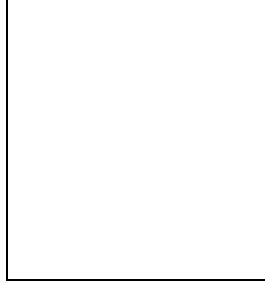


## Prediction of $U_{e3}$ and $\cos 2\theta_{23}$ from discrete symmetry <sup>a</sup>

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We discuss the question why the mixing  $U_{e3}$  is small. The natural answer is  $U_{e3} = 0$  in some symmetric limit, in which two large mixings are realized. It is possible to force  $U_{e3}$  and  $\cos 2\theta_{23}$  to be zero by imposing a discrete symmetry. We investigate a special class of symmetries  $Z_2$  and of the consequences of their perturbative violation.

### 1 Introduction

In the standard model with three families, three mixing angles are free parameters. A lot of studies address the origin of the bi-large mixings of neutrino flavors, which may be a clue to the beyond the standard model, on the other hand, we should also answer the question why the neutrino mixing  $U_{e3}$  is so small.

The natural answer is  $U_{e3} = 0$  in some symmetric limit, in which two large mixings are realized. There are many examples of symmetries which can force  $U_{e3}$  and/or  $\cos 2\theta_{23}$  to vanish. Both quantities vanish in the extensively studied bi-maximal mixing *Ansatz* <sup>2,3,4,5</sup>, which can be realized through a symmetry <sup>6</sup>. One can also make both  $U_{e3}$  and  $\cos 2\theta_{23}$  zero while leaving the solar mixing angle arbitrary <sup>7,8</sup>. Alternatively, it is possible to force only  $U_{e3}$  to be zero by imposing a discrete Abelian <sup>9</sup> or non-Abelian <sup>10</sup> symmetry; conversely, one can obtain maximal atmospheric mixing but a free  $U_{e3}$  in a non-Abelian symmetry or a non-standard CP symmetry <sup>11</sup>.

The symmetries mentioned above need not be exact. It is important to consider perturbations of those symmetries from the phenomenological point of view and to study quantitatively <sup>12</sup> the magnitudes of  $U_{e3}$  and  $\cos 2\theta_{23}$  possibly generated by such perturbations. We discuss a special class of symmetries  $Z_2$  and of the consequences of their perturbative violation. We also

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<sup>a</sup>This talk is based on the work collaborated with W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura and H. Sawanaka <sup>1</sup>.

study of the specific perturbation which is induced by the electroweak radiative corrections to a  $Z_2$ -invariant neutrino mass matrix defined at a high scale. The numerical result of a specific model is presented for this scenario.

## 2 Vanishing $U_{e3}$ and $Z_2$ symmetry

Let us construct the neutrino mass matrix in terms of neutrino masses  $m_1, m_2, m_3$  and mixings:

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (1)$$

where  $c_{ij}$  and  $s_{ij}$  denote  $\cos \theta_{ij}$  and  $\sin \theta_{ij}$ , respectively. The neutrino mass matrix  $M_\nu$  is given in the flavor basis:  $M_\nu = U_{MNS}^* M_{\text{diagonal}} U_{MNS}^\dagger$ .

In the standpoint of naturalness, as a dimensionless small parameter  $s_{13}$  goes down to zero, the symmetry should be enhanced. In  $|U_{e3}| = s_{13} = 0$  limit,  $M_\nu$  is written as

$$M_\nu = \begin{pmatrix} \tilde{X} & \tilde{A} & \tilde{B} \\ \tilde{A} & \tilde{C} & \tilde{D} \\ \tilde{B} & \tilde{D} & \tilde{E} \end{pmatrix}, \quad (2)$$

where matrix elements are given including Majorana phases  $\rho$  and  $\sigma$ :

$$\begin{aligned} \tilde{X} &= c_{12}^2 m_1 e^{-2i\rho} + s_{12}^2 m_2 e^{-2i\sigma}, & \tilde{A} &= c_{12}s_{12}c_{23}(m_2 e^{-2i\sigma} - m_1 e^{-2i\rho}), \\ \tilde{B} &= -c_{12}s_{12}s_{23}(m_2 e^{-2i\sigma} - m_1 e^{-2i\rho}), & \tilde{C} &= s_{12}^2 c_{23}^2 m_1 e^{-2i\rho} + c_{12}^2 c_{23}^2 m_2 e^{-2i\sigma} + s_{23}^2 m_3, \\ \tilde{D} &= c_{23}s_{23}(m_3 - m_1 e^{-2i\rho} s_{12}^2 - m_2 e^{-2i\sigma} c_{12}^2), & \tilde{E} &= s_{12}^2 s_{23}^2 m_1 e^{-2i\rho} + c_{12}^2 s_{23}^2 m_2 e^{-2i\sigma} + c_{23}^2 m_3. \end{aligned} \quad (3)$$

There is no explicit symmetry in this mass matrix if there are no relations among matrix elements. However, we obtain the mass matrix with a  $Z_2$  symmetry by putting  $\sin \theta_{23} = 1/\sqrt{2}$ :

$$M_{\nu f} = \begin{pmatrix} X & A & A \\ A & B & C \\ A & C & B \end{pmatrix}, \quad (4)$$

with

$$\begin{aligned} X &= c_{12}^2 m_1 e^{-2i\rho} + s_{12}^2 m_2 e^{-2i\sigma}, & A &= -\frac{1}{\sqrt{2}} c_{12}s_{12}(m_1 e^{-2i\rho} - m_2 e^{-2i\sigma}), \\ B &= \frac{1}{2}(s_{12}^2 m_1 e^{-2i\rho} + c_{12}^2 m_2 e^{-2i\sigma} + m_3), & C &= \frac{1}{2}(s_{12}^2 m_1 e^{-2i\rho} + c_{12}^2 m_2 e^{-2i\sigma} - m_3), \end{aligned} \quad (5)$$

where

$$SM_{\nu f}S = M_\nu^{(0)}, \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S^2 = 1.$$

The matrix  $S$  is a realization of the discrete group  $Z_2$ . It is emphasized that  $m_1, m_2, m_3, \theta_{12}, \rho, \sigma$  are arbitrary. In order to respect the symmetry,  $\theta_{23}$  is maximal, but  $\theta_{12}$  is not necessarily maximal. The general discussion of this symmetry was given in the previous work<sup>1</sup>.

### 3 Non-zero $U_{e3}$ , $\cos 2\theta_{23}$ from $Z_2$ breaking

Consider a general perturbation  $\delta\mathcal{M}_{\nu f}$  to  $M_{\nu f}$  in eq.(4). The matrix  $\delta\mathcal{M}_{\nu f}$  is a general complex symmetric matrix, but part of it can be absorbed through a redefinition of the parameters in eq.(4). The remaining part can be written, without loss of generality, as

$$\delta\mathcal{M}_{\nu f} = \begin{pmatrix} 0 & \epsilon_1 & -\epsilon_1 \\ \epsilon_1 & \epsilon_2 & 0 \\ -\epsilon_1 & 0 & -\epsilon_2 \end{pmatrix}. \quad (6)$$

The perturbation is controlled by two parameters,  $\epsilon_1$  and  $\epsilon_2$ , which are complex and model-dependent. We want to study their effects perturbatively, i.e. we want to assume  $\epsilon_1$  and  $\epsilon_2$  to be small. We define two dimensionless parameters:

$$\epsilon_1 \equiv \epsilon A, \quad \epsilon_2 \equiv \epsilon' B. \quad (7)$$

Thus, we have the neutrino mass matrix with  $Z_2$  breaking as follows:

$$\mathcal{M}_{\nu f} = \begin{pmatrix} X & A(1+\epsilon) & A(1-\epsilon) \\ A(1+\epsilon) & B(1+\epsilon') & C \\ A(1-\epsilon) & C & B(1-\epsilon') \end{pmatrix}, \quad (8)$$

where we shall assume  $\epsilon$  and  $\epsilon'$  to be small,  $|\epsilon|, |\epsilon'| \ll 1$ .

One finds that, to first order in  $\epsilon$  and  $\epsilon'$ , the only effect of the  $\delta\mathcal{M}_{\nu f}$  is to generate non-zero  $U_{e3}$  and  $\cos 2\theta_{23}$ . The neutrino masses, as well as the solar angle, do not receive any corrections.  $U_{e3}$  and  $\cos 2\theta_{23}$  are of the same order as  $\epsilon$  and  $\epsilon'$ . Define

$$\hat{m}_1 \equiv m_1 e^{-2i\rho}, \quad \hat{m}_2 \equiv m_2 e^{-2i\sigma}, \quad \bar{\epsilon} \equiv (\hat{m}_1 - \hat{m}_2)\epsilon, \quad \bar{\epsilon}' \equiv \frac{\hat{m}_1 s_{12}^2 + \hat{m}_2 c_{12}^2 + m_3}{2} \epsilon', \quad (9)$$

we get

$$\begin{aligned} U_{e3} &= \frac{s_{12}c_{12}}{m_3^2 - m_2^2} \left( \bar{\epsilon} s_{12}^2 \hat{m}_2^* + \bar{\epsilon}^* s_{12}^2 m_3 - \bar{\epsilon}' \hat{m}_2^* - \bar{\epsilon}'^* m_3 \right) \\ &\quad + \frac{s_{12}c_{12}}{m_3^2 - m_1^2} \left( \bar{\epsilon} c_{12}^2 \hat{m}_1^* + \bar{\epsilon}^* c_{12}^2 m_3 + \bar{\epsilon}' \hat{m}_1^* + \bar{\epsilon}'^* m_3 \right), \\ \cos 2\theta_{23} &= \text{Re} \left\{ \frac{2c_{12}^2}{m_3^2 - m_2^2} \left( \bar{\epsilon} s_{12}^2 - \bar{\epsilon}' \right) (\hat{m}_2 + m_3)^* - \frac{2s_{12}^2}{m_3^2 - m_1^2} \left( \bar{\epsilon} c_{12}^2 + \bar{\epsilon}' \right) (\hat{m}_1 + m_3)^* \right\}. \end{aligned} \quad (10)$$

The induced values of  $|U_{e3}|$  and  $|\cos 2\theta_{23}|$  are strongly correlated to neutrino mass hierarchies. This makes it possible to draw some general conclusions even if we do not know the magnitudes of  $\epsilon, \epsilon'$ . Remarks are given as follows:

- $U_{e3}$  gets suppressed by a factor  $\mathcal{O}(\frac{\Delta_{\text{sun}}}{\Delta_{\text{atm}}})$  for the inverted or quasi-degenerate spectrum with  $\rho = \sigma = 0$ . Similar suppression also occurs in the case of the normal neutrino mass hierarchy even when  $\rho \neq \sigma$ .  $U_{e3}$  need not be suppressed in other cases and can be large.
- In contrast to  $U_{e3}$ ,  $\cos 2\theta_{23}$  is almost as large as  $\epsilon, \epsilon'$  if neutrino mass spectrum is normal or inverted. It gets enhanced compared to these parameters if the spectrum is quasi-degenerate.
- In case of the quasi-degenerate spectrum, both  $|\cos 2\theta_{23}|$  and  $|U_{e3}|$  can become quite large and reach the present experimental limits. The parameters  $\epsilon, \epsilon'$  are constrained to be lower than  $10^{-2}$  for the quasi-degenerate spectrum.

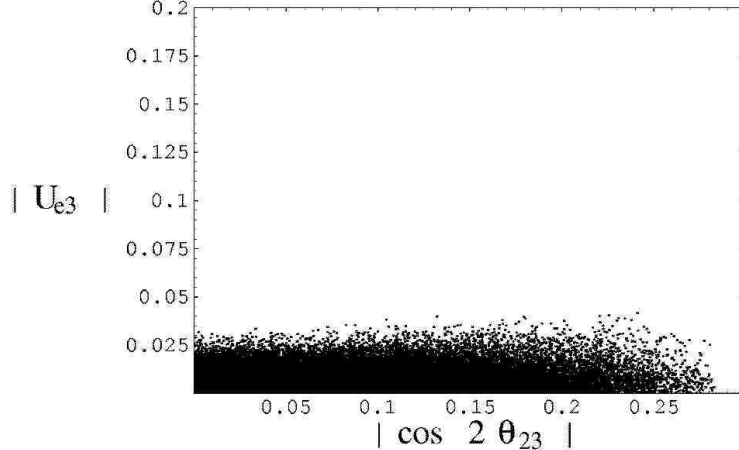


Figure 1: In case of the normal neutrino mass hierarchy with  $\rho = 0$ ,  $\sigma = 0$  and  $\epsilon, \epsilon' = -0.3 \sim 0.3$ .

In our numerical study, the input parameters are randomly varied in the experimentally allowed regions.  $m_1$  was varied up to  $m_2$ . On the other hand,  $\epsilon, \epsilon'$  are unknown unless the symmetry breaking is specified, so these are varied in the range  $-0.3 \sim 0.3$  with the condition that the output parameters should lie in the 90% CL limit of the experimental data.

In Fig.1, we show the numerical result in the case of the normal neutrino mass hierarchy with  $\rho = \sigma = 0$ . The  $|U_{e3}|$  is forced to be small less than 0.025. The value  $\sim 0.025$  at the upper end arises from the (assumed) bound  $|\epsilon|, |\epsilon'| \leq 0.3$ . Since  $|U_{e3}|$  is proportional to  $\epsilon, \epsilon'$ , it increases if the bound on  $\epsilon, \epsilon'$  is loosened. However,  $|\epsilon| \leq 0.3$  is a reasonable bound due to assume if  $Z_2$  breaking is perturbative. On the other hand,  $|\cos 2\theta_{23}|$  can assume large values as seen from Fig. 1. The present bound  $\sin^2 2\theta_{23} > 0.92$  from the atmospheric experiments gets translated to  $|\cos 2\theta_{23}| < 0.28$  which constrains  $|\epsilon'| \leq 0.2$  in our analyses. The phase dependence is found in the prediction of  $|U_{e3}|$ , which increases up to  $0.075$ <sup>1</sup>.

We wish to point out an interesting aspect of this analysis. Since  $U_{e3}$  is zero in the absence of the perturbation, the CP violating Dirac phase  $\delta$  relevant for neutrino oscillations is undefined at this stage. CP violation is present through the Majorana phases  $\rho$  and  $\sigma$ . Turning on perturbation leads to non-zero  $U_{e3}$  and also to a non-zero Dirac phase even if perturbation is real. Moreover,  $\delta$  generated this way can be large and independent of the strength of perturbation parameters<sup>13</sup>.

#### 4 Radiatively generated $U_{e3}$ and $\cos 2\theta_{23}$

The  $\epsilon, \epsilon'$  were treated as independent parameters so far. They can be related in specific models. We now consider one example which is based on the electroweak breaking of the  $Z_2$  symmetry in the MSSM. We assume that neutrino masses are generated at some high scale  $M_X$  and the effective neutrino mass operator describing them is  $Z_2$  symmetric with the result that  $U_{e3} = \cos 2\theta_{23} = 0$  at  $M_X$ . This symmetry is assumed to be broken spontaneously in the Yukawa couplings of the charged leptons. This breaking would radiatively induce non-zero  $U_{e3}$  and  $\cos 2\theta_{23}$ <sup>14</sup>. This can be calculated by using the renormalization group equations (RGEs) of the effective neutrino mass operator<sup>15,16,17</sup>. These equations depend upon the detailed structure of the model below  $M_X$ . We assume here that theory below  $M_X$  is the MSSM and use the RGEs derived in this case. Subsequently we will give an example which realizes our assumptions.

Integration of the RGEs allows us<sup>15,16,17</sup> to relate the neutrino mass matrix  $\mathcal{M}_{\nu f}(M_X)$  to the corresponding matrix at the low scale which we identify here with the  $Z$  mass  $M_Z$ :

$$\mathcal{M}_{\nu f}(M_Z) \approx I_g I_t ( I \mathcal{M}_{\nu f}(M_X) I ) , \quad (11)$$

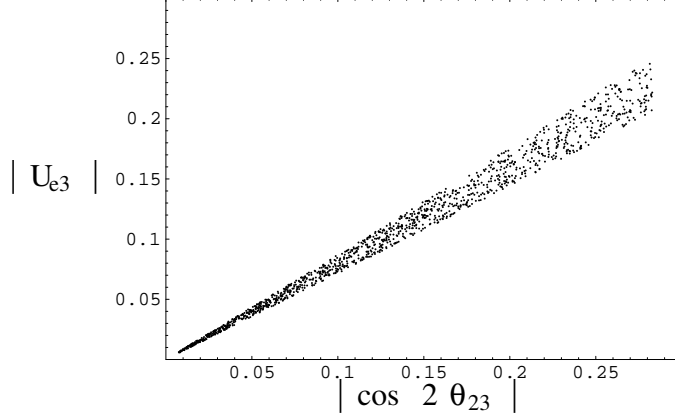


Figure 2: In case of the radiatively broken  $Z_2$  and the quasi-degenerate neutrino masses with  $\rho = 0$ ,  $\sigma = \pi/2$ .

where  $I_{g,t}$  are calculable numbers depending on the gauge and top quark Yukawa couplings.  $I$  is a flavor dependent matrix given by

$$I \approx \text{diag}(1 + \delta_e, 1 + \delta_\mu, 1 + \delta_\tau) \quad \text{with} \quad \delta_\alpha \approx c \left( \frac{m_\alpha}{4\pi v} \right)^2 \ln \frac{M_X}{M_Z}, \quad (12)$$

where  $c = \frac{3}{2}, -\frac{1}{\cos^2 \beta}$  in case of the SM and the MSSM, respectively<sup>15</sup>.  $v$  refers to the VEV for the SM Higgs doublet.

Since  $\mathcal{M}_{\nu f}(M_X)$  is given by eq. (4), we can write  $\mathcal{M}_{\nu f}(M_Z)$  as follows when the muon and the electron Yukawa couplings are neglected:

$$\mathcal{M}_{\nu f}(M_Z) = \begin{pmatrix} X & A' & A' \\ A' & B' & C' \\ A' & C' & B' \end{pmatrix} + \begin{pmatrix} 0 & A'\epsilon & -A'\epsilon \\ A'\epsilon & B'\epsilon' & 0 \\ -A'\epsilon & 0 & -B'\epsilon' \end{pmatrix} + O(\delta_\tau^2), \quad (13)$$

where

$$C' = C(1 + \delta_\tau), \quad A' = A(1 + \frac{\delta_\tau}{2}), \quad B' = B(1 + \delta_\tau), \quad \epsilon = \frac{\epsilon'}{2} = -\frac{\delta_\tau}{2}. \quad (14)$$

Note that  $m_1$ ,  $m_2$  and  $m_3$  defined previously are no longer mass eigenvalues because of the changes  $A \rightarrow A'$ ,  $B \rightarrow B'$  and  $C \rightarrow C'$ . Then we get

$$U_{e3} \simeq -\frac{\delta_\tau s_{12} c_{12}}{2(m_3^2 - m_1^2)} [m_1^2 + 2m_3 \hat{m}_1^* + m_3^2] + \frac{\delta_\tau s_{12} c_{12}}{2m_3^2 - m_2^2} [m_2^2 + 2\hat{m}_2^* m_3 + m_3^2],$$

$$\cos 2\theta_{23} \simeq \frac{\delta_\tau s_{12}^2}{m_3^2 - m_1^2} [m_1^2 + 2m_3 \hat{m}_1^* + m_3^2] + \frac{\delta_\tau c_{12}^2}{m_3^2 - m_2^2} [m_2^2 + 2\hat{m}_2^* m_3 + m_3^2]. \quad (15)$$

It is seen that the effect of the radiative corrections is enhanced in the case of the quasi-degenerate neutrino masses with  $|\rho - \sigma| = \pi/2$  as previous works presented<sup>16,17</sup>. In the MSSM, the parameter  $\delta_\tau$  is negative and its absolute value can become quite large for large  $\tan \beta$ . Results of the numerical analysis are shown in Fig. 2 in case of the quasi-degenerate spectrum with  $m = 0.3$  eV,  $\sigma = \pi/2$ ,  $\rho = 0$ . Both  $|U_{e3}|$  and  $|\cos 2\theta_{23}|$  can reach their respective experimental bound. We find numerically that  $\tan \beta$  is constrained to be lower than 20 in this case. On the other hand,  $|U_{e3}|$  reaches at most 0.025 in the normal-hierarchy and inverted-one. The forthcoming experiments will be able to test this relationship between  $|U_{e3}|$  and  $|\cos 2\theta_{23}|$ .

## 5 Summary

The neutrino mixing matrix contains two small parameters  $|U_{e3}|$  and  $\cos 2\theta_{23}$  which would influence the outcome of the future neutrino experiments. The vanishing of  $|U_{e3}|$  was shown to

follow from a class of  $Z_2$  symmetries of  $\mathcal{M}_{\nu f}$ . This symmetry can be used to parameterize all models with zero  $U_{e3}$ . A specific  $Z_2$  in this class also leads to the maximal atmospheric neutrino mixing angle. We showed that breaking of this can be characterized by two dimensionless parameters  $\epsilon, \epsilon'$  and we studied their effects perturbatively and numerically.

It was found that the magnitudes of  $|U_{e3}|$  and  $|\cos 2\theta_{23}|$  are strongly dependent upon the neutrino mass hierarchies and CP violating phases. The  $|U_{e3}|$  gets strongly suppressed in case of the inverted or quasi-degenerate neutrino spectrum if  $\rho = \sigma$  while similar suppression occurs in the case of normal hierarchy independent of this phase choice. The choice  $\rho \neq \sigma$  can lead to a larger values  $\sim 0.1$  for  $|U_{e3}|$  which could be close to the experimental value in some cases with inverted or quasi-degenerate spectrum. In contrast, the  $|\cos 2\theta_{23}|$  could be large, near its present experimental limit in most cases studied. For the normal and inverted mass spectrum, the magnitude of  $\cos 2\theta_{23}$  is similar to the magnitudes of the perturbations  $\epsilon, \epsilon'$  while it can get enhanced compared to them if the neutrino spectrum is quasi-degenerate.

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