mu\nu}(E_\gamma)$. The current J_μ in Eq. (41) can be written in the effective theory as

$$\begin{aligned}
J_\mu &= iE_\gamma \frac{\bar{n}^\nu}{2} J_{\mu\nu} \\
&= iE_\gamma e^{i(\bar{p}\cdot z - m_b v\cdot z)} \left\{ \sum_i \int d\omega C_i(\omega) j_{i\mu}^{(0)}(\omega) + \sum_i \int d\omega B_i(\omega) j_{i\mu}^{(1)}(\omega) \right. \\
&\quad \left. + \sum_i \int d\omega A_i(\omega) j_{i\mu}^{(m)}(\omega) + \dots \right\},
\end{aligned} \tag{56}$$

where $j_{i\mu}^{(j)}(\omega) = \frac{\bar{n}^\nu}{2} j_{i\mu\nu}^{(j)}(\omega)$, ($j = 0, 1, m$). Here we express $j_{i\mu\nu}^{(j)}(\omega)$ as

$$j_{i\mu\nu}^{(j)}(\omega) = \bar{\xi}_n W \delta(\omega - \bar{n} \cdot \mathcal{P}^\dagger) \Gamma_i^{(j)} h_\nu, \tag{57}$$

with a delta function. In general, the operators $j_{2\mu\nu}^{(1)}(\omega)$ need additional parameters ω' at higher orders in α_s since it consists of at least three external particles including a collinear

gluon, but it is not necessary at one loop since it is sufficient to consider the tree-level Wilson coefficients of $j_{2\mu\nu}^{(1)}(\omega)$.

The forward scattering amplitude $T_{\mu\nu}(E_\gamma)$ can be written as

$$T_{\mu\nu}(E_\gamma) = \frac{E_\gamma^2}{2} \langle \bar{B}_v | \hat{T}_{\mu\nu} | \bar{B}_v \rangle, \quad (58)$$

with the normalization of the B meson states in HQET. $\hat{T}_{\mu\nu}$ is given by

$$\hat{T}_{\mu\nu} = -i \sum_{i,i',k,k'} \int d\omega d\omega' C_{i'}^{(k')}(\omega') C_i^{(k)}(\omega) \int d^4 z e^{-ir \cdot z} T [j_{i'\mu}^{(k')\dagger}(\omega', z), j_{i\nu}^{(k)}(\omega, 0)] + \dots, \quad (59)$$

where $C_i^{(k)}$ ($k = 0, 1, m, \dots$) are the Wilson coefficients C_i , B_i , A_i , and \tilde{A}_i in Eq. (56). The momentum r in the exponential factor is defined as

$$r^\mu = q^\mu + \tilde{p}^\mu - m_b v^\mu. \quad (60)$$

Since the photon momentum q^μ is given by $q^\mu = n \cdot q \bar{n}^\mu / 2$, the label momentum of the collinear strange quark is fixed as

$$\bar{n} \cdot p = m_b, \quad p_\perp^\mu = 0 \quad (61)$$

giving $\bar{n} \cdot r = r_\perp = 0$. $n \cdot r$ can be written as

$$n \cdot r = n \cdot q - m_b = m_B - n \cdot p_X - m_b, \quad (62)$$

where p_X is a momentum of the jet X and we use the momentum conservation $m_B v^\mu = q^\mu + p_X^\mu$. Evidently $n \cdot r$ is of order Λ since the mass difference $\bar{\Lambda} = m_B - m_b$ and $n \cdot p_X$ are of order Λ .

We can express Eq. (59) showing the dependence of the quark mass explicitly as

$$\begin{aligned} \hat{T}_{\mu\nu}^{(m)} = & -i \sum_{i,i'} \left\{ \int d\omega d\omega' C_{i'}(\omega') A_i(\omega) \int d^4 z e^{-ir \cdot z} T [j_{i'\mu}^{(0)\dagger}(\omega', z), j_{i\nu}^{(m)}(\omega, 0)] \right. \\ & + \int d\omega d\omega' C_{i'}(\omega') B_i(\omega) \int d^4 z d^4 x e^{-ir \cdot z} T [j_{i'\mu}^{(0)\dagger}(\omega', z), i\mathcal{L}_m^{(1)}(x), j_{i\nu}^{(1)}(\omega, 0)] \\ & + \int d\omega d\omega' C_{i'}(\omega') C_i(\omega) \left\{ \int d^4 z d^4 x e^{-ir \cdot z} T [j_{i'\mu}^{(0)\dagger}(\omega', z), i\mathcal{L}_m^{(2)}(x), j_{i\nu}^{(0)}(\omega, 0)] \right. \\ & \left. \left. + \int d^4 z d^4 x d^4 y e^{-ir \cdot z} T [j_{i'\mu}^{(0)\dagger}(\omega', z), i\mathcal{L}_m^{(1)}(x), i\mathcal{L}_m^{(1)}(y), j_{i\nu}^{(0)}(\omega, 0)] \right\} + \text{h.c.} \right\}, \quad (63) \end{aligned}$$

where the second and fourth contributions in Eq. (63) start at order α_s since $\mathcal{L}_m^{(1)}$ contains at least one collinear gluon. Note that all these terms are suppressed by λ^2 compared to the leading contributions in SCET_I. Only the third and fourth terms are nonzero due to the spin structure of the currents. The mass term flips the spin of the collinear quark, and therefore there must be even powers of m to conserve spin.

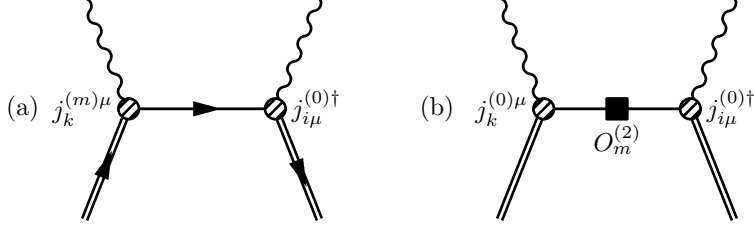


FIG. 5: Tree-level Feynman diagrams for the leading mass corrections to $\overline{B} \rightarrow X_s \gamma$ near the endpoint region in SCET_I. The mirror image of the diagram (a) should be included. The intermediate hard-collinear strange quark is integrated out at $\mu \sim \sqrt{m_b \Lambda}$ to match onto SCET_{II}.

The tree-level Feynman diagrams for the mass corrections to $\overline{B} \rightarrow X_s \gamma$ in the endpoint region are shown in Fig. 5. Fig. 5 (a) is zero as explained above. Fig. 5 (b) yields

$$M_{ik,\mu\nu}^{(b)} = -i \int d^4 z d^4 y e^{-ir \cdot z} T [j_{i\mu}^{(0)\dagger}(z), i\mathcal{L}_m^{(2)}(y), j_{k\nu}^{(0)}(0)], \quad (64)$$

where the leading heavy-to-light currents $j_{k\mu}^{(0)}$ ($k = 1, 2$) are given by

$$j_{1\mu}^{(0)} = i\bar{\xi}_n W \gamma_\mu^\perp (1 - \gamma_5) h_v - \frac{i}{2} \bar{\xi}_n W \bar{n}_\mu (1 + \gamma_5) h_v, \quad j_{2\mu}^{(0)} = i\bar{\xi}_n W \gamma_\mu^\perp (1 - \gamma_5) h_v. \quad (65)$$

The amplitude $M_{ik,\mu\nu}^{(b)}$ is given as

$$\begin{aligned} M_{ik,\mu\nu}^{(b)} &= - \int d^4 z d^4 y e^{-ir \cdot z} T [j_{i\mu}^{(0)\dagger}(z) \bar{\xi}_n \frac{m^2}{\bar{n} \cdot p} \frac{\not{n}}{2} \xi_n(y) j_{k\nu}^{(0)}(0)] \\ &= \frac{m^2}{\bar{n} \cdot p} \int \frac{dn \cdot k d\bar{n} \cdot z}{4\pi} e^{-in \cdot (r+k)\bar{n} \cdot z/2} [J_P(n \cdot k)]^2 T [\bar{h}_v Y(\frac{\bar{n} \cdot z}{2}) \Gamma_{ik,\mu\nu}^{(b)} Y^\dagger h_v(0)] \end{aligned} \quad (66)$$

where the Dirac structure $\Gamma_{ik,\mu\nu}^{(b)}$ is given by

$$\begin{aligned} \Gamma_{11,\mu\nu}^{(b)} &= \gamma_\mu^\perp \not{n} \gamma_\nu^\perp (1 - \gamma_5) - \frac{\bar{n}_\mu}{2} \not{n} \gamma_\nu^\perp (1 - \gamma_5) - \frac{\bar{n}_\nu}{2} \gamma_\mu^\perp \not{n} (1 + \gamma_5) + \frac{\bar{n}_\mu \bar{n}_\nu}{4} \not{n} (1 + \gamma_5), \\ \Gamma_{12,\mu\nu}^{(b)} &= \gamma_\mu^\perp \not{n} \gamma_\nu^\perp (1 - \gamma_5) - \frac{\bar{n}_\mu}{2} \not{n} \gamma_\nu^\perp (1 - \gamma_5), \\ \Gamma_{21,\mu\nu}^{(b)} &= \gamma_\mu^\perp \not{n} \gamma_\nu^\perp (1 - \gamma_5) - \frac{\bar{n}_\nu}{2} \gamma_\mu^\perp \not{n} (1 + \gamma_5), \quad \Gamma_{22,\mu\nu}^{(b)} = \gamma_\mu^\perp \not{n} \gamma_\nu^\perp (1 - \gamma_5). \end{aligned} \quad (67)$$

In Eq. (66), the ultrasoft interactions were decoupled from the collinear field, and the resultant usoft Wilson line is given by

$$Y(x) = \left[\sum_{\text{perms}} \exp\left(-g \frac{1}{n \cdot \mathcal{P}} n \cdot A_{us}(x)\right) \right], \quad (68)$$

and $n \cdot \mathcal{P}$ is of order Λ . In obtaining Eq. (66), we use the definition of the jet function

$$\langle 0 | T [W^\dagger \xi_n(z) \bar{\xi}_n W] | 0 \rangle = i \frac{\not{n}}{2} \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot z} J_P(k), \quad (69)$$

where $P = \bar{n} \cdot p$ is the label momentum, and the jet function is a function of $n \cdot k$ only with $J_P(k) = J_P(n \cdot k)$.

The matrix element of the remaining operators can be written as

$$\begin{aligned}
& \langle \bar{B}_v | \bar{h}_v Y \left(\frac{\bar{n} \cdot z}{2} \right) \Gamma_{ik, \mu\nu}^{(b)} Y^\dagger h_v(0) | \bar{B}_v \rangle \\
&= \int dn \cdot l e^{in \cdot l \bar{n} \cdot z/2} \langle \bar{B}_v | \bar{h}_v Y \Gamma_{ik, \mu\nu}^{(b)} \delta(n \cdot l - n \cdot i\partial) Y^\dagger h_v(0) | \bar{B}_v \rangle \\
&= \int dn \cdot l e^{in \cdot l \bar{n} \cdot z/2} \text{tr} \left(\frac{P_v}{2} \Gamma_{ik, \mu\nu}^{(b)} \right) \langle \bar{B}_v | \bar{h}_v Y \delta(n \cdot l - n \cdot i\partial) Y^\dagger h_v(0) | \bar{B}_v \rangle \\
&= -2(g_{\mu\nu}^\perp - i\epsilon_{\mu\nu}^\perp) \int dn \cdot l e^{in \cdot l \bar{n} \cdot z/2} f^{(0)}(n \cdot l), \tag{70}
\end{aligned}$$

where $P_v = (1 + \not{v})/2$ is the projection operator for the heavy quark, and $f^{(0)}$ is the leading shape function of the B meson, which is defined as

$$\begin{aligned}
f^{(0)}(n \cdot l) &= \frac{1}{2} \int \frac{d\bar{n} \cdot z}{4\pi} e^{-in \cdot l \frac{\bar{n} \cdot z}{2}} \langle \bar{B}_v | \bar{h}_v Y \left(\frac{\bar{n} \cdot z}{2} \right) Y^\dagger h_v(0) | \bar{B}_v \rangle \\
&= \frac{1}{2} \langle \bar{B}_v | \bar{h}_v Y \delta(n \cdot l - n \cdot i\partial) Y^\dagger h_v | \bar{B}_v \rangle, \tag{71}
\end{aligned}$$

with $\epsilon_{\mu\nu}^\perp = \epsilon_{\mu\nu\alpha\beta} \bar{n}^\alpha n^\beta / 2$. Note that the final result of Eq. (70) is independent of $\Gamma_{ik}^{(b)}$ since only the first term in each Dirac structure in Eq. (67) contributes.

The forward scattering amplitude with the mass correction at tree level can be written as

$$\begin{aligned}
T_{\mu\nu}^{(m)}(\omega) &= -\tilde{H}(\omega, m_b, \mu_0) E_\gamma^2 (g_{\mu\nu}^\perp - i\epsilon_{\mu\nu}^\perp) \int dn \cdot k J_\omega^{(m)}(n \cdot k) f^{(0)}(n \cdot k - m_b(1 - x_\gamma)) \tag{72} \\
&= -\tilde{H}(\omega, m_b, \mu_0) E_\gamma^2 (g_{\mu\nu}^\perp - i\epsilon_{\mu\nu}^\perp) \int dn \cdot l f^{(0)}(n \cdot l) J_\omega^{(m)}(m_b(1 - x_\gamma) + n \cdot l, \mu_0, \mu),
\end{aligned}$$

where μ_0 is a typical scale where SCET_I can be matched onto SCET_{II}, and ω is fixed as $\bar{n} \cdot p$ by the delta function in Eq. (57). Here we use the relation $n \cdot l = n \cdot k + n \cdot r = n \cdot k + \bar{\Lambda} - n \cdot p_X$. $J_\omega^{(m)}(n \cdot k)$ is the jet function obtained from the matching between SCET_I and SCET_{II}, with the tree-level result given by

$$J_\omega^{(m)}(n \cdot k)|_{\text{tree}} = \frac{m^2}{\omega} \left(J_\omega(n \cdot k) \right)^2 = \frac{m^2}{\omega} \frac{1}{(n \cdot k + i\epsilon)^2}. \tag{73}$$

The hard factor $\tilde{H}(\omega, m_b, \mu_0)$ is obtained by matching the heavy-to-light currents between the full theory and SCET_I and is evolved to the scale $\mu \sim \mu_0$. At $\mu = m_b$, it is given by

$$\tilde{H}(\omega, m_b, \mu = m_b) = |C_1(\omega, m_b) + C_2(\omega, m_b)|^2. \tag{74}$$

Since the invariant mass of the jet is given by $p_X^2 \geq 0$, the range of the residual momentum $n \cdot k$ is $0 \leq n \cdot k \leq n \cdot p_X$. Also the residual momentum of the heavy quark $n \cdot l$ is smaller than the B meson residual mass $\bar{\Lambda} = m_B - m_b$.

To proceed further, there are two possible types of formulations. One, based on the second expression in Eq. (72), is useful in the moment analysis and will be illustrated in the next subsection. Here, we begin with the first expression in Eq. (72), in which we write $n \cdot l$ as [30]

$$n \cdot l = \bar{\Lambda} - (1 - z)n \cdot p_X, \quad (75)$$

where z is given by

$$n \cdot k = zn \cdot p_X, \quad 0 \leq z \leq 1. \quad (76)$$

Then $T_{\mu\nu}^{(m)}$ can be written as

$$\begin{aligned} T_{\mu\nu}^{(m)} &= -E_\gamma^2 (g_{\mu\nu}^\perp - i\epsilon_{\mu\nu}^\perp) \tilde{H}(\bar{n} \cdot p_X, m_b, \mu_0) n \cdot p_X \\ &\times \int_0^1 dz J_{\bar{n} \cdot p_X}^{(m)}(zn \cdot p_X, \mu_0, \mu) f^{(0)}(\bar{\Lambda} - (1 - z)n \cdot p_X, \mu) \\ &\equiv -E_\gamma^2 (g_{\mu\nu}^\perp - i\epsilon_{\mu\nu}^\perp) \tilde{H}(\bar{n} \cdot p_X, m_b, \mu_0) \int_0^1 dz \mathcal{J}_m(z, p_X^2, \mu) f^{(0)}(\bar{\Lambda} - (1 - z)n \cdot p_X, \mu), \end{aligned} \quad (77)$$

where we use the relation $\omega = \bar{n} \cdot p = \bar{n} \cdot p_X$ at leading order. The dimensionless jet function \mathcal{J}_m is given as

$$\begin{aligned} \mathcal{J}_m(z, p_X^2, \mu) &= n \cdot p_X J_{\bar{n} \cdot p_X}^{(m)}(zn \cdot p_X, \mu_0, \mu), \\ \mathcal{J}_m^{(0)}(z, p_X^2, \mu) &= \frac{m^2}{p_X^2} \frac{1}{(z + i\epsilon)^2}, \end{aligned} \quad (78)$$

where $\mathcal{J}_m^{(0)}$ is the jet function at tree level. When we take a discontinuity of the forward scattering amplitude, the imaginary part entirely comes from the jet function, which is given by

$$\text{Im} \left(\frac{1}{\pi} \mathcal{J}_m^{(0)} \right) = \frac{m^2}{p_X^2} \frac{d}{dz} \delta(z). \quad (79)$$

As shown in Eq. (77), the forward scattering amplitude can be expressed as a convolution of the jet function and the shape function; that is, it is given by the factorized form with the hard factor, the jet function with the mass correction, and the leading-order shape function. Note that the effect of the quark mass, even at higher orders in m^2/p_X^2 , resides only in the jet functions as it should, and it does not affect the B -meson shape function. In order to obtain the full subleading contributions to the decay rate, we should include the mass correction in the jet functions, the contribution of the subleading B -meson shape functions induced by the high-dimensional heavy-quark bilinears, which can be studied in the framework of HQET, and the effects of the subleading heavy-to-light currents in taking the time-ordered products.

As can be seen in Eqs. (78) and (79), the subleading jet function from the mass correction is suppressed by m^2/p_X^2 or $m^2/(m_b\Lambda)$ compared to the leading jet function. This contribution is of order Λ/m_b if we treat the strange quark mass to be of order Λ . This is one of the examples in which the subleading terms are formally suppressed by Λ^2/m_b^2 , but they are actually suppressed by Λ/m_b near the endpoint.

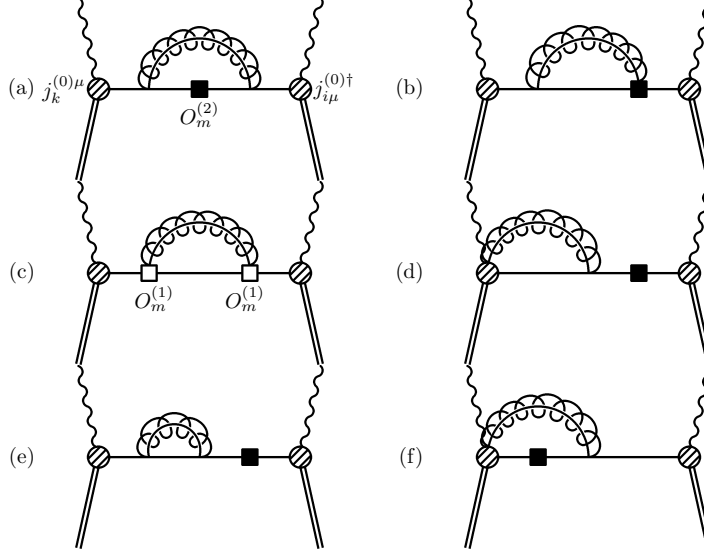


FIG. 6: Feynman diagrams for the jet function at one loop. The mirror diagrams of (b), (d), (e), and (f) are omitted. The Sudakov double logarithms are produced only in the diagram (d).

We can expand the jet functions in powers of m^2/p_X^2 and α_s . The jet function at first order in m^2/p_X^2 and in α_s can be computed, with the relevant Feynman diagrams shown in Fig. 6. The contributions of Fig. 6 (a), (b), (c) are obtained from Eq. (29), and Fig. 6 (d), (e), (f) with their hermitian conjugates are given as

$$\begin{aligned}
\sum_{i=a,b,c} \mathcal{J}_{m,i}^{(1)} &= \frac{\alpha_s C_F m^2}{4\pi p_X^2} \frac{1}{(z+i\epsilon)^2} (-p_X^2)^{-\epsilon} \left(\frac{7}{\epsilon} + 9 \right), \\
\mathcal{J}_{m,d}^{(1)} &= \frac{\alpha_s C_F m^2}{4\pi p_X^2} \frac{1}{(z+i\epsilon)^2} (-p_X^2)^{-\epsilon} \left(\frac{4}{\epsilon^2} + \frac{4}{\epsilon} + 8 - \frac{\pi^2}{3} \right), \\
\mathcal{J}_{m,e}^{(1)} &= \frac{\alpha_s C_F m^2}{4\pi p_X^2} \frac{1}{(z+i\epsilon)^2} (-p_X^2)^{-\epsilon} \left(-\frac{2}{\epsilon} - 2 \right), \\
\mathcal{J}_{m,f}^{(1)} &= \frac{\alpha_s C_F m^2}{4\pi p_X^2} \frac{1}{(z+i\epsilon)^2} (-p_X^2)^{-\epsilon} \frac{4}{\epsilon}.
\end{aligned} \tag{80}$$

Therefore the jet function at order α_s is given as

$$\begin{aligned}
\mathcal{J}_m^{(1)}(z, p_X^2, \mu) &= \frac{\alpha_s C_F m^2}{4\pi p_X^2} \frac{1}{(z+i\epsilon)^2} \left[2 \left(\ln \frac{p_X^2}{\mu^2} + \ln(-z-i\epsilon) \right)^2 \right. \\
&\quad \left. - 13 \left(\ln \frac{p_X^2}{\mu^2} + \ln(-z-i\epsilon) \right) + 15 - \frac{\pi^2}{3} \right].
\end{aligned} \tag{81}$$

The discontinuity of the jet function to order α_s is given by

$$\begin{aligned}
\text{Im} \left(\frac{1}{\pi} \mathcal{J}_m \right) &= \frac{m^2}{p_X^2} \frac{d}{dz} \left\{ \delta(z) + \frac{\alpha_s C_F}{4\pi} \left[\left(2 \ln^2 \frac{p_X^2}{\mu^2} - 9 \ln \frac{p_X^2}{\mu^2} + 10 - \pi^2 \right) \delta(z) \right. \right. \\
&\quad \left. \left. + \left(4 \ln \frac{p_X^2}{\mu^2} - 9 \right) \frac{1}{(z)_+} + 4 \left(\frac{\ln(z)}{z} \right)_+ \right] \right\}.
\end{aligned} \tag{82}$$

C. Moment analysis of the mass correction

In the endpoint region $x_\gamma \rightarrow 1$ ($x_\gamma = 2E_\gamma/m_b$), it is useful to consider the moments of the differential decay rate and take the large N limit. In the limit $1 - x_\gamma \sim 1/N \sim \Lambda/m_b$, the moments of the mass correction to the differential decay rate can be studied. In doing the moment analysis, it is convenient to employ the second expression for $T_{\mu\nu}^{(m)}$ in Eq. (72). Consider the quantity

$$S = \int dn \cdot l f^{(0)}(n \cdot l) J_\omega^{(m)}(m_b(1 - x_\gamma) + n \cdot l, \mu_0, \mu), \quad (83)$$

with the momentum of the strange quark $p_s^\mu = m_b n^\mu/2 + k^\mu + m_b(1 - x_\gamma)\bar{n}^\mu/2$. The jet function has support for $-m_b(1 - x_\gamma) \leq n \cdot l \leq \bar{\Lambda}$, and if we let $n \cdot l = -(1 - y)m_b$ Eq. (83) can be written as

$$S = m_b \int_{x_\gamma}^{m_B/m_b} dy f^{(0)}(-m_b(1 - y)) J_{m_b}^{(m)}(y, z_\gamma), \quad (84)$$

where the jet function proportional to m^2 to order α_s is given by

$$J_\omega^{(m)}(y, z_\gamma) = \frac{m^2}{\omega m_b^2 y^2 (1 - z_\gamma + i\epsilon)^2} \times \left[1 + \frac{\alpha_s C_F}{4\pi} \left(2 \ln^2 \frac{-\omega m_b y (1 - z_\gamma)}{\mu^2} - 13 \ln \frac{-\omega m_b y (1 - z_\gamma)}{\mu^2} + 15 - \frac{\pi^2}{3} \right) \right], \quad (85)$$

where $z_\gamma = x_\gamma/y$. Taking the imaginary part, we obtain

$$\begin{aligned} -\frac{1}{\pi} \text{Im} J_{m_b}^{(m)}(y, z_\gamma) &= \frac{m^2}{m_b^3 y^2} \left\{ \left[1 + \frac{\alpha_s C_F}{4\pi} \left(2 \ln^2 \frac{m_b^2 y}{\mu^2} - 9 \ln \frac{m_b^2 y}{\mu^2} + 10 - \pi^2 \right) \right] \frac{d}{dz_\gamma} \delta(1 - z_\gamma) \right. \\ &\quad \left. + \frac{\alpha_s C_F}{4\pi} \left[\left(4 \ln \frac{m_b^2 y}{\mu^2} - 9 \right) \frac{d}{dz_\gamma} \frac{1}{(1 - z_\gamma)_+} + 4 \frac{d}{dz_\gamma} \left(\frac{\ln(1 - z_\gamma)}{1 - z_\gamma} \right)_+ \right] \right\} \\ &\equiv \frac{1}{m_b y^2} \bar{\mathcal{J}}^{(m)}(z_\gamma), \end{aligned} \quad (86)$$

where we neglect $\ln y$ terms near the endpoint $y \rightarrow 1$, and $\bar{\mathcal{J}}^{(m)}(z_\gamma)$ is a dimensionless function suppressed by m^2/m_b^2 .

Finally the mass correction to the differential decay rate can be written as

$$\frac{1}{\Gamma_0} \frac{d\Gamma^{(m)}}{dx_\gamma} = \tilde{H}(m_b, \mu_0) x_\gamma^3 \int_{x_\gamma}^1 \frac{dy}{y^2} \bar{f}^{(0)}(y, \mu) \bar{\mathcal{J}}^{(m)}\left(\frac{x_\gamma}{y}, \mu_0, \mu\right), \quad (87)$$

where $\bar{f}^{(0)}(y, \mu) = f^{(0)}(-(1-y)m_b, \mu)/m_b$, and the difference between m_b and m_B is neglected because it is subleading. The moments of the mass correction to the differential decay rate are given by

$$\begin{aligned} \int_0^1 dx_\gamma x_\gamma^{N-1} \frac{1}{\Gamma_0} \frac{d\Gamma^{(m)}}{dx_\gamma} &= \tilde{H}(m_b, \mu_0) \int_0^1 dx_\gamma x_\gamma^{N+2} \int_{x_\gamma}^1 \frac{dy}{y^2} \bar{f}^{(0)}(y, \mu) \bar{\mathcal{J}}^{(m)}\left(\frac{x_\gamma}{y}, \mu_0, \mu\right) \\ &= \tilde{H}(m_b, \mu_0) \int_0^1 dy y^{N+1} \bar{f}^{(0)}(y) \int_0^1 dz_\gamma z_\gamma^{N+2} \bar{\mathcal{J}}^{(m)}(z_\gamma). \end{aligned} \quad (88)$$

Therefore the moment of the differential decay rate is given by the product of the moments of the shape function and the moments of the jet function. The moments $\overline{\mathcal{J}}_N^{(m)}$ to order α_s are given by

$$\begin{aligned}\overline{\mathcal{J}}_N^{(m)} &= \int_0^1 dz_\gamma z_\gamma^{N-1} \overline{\mathcal{J}}^{(m)}(z_\gamma) \\ &= -\frac{m^2}{m_b^2} (N-1) \left[1 + \frac{\alpha_s C_F}{4\pi} \left(2 \ln^2 \frac{m_b^2}{\mu^2} + (4H_{N-2} - 9) \ln \frac{m_b^2}{\mu^2} \right. \right. \\ &\quad \left. \left. + 4 \sum_{j=1}^{N-2} \frac{H_j}{j} - 9H_{N-2} + 10 - \pi^2 \right) \right],\end{aligned}\quad (89)$$

where $\ln y$ is neglected in the limit $y \rightarrow 1$, and $H_j = \sum_{k=1}^j 1/k$. In the large N limit, this becomes

$$\overline{\mathcal{J}}_N^{(m)}(\mu) = -\frac{m^2}{m_b^2} N \left[1 + \frac{\alpha_s}{4\pi} C_F \left(2 \ln^2 \frac{m_b^2}{\mu^2 \overline{N}} - 9 \ln \frac{m_b^2}{\mu^2 \overline{N}} + 10 - \frac{2\pi^2}{3} \right) \right], \quad (90)$$

where $\overline{N} = N e^{\gamma_E}$.

For comparison, the leading result without the quark mass term can be written in the same way as Eq. (72) with the jet function replaced by

$$\begin{aligned}J_\omega^{(0)}(y, z_\gamma) &= \frac{1}{m_b y (1 - z_\gamma + i\epsilon)} \left[1 + \frac{\alpha_s C_F}{4\pi} \left(2 \ln^2 \frac{-\omega m_b y (1 - z_\gamma + i\epsilon)}{\mu^2} \right. \right. \\ &\quad \left. \left. - 3 \ln \frac{-\omega m_b y (1 - z_\gamma + i\epsilon)}{\mu^2} + 7 - \frac{\pi^2}{3} \right) \right],\end{aligned}\quad (91)$$

with discontinuity

$$\begin{aligned}-\frac{1}{\pi} \text{Im} J_{m_b}^{(0)}(y, z_\gamma) &= \frac{1}{m_b y} \left\{ \delta(1 - z_\gamma) \left[1 + \frac{\alpha_s C_F}{4\pi} \left(2 \ln^2 \frac{m_b^2 y}{\mu^2} - 3 \ln \frac{m_b^2 y}{\mu^2} + 7 - \pi^2 \right) \right] \right. \\ &\quad \left. + \frac{\alpha_s C_F}{4\pi} \left[4 \left(\frac{\ln(1 - z_\gamma)}{1 - z_\gamma} \right)_+ + \left(4 \ln \frac{m_b^2 y}{\mu^2} - 3 \right) \frac{1}{(1 - z_\gamma)_+} \right] \right\} \theta(z_\gamma) \theta(1 - z_\gamma) \\ &\equiv \frac{1}{m_b y} \overline{\mathcal{J}}^{(0)}(z_\gamma, \mu),\end{aligned}\quad (92)$$

where we again neglect $\ln y$ terms near the endpoint $y \rightarrow 1$. This is consistent with the result in Ref. [30].

The leading differential decay rate is given by

$$\frac{1}{\Gamma_0} \frac{d\Gamma}{dx_\gamma} = x_\gamma^3 \left(\frac{m_b}{m_B} \right)^3 \tilde{H}(m_b, \mu_0) \int_{x_\gamma}^{m_B/m_b} \frac{dy}{y} \overline{f}^{(0)}(y, \mu) \overline{\mathcal{J}}^{(0)}\left(\frac{x_\gamma}{y}, \mu_0, \mu\right), \quad (93)$$

with moments

$$\begin{aligned}&\int_0^{m_B/m_b} dx_\gamma x_\gamma^{N-1} \frac{1}{\Gamma_0} \frac{d\Gamma}{dx_\gamma} = \left(\frac{m_B}{m_b} \right)^3 \tilde{H}(m_b, \mu_0) \int_0^{m_B/m_b} dx_\gamma x_\gamma^{N+1} \\ &\quad \times \int_{x_\gamma}^{m_B/m_b} \frac{dy}{y} \overline{f}^{(0)}(y, \mu) \overline{\mathcal{J}}^{(0)}\left(\frac{x_\gamma}{y}, \mu_0, \mu\right) \\ &= \left(\frac{m_B}{m_b} \right)^3 \int_0^{m_B/m_b} dy y^{N+2} \overline{f}^{(0)}(y, \mu) \int_0^1 dz_\gamma z_\gamma^{N+2} \overline{\mathcal{J}}^{(0)}(z_\gamma, \mu_0, \mu).\end{aligned}\quad (94)$$

The moments of the differential decay rate is factorized into a product of the moments of the jet function and the moments of the shape function of the B meson. The moments of the leading-order jet function become

$$\begin{aligned}
\overline{\mathcal{F}}_N^{(0)} &= \int_0^1 dz_\gamma z_\gamma^{N-1} \overline{\mathcal{F}}^{(0)}(z_\gamma) \\
&= 1 + \frac{\alpha_s C_F}{4\pi} \left[2 \ln^2 \frac{m_b^2}{\mu^2} - (4H_{N-1} + 3) \ln \frac{m_b^2}{\mu^2} + 3H_{N-1} + 4 \sum_{j=1}^{N-1} \frac{H_j}{j} + 7 - \pi^2 \right] \\
&\rightarrow 1 + \frac{\alpha_s C_F}{4\pi} \left(2 \ln^2 \frac{m_b^2}{\mu^2 \overline{N}} - 3 \ln \frac{m_b^2}{\mu^2 \overline{N}} + 7 - \frac{2\pi^2}{3} \right).
\end{aligned} \tag{95}$$

In Eq. (88), the typical matching scale μ_0 between SCET_I and SCET_{II} can be considered as $m_b/\sqrt{\overline{N}}$ and the hard coefficients \tilde{H} evolves from the scale $\mu = m_b$ to $m_b/\sqrt{\overline{N}}$. The jet function, determined at the scale $\mu = m_b/\sqrt{\overline{N}}$, scales down to the arbitrary scale $\mu < m_b/\sqrt{\overline{N}}$. Since the product of $\overline{\mathcal{F}}_N^{(0)}(\mu)$ and $\overline{\mathcal{F}}_N^{(m)}(\mu_0, \mu)$ is independent of the renormalization scale μ , the renormalization behavior of $\overline{\mathcal{F}}_N^{(m)}(\mu_0, \mu)$ is easily determined from the scaling of $\overline{\mathcal{F}}_N^{(0)}(\mu)$, which is given as [1, 13]

$$\mu \frac{d}{d\mu} \overline{\mathcal{F}}_N^{(0)}(\mu) = -\gamma_N(\mu) \overline{\mathcal{F}}_N^{(0)}(\mu), \tag{96}$$

where γ_N is the anomalous dimension of the shape function. For large N and at order α_s , it is given by

$$\gamma_N(\mu) = -\frac{\alpha_s C_F}{\pi} \left(1 + 2 \ln \frac{m_b}{\mu \overline{N}} \right). \tag{97}$$

Finally, since the hard part and the shape function in the leading mass correction of the moments of $\overline{B} \rightarrow X_s \gamma$ are the same as those of the leading-order moments [1], we find the resummation for the moments of the leading mass correction at the scale $\mu = m_b/\overline{N}$ to order α_s can be written as

$$\begin{aligned}
\int_0^1 x_\gamma^{N-1} \frac{1}{\Gamma_0} \frac{d\Gamma^{(m)}}{dx_\gamma} \left(\frac{m_b}{\overline{N}} \right) &= \overline{\mathcal{F}}_N^{(0)} \left(\frac{m_b}{\overline{N}} \right) \left(\frac{\alpha_s(m_b/\sqrt{\overline{N}})}{\alpha_s(m_b)} \right)^{\frac{C_F}{\beta_0} \left(\frac{5-8\pi}{\beta_0 \alpha_s} \right)} \left(-N \frac{m^2(m_b/\sqrt{\overline{N}})}{m_b^2} \right) \\
&\quad \times \left(\frac{\alpha_s(m_b/\overline{N})}{\alpha_s(m_b/\sqrt{\overline{N}})} \right)^{\frac{2C_F}{\beta_0} \left(1 + \frac{4\pi}{\beta_0 \alpha_s} - 2 \ln \overline{N} \right)}.
\end{aligned} \tag{98}$$

This result represents that the leading mass corrections are of order $(m^2/m_b^2)N \ln^k N$ in the large N limit, and they are resummed in moment space. Compared with the leading-order moments, the leading mass correction is always suppressed by $Nm^2/m_b^2 \sim \Lambda/m_b$.

A note is in order for Eq. (98), in which the quark mass m is evaluated at $m_b/\sqrt{\overline{N}}$ instead of m_b/\overline{N} . This means that the strange quark mass is frozen at $\mu = m_b/\sqrt{\overline{N}}$, and the scaling behavior below that scale to m_b/\overline{N} resides in the jet function. This is motivated by the fact that the effects of the quark mass reside only in the jet function and it looks more

transparent to consider the scaling behavior of the jet function as a whole including the quark mass. As can be seen in Fig. 6 (a)-(c), the radiative correction for the quark mass is included in computing the jet function. Another equivalent way of expressing Eq. (98) is to separate the effects of the quark mass from the remainder of the jet function and to scale each of them. Then the result can be expressed with m^2 evaluated at m_b/\sqrt{N} . These two methods are equivalent, and the latter method may be useful in considering the effects of the quark mass in exclusive B decays.

From this analysis we find that the quark mass effect to the leading decay rate is of order $Nm^2/m_b^2 \sim m^2/m_b^2(1-x_\gamma) \sim m^2/m_b\Lambda$ near the endpoint region $1-x_\gamma \sim \Lambda/m_b$. The size of the subleading corrections in Ref. [30] is of order Λ/m_b . The quark mass corrections and the subleading corrections are of the same order if we regard the strange quark mass as of order $m \sim \Lambda$. However, the strange quark mass is numerically about 80–130 MeV. Taking $\Lambda \sim 500$ MeV, the quark mass correction is about 2–7% of other subleading corrections, and less than 1% compared to the leading decay rate. Therefore the mass effect can be regarded as small compared to other subleading corrections, but the important point is that the effect of the light quark mass can be systematically implemented in the theoretical framework of SCET, and as experimental uncertainties become smaller, this effect should also be included.

V. CONCLUSION

We have considered the contribution of a quark mass of order Λ in SCET and its application to $\overline{B} \rightarrow X_s\gamma$ in the endpoint region. The quark mass can be included in the SCET Lagrangian systematically by integrating out hard degrees of freedom. We can find an extended reparameterization invariance including the quark mass, in which we modify only the reparameterization transformation of the spinor for the transformation of type-II. As a result, the SCET Lagrangian can be separated into two reparameterization-invariant combinations. The subleading operators in each combination are related to the leading operators in that combination by the reparameterization invariance, and they have the same Wilson coefficients as those of the leading operators. In particular, we find that the mass operators in the SCET Lagrangian have trivial Wilson coefficients and are not renormalized. These results are explicitly confirmed by the calculation of the corrections to the mass operators in SCET to one loop. The extended reparameterization invariance also constrains some of the Wilson coefficients for the heavy-to-light current operators with the quark mass. It plays an important role in the matching process of the subleading heavy-to-light currents and the higher-order calculations of the time-ordered products of the mass operators.

When we consider $\overline{B} \rightarrow X_s\gamma$ in the endpoint region, treating the strange quark mass to be of order Λ , the subleading contribution is of order Λ/m_b . We have verified this by matching the heavy-to-light current onto SCET_I with the mass operators. Many of the currents with the mass are related to the leading-order currents by the extended reparameterization

symmetry. There are also subleading operators which are independent of the leading current, and these are obtained at higher orders in α_s . There are no contributions with odd powers of m to the decay rate because of spin conservation. The subleading contributions of order $m^2/p_X^2 \sim \Lambda/m_b$ come from the time-ordered products of the double spin-flipped currents with the leading heavy-to-light currents, and the double spin-flipped currents are obtained by the time-ordered products of the leading currents with the mass operators in the SCET Lagrangian. The mass correction to the forward scattering amplitude is given by the factorized form which is expressed as a convolution of the m^2/p_X^2 suppressed jet function and the leading shape function of the B meson. The jet functions which are obtained from the matching between SCET_I and SCET_{II} can be always expanded by m^2/p_X^2 , and can be computed perturbatively in α_s .

In some B decays, the subleading effects can be important to extract the CKM matrix elements. We have shown that the strange quark mass corrections give nonnegligible contributions of order Λ/m_b in $\bar{B} \rightarrow X_s \gamma$, and it would be interesting to see if the mass corrections can give significant contribution to other B decays. The results in this paper can be a basis on how to explain the $SU(3)$ flavor-symmetry breaking effects, as in $B \rightarrow K^* \gamma$ and $B \rightarrow \rho \gamma$, in which the mass effects could be a leading result.

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- [1] C. W. Bauer, S. Fleming and M. E. Luke, Phys. Rev. D **63**, 014006 (2001) [arXiv:hep-ph/0005275].
 - [2] C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001) [arXiv:hep-ph/0011336].
 - [3] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **65**, 054022 (2002) [arXiv:hep-ph/0109045].
 - [4] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. Lett. **87**, 201806 (2001) [arXiv:hep-ph/0107002].
 - [5] C. W. Bauer, S. Fleming, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D **66**, 014017 (2002) [arXiv:hep-ph/0202088].
 - [6] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **66**, 054005 (2002)

- [arXiv:hep-ph/0205289]; I. W. Stewart, [arXiv:hep-ph/0308185].
- [7] J. Chay and C. Kim, Phys. Rev. D **65**, 114016 (2002) [arXiv:hep-ph/0201197].
- [8] M. Beneke, A. P. Chapovsky, M. Diehl and T. Feldmann, Nucl. Phys. B **643**, 431 (2002) [arXiv:hep-ph/0206152]; R. J. Hill and M. Neubert, Nucl. Phys. B **657**, 229 (2003) [arXiv:hep-ph/0211018]; C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **67**, 071502(R) (2003) [arXiv:hep-ph/0211069]; D. Pirjol and I. W. Stewart, Phys. Rev. D **67**, 094005 (2003) [arXiv:hep-ph/0211251].
- [9] S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. B **650**, 356 (2003) [arXiv:hep-ph/0209216]; E. Lunghi, D. Pirjol and D. Wyler, Nucl. Phys. B **649**, 349 (2003) [arXiv:hep-ph/0210091]; S. W. Bosch, R. J. Hill, B. O. Lange and M. Neubert, Phys. Rev. D **67**, 094014 (2003) [arXiv:hep-ph/0301123].
- [10] J. Chay and C. Kim, Phys. Rev. D **68**, 071502(R) (2003) [arXiv:hep-ph/0301055]; Nucl. Phys. B **680**, 302 (2004) [arXiv:hep-ph/0301262]; C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D **70**, 054015 (2004) [arXiv:hep-ph/0401188].
- [11] S. Mantry, D. Pirjol and I. W. Stewart, Phys. Rev. D **68**, 114009 (2003) [arXiv:hep-ph/0306254].
- [12] J. Chay and C. Kim, Phys. Rev. D **68**, 034013 (2003) [arXiv:hep-ph/0305033]; S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. B **693**, 103 (2004) [arXiv:hep-ph/0403277]; T. Becher, R. J. Hill and M. Neubert, arXiv:hep-ph/0503263.
- [13] C. W. Bauer and A. V. Manohar, Phys. Rev. D **70**, 034024 (2004) [arXiv:hep-ph/0312109]; S. W. Bosch, B. O. Lange, M. Neubert and G. Paz, Nucl. Phys. B **699**, 335 (2004) [arXiv:hep-ph/0402094].
- [14] S. Fleming and A. K. Leibovich, Phys. Rev. Lett. **90**, 032001 (2003) [arXiv:hep-ph/0211303]; Phys. Rev. D **67**, 074035 (2003) [arXiv:hep-ph/0212094]; Phys. Rev. D **70**, 094016 (2004) [arXiv:hep-ph/0407259]; S. Fleming, A. K. Leibovich and T. Mehen, Phys. Rev. D **68**, 094011 (2003) [arXiv:hep-ph/0306139]; S. Fleming, C. Lee and A. K. Leibovich, [arXiv:hep-ph/0411180].
- [15] A. V. Manohar, Phys. Rev. D **68**, 114019 (2003) [arXiv:hep-ph/0309176].
- [16] C. W. Bauer, A. V. Manohar and M. B. Wise, Phys. Rev. Lett. **91**, 122001 (2003) [arXiv:hep-ph/0212255]; C. W. Bauer, C. Lee, A. V. Manohar and M. B. Wise, Phys. Rev. D **70**, 034014 (2004) [arXiv:hep-ph/0309278].
- [17] I. Z. Rothstein, Phys. Rev. D **70**, 054024 (2004) [arXiv:hep-ph/0301240].
- [18] A. K. Leibovich, Z. Ligeti and M. B. Wise, Phys. Lett. B **564**, 231 (2003) [arXiv:hep-ph/0303099].
- [19] J. W. Chen and I. W. Stewart, Phys. Rev. Lett. **92**, 202001 (2004) [arXiv:hep-ph/0311285].
- [20] H. Boos, T. Feldmann, T. Mannel and B. D. Pecjak, arXiv:hep-ph/0504005.
- [21] A. V. Manohar, T. Mehen, D. Pirjol and I. W. Stewart, Phys. Lett. B **539**, 59 (2002) [arXiv:hep-ph/0204229].
- [22] E. Eichten and B. Hill, Phys. Lett. B **243**, 427 (1990).
- [23] A. F. Falk, B. Grinstein and M. E. Luke, Nucl. Phys. B **357**, 185 (1991).
- [24] G. Amoros, M. Beneke and M. Neubert, Phys. Lett. B **401**, 81 (1997) [arXiv:hep-ph/9701375].

- [25] M. Beneke and T. Feldmann, Nucl. Phys. B **685**, 249 (2004) [arXiv:hep-ph/0311335].
- [26] L. F. Abbott, Nucl. Phys. B **185**, 189 (1981).
- [27] T. Becher, R. J. Hill, B. O. Lange and M. Neubert, arXiv:hep-ph/0309227.
- [28] J. Chay and C. Kim, arXiv:hep-ph/0401089.
- [29] J. Chay, H. Georgi and B. Grinstein, Phys. Lett. B **247**, 399 (1990); I. I. Y. Bigi, M. A. Shifman, N. G. Uraltsev and A. I. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993) [arXiv:hep-ph/9304225]; B. Blok, L. Koyrakh, M. A. Shifman and A. I. Vainshtein, Phys. Rev. D **49**, 3356 (1994) [Erratum-ibid. D **50**, 3572 (1994)] [arXiv:hep-ph/9307247]; A. V. Manohar and M. B. Wise, Phys. Rev. D **49**, 1310 (1994) [arXiv:hep-ph/9308246].
- [30] K. S. M. Lee and I. W. Stewart, arXiv:hep-ph/0409045.
- [31] S. W. Bosch, M. Neubert and G. Paz, JHEP **0411**, 073 (2004) [arXiv:hep-ph/0409115].
- [32] M. Beneke, F. Campanario, T. Mannel and B. D. Pecjak, arXiv:hep-ph/0411395.
- [33] B. Grinstein, R. P. Springer and M. B. Wise, Nucl. Phys. B **339**, 269 (1990).
- [34] M. Beneke, Y. Kiyo and D. s. Yang, Nucl. Phys. B **692**, 232 (2004) [arXiv:hep-ph/0402241].
- [35] R. J. Hill, T. Becher, S. J. Lee and M. Neubert, JHEP **0407**, 081 (2004) [arXiv:hep-ph/0404217].