# B meson wave function from the $B \rightarrow \gamma l \nu$ decay

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We show that the leading-power B meson wave function can be extracted reliably from the photon energy spectrum of the  $B \to \gamma l \nu$  decay up to  $O(1/m_b^2)$  and  $O(\alpha_s^2)$  uncertainty,  $m_b$  being the b quark mass and  $\alpha_s$  the strong coupling constant. The  $O(1/m_b)$  corrections from heavy-quark expansion can be absorbed into a redefined leading-power B meson wave function. The two-parton  $O(1/m_b)$ corrections cancel exactly, and the three-parton B meson wave functions turn out to contribute at  $O(1/m_b^2)$ . The constructive long-distance contribution through the  $B \to V \to \gamma$  transition, Vbeing a vector meson, almost cancels the destructive  $O(\alpha_s)$  radiative correction. Using models of the leading-power B meson wave function available in the literature, we obtain the photon energy spectrum in the perturbative QCD framework, which is then compared with those from other approaches.

#### I. INTRODUCTION

The two-parton leading-power (LP) B meson wave function (distribution amplitude)  $\phi_+$  plays an essential role in a perturbative analysis of exclusive B meson decays based on  $k_T$  factorization theorem [1, 2, 3] (collinear factorization theorem [4, 5, 6, 7, 8, 9]). Its behavior certainly matters, and has been investigated in various approaches recently. Models of the distribution amplitude  $\phi_+(x)$  with an exponential tail in the large x region have been proposed [10], where x is the longitudinal momentum fraction carried by the light spectator quark. Neglecting three-parton distribution amplitudes in a study by means of equations of motion [11, 12],  $\phi_+(x)$  was found to be proportional to a step function with a sharp drop at large x [13]. The wave function  $\phi_+(x, k_T)$ , where  $k_T$  is the transverse momentum carried by the light spectator quark, was also derived in the same framework [13]. All these models depend on at least one shape parameter, whose determination requires experimental inputs from exclusive B meson decays.

In this paper we shall show that the radiative decay  $B \to \gamma l\nu$  provides the cleanest information of the LP *B* meson wave function  $\phi_+$ . This mode has been widely studied in [3, 8, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28] due to different motivations: for extracting the *B* meson decay constant  $f_B$  and the Cabibbo-Kobayashi-Maskawa (CKM) matrix element  $|V_{ub}|$ , for demonstrating the next-to-leading-order (NLO) calculation and the proof of QCD factorization theorem, for deriving resummation of large logarithmic corrections, for studying long-distance effect and annihilation mechanism,.... The subject on the extraction of the *B* meson wave function from the  $B \to \gamma l\nu$  data has not yet been discussed. It will be shown that two-parton next-to-leading-power (NLP)  $[O(1/m_b)]$  corrections cancel exactly,  $m_b$  being the *b* quark mass. The contributions from higher Fock states, the three-parton *B* meson wave functions, turn out to be of  $O(1/m_b^2)$ . The constructive long-distance contribution through the  $B \to V \to \gamma$ transition, *V* being a vector meson, almost cancels the destructive  $O(\alpha_s)$  radiative correction,  $\alpha_s$  being the strong coupling constant. The effect from bremsstrahlung photon emissions vanishes like the lepton mass because of helicity suppression. Therefore, the extraction of  $\phi_+$  from the measured photon energy spectrum of the  $B \to \gamma l\nu$  decay suffers only  $O(1/m_b^2)$  and  $O(\alpha_s^2)$  uncertainty.

We identify and discuss the higher-power corrections to the  $B \to \gamma l\nu$  decay in Sec. II, and calculate the longand short-distance effects in Sec. III. Section IV is the conclusion. The hard kernel associated with the threeparton distribution amplitudes is derived in the Appendix, whose explicit expression is necessary for demonstrating the smallness of the higher-Fock-state contribution. Our conclusion differs from that drawn in [29], in which the semileptonic decay  $B \to \pi l\nu$  was regarded as a more ideal process for extracting the *B* meson wave function. The argument is that the radiative decay  $B \to \gamma l\nu$ , receiving a large long-distance uncertainty, does not serve the purpose. As stated above, this long-distance effect is in fact cancelled by the  $O(\alpha_s)$  short-distance one almost exactly.

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FIG. 1: Lowest-order diagrams for the  $B \rightarrow \gamma l \nu$  decay.

#### **II. HIGHER-POWER CORRECTIONS**

In this section we identify and discuss higher-power corrections to the  $B \to \gamma l \nu$  decay. The B meson momentum  $P_1$  and the photon momentum  $P_2$  are parameterized, in the light-cone coordinates, as

$$P_1 = \frac{m_B}{\sqrt{2}} (1, 1, \mathbf{0}_T) , \quad P_2 = \frac{m_B}{\sqrt{2}} (0, \eta, \mathbf{0}_T) , \qquad (1)$$

respectively, where  $\eta \equiv 2E_{\gamma}/m_B$ ,  $m_B$  being the *B* meson mass, denotes the photon energy fraction. The decay amplitude is decomposed into

$$\frac{1}{e}\left\langle\gamma\left(P_{2},\epsilon_{T}\right)\left|\bar{u}\gamma_{\mu}(1-\gamma_{5})b\right|\bar{B}\left(P_{1}\right)\right\rangle = \epsilon_{\mu\nu\alpha\beta}\epsilon_{T}^{*\nu}v^{\alpha}P_{2}^{\beta}F_{V}(q^{2}) + i\left[\epsilon_{T\mu}^{*}\left(v\cdot P_{2}\right) - \left(\epsilon_{T}^{*}\cdot v\right)P_{2\mu}\right]F_{A}(q^{2}),\tag{2}$$

where e is the electron charge,  $\epsilon_T$  the polarization vector of the photon,  $v = P_1/m_B$  the B meson velocity, and  $q^2 \equiv (P_1 - P_2)^2 = (1 - \eta)m_B^2$  the lepton-pair invariant mass. The decay spectrum is then given, in terms of the form factors  $F_{V,A}$ , by

$$\frac{d\Gamma}{d\eta} = \frac{\alpha G_F^2 |V_{ub}|^2}{96\pi^2} m_B^5 (1-\eta) \eta^3 \left[ F_V^2(q^2) + F_A^2(q^2) \right] , \qquad (3)$$

with  $\alpha \equiv e^2/(4\pi)$  and the Fermi constant  $G_F$ .

The collinear factorization theorem for the form factors  $F_{V,A}$  in the large  $\eta$  region has been proved in [24, 27], which are expressed as the convolution of hard kernels with the *B* meson distribution amplitudes in the momentum fractions *x* of the light spectator quark. A hard kernel, being infrared-finite, is calculable in perturbation theory. The *B* meson distribution amplitudes, collecting the soft dynamics in exclusive *B* meson decays, are not calculable but universal. In the framework of factorization theorem, there are four sources of higher-power corrections to the  $B \to \gamma l\nu$  decay:

1. The heavy-quark expansion of the heavy-light current in Eq. (2),

$$\bar{u}\gamma_{\mu}(1-\gamma_{5})b \to \bar{u}\gamma_{\mu}(1-\gamma_{5})h + \frac{1}{2m_{b}}\bar{u}\gamma_{\mu}(1-\gamma_{5})i \not\!\!Dh + O(1/m_{b}^{2}) ,$$
 (4)

where the operator D represents the covariant derivative, and the rescaled b quark field h is related to the full field b by

The factorization of the transition matrix element associated with the first (second) term in the above expansion leads to the LP (NLP) B meson distribution amplitudes.

2. The higher-power interactions in the Lagrangian of the heavy-quark effective theory (HQET). The insertion of the HQET interactions,

$$O_1 = \frac{1}{m_b} \bar{h} (iD)^2 h , \quad O_2 = \frac{g}{2m_b} \bar{h} \sigma^{\mu\nu} G_{\mu\nu} h , \qquad (6)$$

into the transition matrix element associated with the first term in Eq. (4) yields  $O(1/m_b)$  corrections. We mention that there exists an alternative heavy-quark effective theory, in which the higher-power corrections are formulated in a different way [30].

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3. The higher Fock states of the B meson. The nonlocal matrix element,

$$\langle 0|\bar{u}(z)gG_{\alpha\beta}(uz)h(0)|\bar{B}(P_1)\rangle , \qquad (7)$$

defines the three-parton distribution amplitudes, where  $G_{\alpha\beta}(uz)$  is the gluon field strength evaluated at the coordinate uz,  $0 \le u \le 1$ . The additional valence gluon, attaching internal off-shell quark lines, introduces one more hard propagator, i.e., one more power of  $1/m_b$ .

4. The subleading parton-level diagrams (hard kernels). The two-parton lowest-order hard kernels are displayed in Fig. 1, where the upper quark line represents a *b* quark. It is easy to observe that Fig. 1(a) [(b)] represents the LP (NLP) hard kernel, since the internal quark line is off-shell by  $m_b \bar{\Lambda} (m_b^2)$  with  $\bar{\Lambda}$  being a hadronic scale, such as the mass difference  $m_B - m_b$ .

## A. Heavy-quark Expansion

The factorization of soft dynamics from the transition matrix element associated with the first term on the righthand side of Eq. (4),

$$\left\langle \gamma\left(P_{2},\epsilon_{T}\right)\left|\bar{u}\gamma_{\mu}(1-\gamma_{5})h\right|\bar{B}\left(P_{1}\right)\right\rangle , \tag{8}$$

leads to the nonlocal matrix element [10],

$$\int \frac{dz^{-}d^{2}z_{T}}{(2\pi)^{3}} e^{i(k^{+}z^{-}-\mathbf{k}_{T}\cdot\mathbf{z}_{T})} \langle 0|\bar{u}_{\rho}(z)h_{\delta}(0)|\bar{B}(P_{1})\rangle = i\frac{f_{B}}{\sqrt{2}} \left\{ (\mathcal{P}_{1}+m_{B})\gamma_{5}\left[\not\!\!\!/a_{+}\Phi_{+}(k)+\not\!\!\!/a_{-}\Phi_{-}(k)\right] \right\}_{\delta\rho} , \qquad (9)$$

which defines the two-parton LP *B* meson wave functions  $\Phi_{\pm}$ , with the null vectors  $n_{+} = (1, 0, 0_T)$  and  $n_{-} = (0, 1, 0_T)$ , and the light quark momentum *k*. Because the photon momentum  $P_2$  has been chosen in the minus direction, the hard kernels for the form factors  $F_{V,A}$  are independent of the component  $k^-$ , which becomes irrelevant. We construct the *B* meson distribution amplitudes  $\phi_{\pm}(x), x \equiv k^+/P_1^+$ , from the *B* meson wave functions  $\phi_{\pm}(x, k_T) \equiv P_1^+ \Phi_{\pm}(xP_1^+, k_T)$ by integrating the latter over  $k_T$ ,

$$\phi_{\pm}(x) = \int d^2 k_T \phi_{\pm}(x, k_T) \ . \tag{10}$$

The dependence of  $\phi_{\pm}(x)$  and of  $\phi_{\pm}(x, k_T)$  on the renormalization scale  $\mu$  has been suppressed.

Define the moments of the B meson distribution amplitude  $\phi_{+}(x)$ ,

$$\Lambda_0 \equiv \int dx \frac{\phi_+(x)}{x} , \quad \Lambda_1 \equiv \int dx \phi_+(x) . \tag{11}$$

The asymptotic behavior of  $\phi_+(x)$  has been extracted from a renormalization-group equation, which exhibits a decrease slower than 1/x [31, 32]. That is, the normalization  $\Lambda_1$  of the *B* meson distribution amplitude is divergent after taking into account the evolution effect. It has been argued that a non-normalizable *B* meson distribution amplitude does not cause a trouble in practice [33], since only the inverse moment  $\Lambda_0$  matters at LP [25, 34], which is convergent. Note that a hard kernel would not be as simple as 1/x at higher orders in  $\alpha_s$ , and information of more moments is also necessary. In the following discussion we shall neglect the evolution effect, and assume that  $\phi_+(x)$  is normalized to unity, i.e.,  $\Lambda_1 = 1$ . Since the *B* meson distribution amplitudes absorb soft dynamics, the light quark momentum *k* is of  $O(\overline{\Lambda})$ . We then have the relative importance  $\Lambda_1/\Lambda_0 \sim \overline{\Lambda}/m_b$  for  $x \sim O(\overline{\Lambda}/m_b)$ .

The factorization of soft dynamics from the transition matrix element associated with the second term on the right-hand side of Eq. (4) gives the nonlocal matrix element,

$$\langle 0|\bar{u}_{\rho}(z)i \not\!\!Dh_{\delta}(0)|\bar{B}(P_1)\rangle$$
 (12)

The factorization of the transition matrix elements with the insertion of the  $O(1/m_b)$  interactions in Eq. (6) into Eq. (8) leads to,

$$\langle 0|i \int d^4 y T[\bar{u}_{\rho}(z)h_{\delta}(0)O_{1,2}(y)]|\bar{B}(P_1)\rangle$$
 (13)

The contributions from Eqs. (12) and (13) can be absorbed into the nonlocal matrix element,

$$\langle 0|\bar{u}_{\rho}(z)b_{\delta}(0)|\bar{B}(P_{1})\rangle , \qquad (14)$$



FIG. 2: Three-parton contribution to the  $B \rightarrow \gamma l \nu$  decay.

where the rescaled b quark field h has been replaced by the full field b. It is easy to check that the heavy-quark expansion of Eq. (14) generates Eqs. (12) and (13). This absorption makes sense, because Eqs. (12) and (13), concerning only the initial b quark, are universal for all exclusive B meson decays. The decomposition in Eq. (9) still holds, but the B meson distribution amplitudes  $\Phi_{\pm}$ , redefined by Eq. (14) in terms of the full field b, exhibit a renormalization-group evolution different from that in Eq. (9) [35].

### B. Three-parton Distribution Amplitudes

We explain that the nonlocal matrix element in Eq. (7) is negligible in the current accuracy: the three-parton distribution amplitudes, whose contributions to the form factors are supposed to be of  $O(1/m_b)$ , turn out to appear at  $1/m_b^2$ . The three-parton distribution amplitudes  $\tilde{\Phi}_V$ ,  $\tilde{\Phi}_A$ ,  $\tilde{X}_A$ , and  $\tilde{Y}_A$  in coordinate space are defined via the decomposition,

$$\langle 0|\bar{u}_{\rho}(z)gG_{\alpha\beta}(uz)n_{-}^{\beta}h_{\delta}(0)|\bar{B}(P_{1})\rangle = f_{B}\left\{ \left(\mathcal{P}_{1}+m_{B}\right)\gamma_{5}\left[ \left(v_{\alpha}\not n_{-}-v\cdot n_{-}\gamma_{\alpha}\right)\left(\tilde{\Phi}_{V}(t,u)-\tilde{\Phi}_{A}(t,u)\right)\right. \\ \left. \left. \left. \left. -i\sigma_{\alpha\beta}n_{-}^{\beta}\tilde{\Phi}_{V}(t,u)-n_{-\alpha}\tilde{X}_{A}(t,u)+\frac{n_{-\alpha}}{v\cdot n_{-}}\not n_{-}\tilde{Y}_{A}(t,u)\right] \right\}_{\delta\rho} \right\}_{\delta\rho},$$

$$(15)$$

with the variable  $t = v \cdot z$ . The corresponding hard kernels arise from the contraction of all the structures  $\Gamma = v_{\alpha} \not n_{-}$ ,  $v \cdot n_{-} \gamma_{\alpha}, \dots$ , in Eq. (15) with Fig. 2, written as

$$\mathcal{M}_{a}^{(3)} \propto \frac{tr\{\not \in_{T}^{\ast}[u \not h_{+}\gamma^{\alpha}(\not P_{2}-\not k_{1}-u \not k_{2})-\bar{u}(\not P_{2}-\not k_{1}-u \not k_{2})\gamma^{\alpha} \not h_{+}]\gamma_{\mu}(1-\gamma_{5})(\not P_{1}+m_{B})\gamma_{5}\Gamma\}}{[(P_{2}-k_{1}-uk_{2})^{2}]^{2}},$$
(16)

where  $k_1$  ( $k_2$ ) is the momentum carried by the light quark (gluon). The derivation of the above expression is referred to the Appendix.

For  $\Gamma = v \cdot n_{-}\gamma_{\alpha}$ , Eq. (16) vanishes because of  $\epsilon_{T}^{*} \cdot n_{+} = \epsilon_{T}^{*} \cdot (P_{2} - k_{1} - uk_{2}) = 0$ . Express  $\sigma_{\alpha\beta}n_{-}^{\beta} = i(n_{-\alpha} - \not n_{-}\gamma_{\alpha})$ , in which the first term has the same structure as of  $\tilde{X}_{A}$ . The second term  $\not n_{-}\gamma_{\alpha}$  renders Eq. (16) vanish for the same reason. For the other structures  $v_{\alpha} \not n_{-}$ ,  $n_{-\alpha}$ , and  $n_{-\alpha} \not n_{-}$ , we always have  $\gamma^{\alpha} = \gamma^{+}$ . Once  $\gamma^{\alpha} = \gamma^{+}$ , Eq. (16) is proportional to

$$\mathcal{M}_{a}^{(3)} \sim \frac{P_1 \cdot (k_1 + uk_2)}{[(P_2 \cdot (k_1 + uk_2)]^2} \,. \tag{17}$$

Note that  $k_1^+$  and  $k_2^+$  are of  $O(\bar{\Lambda})$ , and that the moments of the three-parton B meson distribution amplitudes are at most of  $O(\bar{\Lambda}^2)$  [10]. Therefore, when convoluting Eq. (17) with the three-parton distribution amplitudes, the resultant contribution to the form factors  $F_{V,A}$  is of  $O(\bar{\Lambda}^2/m_b^2)$  compared to the LP one from Fig. 1(a). The same higher Fock state has been shown to give a power-suppressed correction to the  $B \to \gamma l\nu$  decay in the framework of soft-collinear effective theory (SCET) [23]. With a similar reasoning, the three-parton B meson wave functions also contribute at  $O(1/m_b^2)$  in  $k_T$  factorization theorems. We emphasize that the three-parton B meson wave functions are relevant in the NLP analysis of the  $B \to \pi$  transition form factors. This is the reason the  $B \to \gamma l\nu$  decay is a cleaner mode than the  $B \to \pi l\nu$  decay for determining the LP B meson wave function.

#### C. NLP Hard Kernels

Contracting Fig. 1 with the two structures in Eq. (9), we get the quark-level amplitudes,

$$\mathcal{M}_{a}^{+} = \frac{i}{4\sqrt{2}} \frac{tr[\mathscr{A}^{*}(\mathscr{P}_{2}-\mathscr{K})\gamma_{\mu}(1-\gamma_{5})(\mathscr{P}_{1}+m_{B})\gamma_{5}\mathscr{N}_{+}]}{(P_{2}-k)^{2}},$$
  
$$\mathcal{M}_{b}^{+} = \frac{i}{4\sqrt{2}} \frac{tr[\gamma_{\mu}(1-\gamma_{5})(\mathscr{A}-\mathscr{K}+m_{b})\mathscr{A}^{*}(\mathscr{P}_{1}+m_{B})\gamma_{5}\mathscr{N}_{+}]}{(q-k)^{2}-m_{b}^{2}},$$
(18)

and  $\mathcal{M}_{a,b}^-$  with the null vector  $n_+$  in  $\mathcal{M}_{a,b}^+$  being replaced by  $n_-$ . As stated above, Fig. 1(a) is LP, because of  $(P_2 - k)^2 = -2P_2 \cdot k \sim O(m_b\bar{\Lambda})$ , and Fig. 1(b) is NLP, because of  $(q - k)^2 - m_b^2 = -2P_1 \cdot P_2 \sim O(m_b^2)$ . The contribution from Fig. 1(b) has not yet been considered in the literature. We shall neglect the mass difference between the *B* meson and the *b* quark in  $\mathcal{M}_b^{+,-}$  in our analysis accurate up to NLP.

The collinear factorization formulas for  $F_{V,A}$  are written as

$$F_{V(A)}(q^2) = f_B \int dx \left[ \phi_+(x) H^+_{V(A)}(x,\eta) + \phi_-(x) H^-_{V(A)}(x,\eta) \right] , \qquad (19)$$

where the hard kernels H are extracted according to Eq. (2) by keeping only the longitudinal component  $k^+$  in Eq. (18). In terms of the LP and NLP moments in Eq. (11), Eq. (19) becomes

$$F_{V,A}(q^2) = \frac{f_B}{\eta m_B} \left[ \Lambda_0 \pm \left( 1 + \frac{1}{\eta} \right) \Lambda_1 \right] , \qquad (20)$$

in which the coefficient 1 of  $\Lambda_1$  comes from Fig. 1(a) and  $1/\eta$  from Fig. 1(b). It has been mentioned that the equality of  $F_V$  and  $F_A$  at LP is attributed to the spin symmetry in the large-recoil region [20]. The coefficient  $1/\eta$  implies the increase of the subleading-power correction with the decrease of the photon energy. This is why a perturbation theory is reliable only in the large  $\eta$  region. The distribution amplitude  $\phi_-(x)$ , contributing only through the normalization of the combination,

$$\int dx [\phi_+(x) - \phi_-(x)] = 0 , \qquad (21)$$

disappears from Eq. (20). As shown in Eq. (20), the normalization  $\Lambda_1$  does appear at NLP, which is divergent under the evolution. This is another example that the QCD-improved factorization (QCDF) approach based on collinear factorization theorem breaks down at NLP [34, 36].

The decay spectrum in Eq. (3) becomes

$$\frac{d\Gamma}{d\eta} = \frac{\alpha G_F^2 |V_{ub}|^2}{48\pi^2} f_B^2 m_B^3 (1-\eta) \eta \left[ \Lambda_0^2 + \left( 1 + \frac{1}{\eta} \right)^2 \Lambda_1^2 \right] .$$
(22)

The above expression indicates that the NLP terms for the spectrum have cancelled, and only the  $O(1/m_b^2)$  term  $\Lambda_1^2$  is left. In this case we can estimate the  $O(1/m_b^2)$  effect using the models for the *B* meson distribution amplitudes available in the literature [13, 37],

$$\phi_{\pm}(x) = \frac{\lambda \pm (x - \lambda)}{2\lambda^2} \theta(x) \theta(2\lambda - x) , \qquad (23)$$

with the shape parameter  $\lambda \equiv \overline{\Lambda}/m_b$ . The value of  $\overline{\Lambda}$  has been found to range between 0.5 and 0.7 GeV [25, 38, 39], which corresponds to  $\lambda = 0.1 \sim 0.15$  approximately. Certainly, there are other models of the *B* meson distribution amplitudes (see [40]).

Employing the inputs  $\alpha = 1/137$ ,  $G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$ ,  $|V_{ub}| = 3.9 \times 10^{-3}$ ,  $f_B = 190 \text{ MeV}$ , and  $m_B = 5.28 \text{ GeV}$ , we derive the photon energy spectra of the  $B \to \gamma l \nu$  decay for  $\lambda = 0.1$  and for  $\lambda = 0.15$  in Fig. 3. The specific models in Eq. (23) lead to the relation  $\Lambda_1/\Lambda_0 = \lambda$ . Therefore, the subleading-power term is indeed negligible at large  $\eta$ , whose contribution is around 5%. However, this term diverges quickly at small  $\eta$ , breaking the perturbative expansion in  $1/m_b$ . The form factors  $F_{V,A}$  in Eq. (20) contain a dominant monopole component proportional to  $\Lambda_0/\eta$ , and a small dipole component proportional to  $\Lambda_1/\eta^2$ , which is important only at small  $\eta$ . This is the reason one always obtains a symmetric spectrum in  $\eta$  at LP from a perturbation theory [20] as shown in Fig. 3. To generate an



FIG. 3: Spectra in units of GeV<sup>-1</sup> from collinear factorization with the solid (dashed-dotted)line corresponding to the LP contribution for  $\lambda = 0.1$  ( $\lambda = 0.15$ ), and the dashed (dotted)line to the inclusion of the NLP contribution for  $\lambda = 0.1$  ( $\lambda = 0.15$ ).

asymmetric spectrum, the dipole component must be enhanced as postulated in [16, 26]. Therefore, an asymmetric spectrum signals an important NLP contribution, i.e., a breakdown of factorization theorem.

It has been explained that the undesirable feature of the B meson distribution amplitude under evolution is a consequence of collinear factorization, which can be removed in  $k_T$  factorization [35]. The evolution effect on the  $k_T$ -dependent B meson wave function was also studied in [41]. Moreover, applying  $k_T$  factorization theorem to the  $B \rightarrow \gamma l \nu$  decay, which has been proved in [3], we can extend the spectrum to lower  $\eta$  as demonstrated below. Keeping both the longitudinal momentum  $k^+$  and the transverse momentum  $k_T$  in Eq. (18), the hard kernels in  $k_T$  factorization theorem are derived. Defining the LP and NLP functions,

$$\Lambda_{0}(\eta) \equiv m_{B}^{2} \int dx \int d^{2}k_{T} \frac{\phi_{+}(x,k_{T})}{\eta x m_{B}^{2} + k_{T}^{2}} ,$$
  

$$\Lambda_{1}(\eta) \equiv m_{B}^{2} \int dx \int d^{2}k_{T} \left[ \frac{\phi_{+}(x,k_{T})}{\eta m_{B}^{2} + k_{T}^{2}} + \frac{x\phi_{-}(x,k_{T})}{\eta(\eta x m_{B}^{2} + k_{T}^{2})} \right] ,$$
(24)

respectively, we obtain the form factors,

$$F_{V,A}(q^2) = \frac{f_B}{m_B} \left[ \Lambda_0(\eta) \pm \Lambda_1(\eta) \right] .$$
<sup>(25)</sup>

Because of  $k_T \sim O(\bar{\Lambda})$  in the *B* meson,  $\Lambda_1(\eta)$  is of  $O(\bar{\Lambda}/m_b)$  relative to  $\Lambda_0(\eta)$  in the large  $\eta$  region. Again, only a single *B* meson wave function is relevant in the LP analysis of the  $B \to \gamma l\nu$  decay, consistent with the observation in [49]. Compared to Eq. (20), both  $\phi_{\pm}$  appear in the  $k_T$  factorization theorem at NLP.

The decay spectrum is then given, according to Eq. (3), by

$$\frac{d\Gamma}{d\eta} = \frac{\alpha G_F^2 |V_{ub}|^2}{48\pi^2} f_B^2 m_B^3 (1-\eta) \eta^3 \left[ \Lambda_0^2(\eta) + \Lambda_1^2(\eta) \right] \,. \tag{26}$$

Similarly, the NLP terms have cancelled, and only the  $O(\bar{\Lambda}^2/m_b^2)$  term  $\Lambda_1^2(\eta)$  is left. We adopt the models for the *B* meson wave functions in [13], whose  $k_T$  dependence is coupled to the *x* dependence through a  $\delta$ -function,

$$\phi_{\pm}(x,k_T) = \phi_{\pm}(x)\frac{1}{\pi}\delta\left(k_T^2 - x(2\lambda - x)m_B^2\right) .$$
(27)

Using the same input parameters, we obtain the photon energy spectra from  $k_T$  factorization theorem in Fig. 4 for  $\lambda = 0.1$  and for  $\lambda = 0.15$ . These spectra are symmetric in  $\eta$ , and modified only slightly by the higher-power correction. Hence, the higher-power correction is under control in  $k_T$  factorization theorem compared to that in collinear factorization theorem: the power behavior  $1/\eta$  of the spectrum in the small  $\eta$  region has been smeared into  $\eta \ln^2 \eta$ . It implies that the perturbative QCD (PQCD) approach based on  $k_T$  factorization theorem [42, 43, 44, 45] has a better convergence at subleading level.



FIG. 4: Spectra in units of GeV<sup>-1</sup> from  $k_T$  factorization with the solid (dashed-dotted)line corresponding to the LP contribution for  $\lambda = 0.1$  ( $\lambda = 0.15$ ), and the dashed (dotted)line to the inclusion of the NLP contribution for  $\lambda = 0.1$  ( $\lambda = 0.15$ ).

# **III. LONG- AND SHORT-DISTANCE CORRECTIONS**

In this section we discuss the long-distance and short-distance corrections to the  $B \rightarrow \gamma l \nu$  decay spectrum. For this purpose, the form factors are written, in  $k_T$  factorization theorem, as

$$F_{V,A}(q^2) = \frac{f_B}{m_B} \left[ \Lambda_0(\eta) + \Lambda_0^{(1)}(\eta) \right] + F_{V,A}^{\text{LD}}(q^2) , \qquad (28)$$

where  $\Lambda_0^{(1)}$  and  $F_{V,A}^{\text{LD}}$  denote the  $O(\alpha_s)$  and long-distance correction to the leading result, respectively. We shall estimate the latter by considering the  $B \to V \to \gamma$  transition. This correction is certainly significant in the small  $\eta$ (large  $q^2$ ) region, where the internal quark becomes soft, and easily form a resonance with the spectator quark. Hence, it could break the QCD factorization of the form factors  $F_{V,A}$  at small  $\eta$ . At large  $\eta$ , the long-distance contribution may be suppressed by the values of the  $B \to V$  transition form factors [15].

The long-distance amplitude is written as [46]

$$\frac{1}{e} \left\langle \gamma \left( P_2, \epsilon_T \right) \left| \bar{u} \gamma_\mu (1 - \gamma_5) b \right| \bar{B} \left( P_1 \right) \right\rangle = \sum_V \left\langle \gamma \left( P_2, \epsilon_T \right) \left| J_{em}^\alpha \right| V \left( P_2, \epsilon_T \right) \right\rangle \frac{-i\epsilon_{T\alpha}^*}{P_2^2 - m_V^2 + im_V \Gamma_V} \times \left\langle V \left( P_2, \epsilon_T \right) \left| \bar{u} \gamma_\mu (1 - \gamma_5) b \right| \bar{B} \left( P_1 \right) \right\rangle ,$$
(29)

with the vector mesons  $V = \rho, \omega, \dots$ , and their masses  $m_V$  and widths  $\Gamma_V$ . Take the *B* meson transition into a transversely polarized  $\rho$  meson as an example, for which the first matrix element on the right-hand side of Eq. (29) gives

$$\langle \gamma(P_2, \epsilon_T) | J^{\alpha}_{em} | \rho(P_2, \epsilon_T) \rangle = -\frac{i}{2} m_{\rho} f_{\rho} \epsilon^{\alpha}_T , \qquad (30)$$

 $f_\rho$  being the  $\rho$  meson decay constant. The second matrix element is decomposed into

$$\langle \rho(P_2, \epsilon_T) | \bar{u} \gamma_\mu (1 - \gamma_5) b | \bar{B}(P_1) \rangle = -\frac{2V(q^2)}{m_B + m_\rho} \epsilon_{\mu\nu\rho\sigma} \epsilon_T^{*\nu} P_1^{\rho} P_2^{\sigma} - i(m_B + m_\rho) A_1(q^2) \epsilon_{T\mu}^* , \qquad (31)$$

with the  $B \to \rho$  form factors  $V(q^2)$  and  $A_1(q^2)$ . Combining Eqs. (30) and (31), we extract from Eq. (29),

$$F_V^{\rm LD}(q^2) = \frac{f_\rho}{m_\rho - i\Gamma_\rho} \frac{m_B}{m_B + m_\rho} V(q^2) ,$$
  

$$F_A^{\rm LD}(q^2) = \frac{f_\rho}{m_\rho - i\Gamma_\rho} \frac{(m_B + m_\rho)}{\eta m_B} A_1(q^2) .$$
(32)

For the long-distance contribution through the  $B \to \omega$  transition, we have the similar expressions to Eq. (32), but with the charge factor 1/2 in Eq. (30) being replaced by 1/6, and the appropriate replacement of the vector meson mass and of the decay constant. The  $B \rightarrow \psi$  transitions do not contribute in this case.

For the  $\rho$  and  $\omega$  mesons, we employ the inputs [46]

$$m_{\rho} = 0.771 \text{ GeV}, \quad \Gamma_{\rho}/m_{\rho} = 0.21, \quad f_{\rho} = 0.217 \text{ GeV}, m_{\omega} = 0.783 \text{ GeV}, \quad \Gamma_{\omega}/m_{\omega} \approx 0, \quad f_{\omega} = 0.195 \text{ GeV}.$$
 (33)

For the  $B \to \rho$ ,  $\omega$  form factors, we adopt the models derived from the light-front QCD [47], which have been parameterized as

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2},$$
(34)

with the constants,

$$V(q^2): \quad F(0) = 0.27 , \quad a = 1.84 , \quad b = 1.28 ,$$
  

$$A_1(q^2): \quad F(0) = 0.22 , \quad a = 0.95 , \quad b = 0.21 .$$
(35)

We restrict the above formalism in the region,

$$\eta > 1 - \frac{q_{\max}^2}{m_B^2} = 0.275 , \qquad (36)$$

 $q_{\text{max}}^2$  being the maximal value of  $q^2$  in the  $B \to \omega$  transition, in which Eq. (34) holds. The long-distance contribution increases  $F_{V,A}$  by about 30 ~ 50% for  $\lambda = 0.1 \sim 0.15$  at large  $\eta$ , consistent with the observation in [15, 25, 48]. Its effect to the decay spectrum is quite important, especially for  $\eta < 0.8$ , as shown in Fig. 5.

The  $B \to \rho, \omega$  transition form factors at large recoil could be regarded as an  $O(\alpha_s)$  object [49]. This observation hints that we should attempt to take into account the NLO short-distance correction to  $F_{V,A}$ . The NLO correction to the  $B \to \gamma l \nu$  decay has been computed by several groups [22, 23, 24] in collinear factorization theorem (SCET or QCDF). However, we need the result from  $k_T$  factorization theorem (with the parton transverse momenta  $k_T$  being included), which is quoted from [20]:

$$\Lambda_{0}^{(1)}(\eta) = -\frac{\alpha_{s}(2E_{\gamma})}{4\pi}C_{F}m_{B}^{2}\int dx \int d^{2}k_{T}\frac{\phi_{+}(x,k_{T})}{\eta x m_{B}^{2} + k_{T}^{2}} \times \left[\ln^{2}\frac{\eta}{x} - \frac{5}{2}\ln\frac{\eta}{x} + \frac{4\pi^{2}}{3} - \ln^{2}\left(1 + \frac{k_{T}^{2}}{2k^{+2}}\right) + 2\pi i\ln\left(1 + \frac{k_{T}^{2}}{2k^{+2}}\right)\right].$$
(37)

The weaker evolution of  $f_B$  will be neglected for simplicity. Due to the large negative double logarithm, the NLO correction to the form factors  $F_{V,A}$  is destructive, and about 30% of the leading result for both  $\lambda = 0.1$  and  $\lambda = 0.15$ at large  $\eta$ . The resummation of this double logarithm to all orders has been discussed in [20, 22, 23, 24, 28].

We emphasize that the NLO hard kernel depends on a factorization scheme, in which the B meson wave function is defined [23]. Therefore, it is not very legitimate to adopt an expression straightforwardly from some other works in the literature. The calculation of the NLO hard kernel for the  $B \rightarrow \gamma l \nu$  decay in the factorization scheme specified in [35] is in progress, which will be published elsewhere. The NLO correction in SCET has been further factorized into a function characterized by the scale  $m_b$ , and another by  $\sqrt{m_b\Lambda}$ . As stated in [23], this further factorization is not numerically essential for  $m_b \approx 5$  GeV. On the other hand, the model-dependent estimate of the long-distance contribution also suffers large uncertainty. Hence, we just intend to point out the potential strong cancellation between the long-distance and short-distance corrections in this mode. As shown in Fig. 5, after combining both subleading contributions, the net effect has been greatly reduced. Especially, for the shape parameter  $\lambda = 0.1$ , the cancellation is almost exact for  $\eta > 0.8$ . We conclude that the leading result in the large  $\eta$  region is stable under these corrections. Using the lifetime of a charged B meson  $\tau_{B^{\pm}} = 1.674 \times 10^{-12}$  s and considering only the leading contribution, we

obtain the branching ratios for  $\lambda = 0.15 \sim 0.1$ ,

$$B(B \to \gamma l \nu) = (1.8 \sim 4.8) \times 10^{-6}$$
, (38)

from Eq. (26) in  $k_T$  factorization theorem (PQCD), with only the  $O(\bar{\Lambda}^2/m_b^2)$  and  $O(\alpha_s^2)$  uncertainty. The values in Eq. (38) are more or less consistent with other estimates in the literature: a model-dependent evaluation of the structure-dependent photon emission contribution gave the branching ratio  $10^{-7} \sim 10^{-6}$  [14]. Using the B meson



FIG. 5: Spectra in units of  $\text{GeV}^{-1}$  for  $\lambda = 0.1$  and  $\lambda = 0.15$  with the solid lines corresponding to the LP contribution only, the dashed-dotted lines to the inclusion of long-distance contribution, the dashed lines to the inclusion of the NLO correction, and the dotted lines to the inclusion of both the long-distance and NLO contributions.

bound-state wave function from a Salpeter equation,  $0.9 \times 10^{-6}$  has been obtained [17, 21]. Both a simple non-relativistic model and light-front QCD led to  $3.5 \times 10^{-6}$  [18, 19]. Light-cone sum rules and the pole-model calculation gave  $2 \times 10^{-6}$  [16] and  $2.26 \times 10^{-6}$  [26], respectively. At last, the experimental upper bound at 90% confidence level was [50]

$$B(B \to \gamma l\nu) < 2.0 \times 10^{-6} . \tag{39}$$

# IV. CONCLUSION

In this paper we have studied the  $B \to \gamma l\nu$  decay in the PQCD approach based on  $k_T$  factorization theorem. This formalism is well-defined at subleading level, since the two-parton LP *B* meson wave functions remain normalizable even after including the evolution effect. Note that the QCDF approach based on collinear factorization theorem fails at NLP. We have shown that the  $O(1/m_b)$  corrections from the heavy-quark expansion can be absorbed into the LP *B* meson wave functions redefine by the nonlocal matrix element in Eq. (14). The NLP contributions from the hard kernels to the decay spectrum cancel. The three-parton *B* meson wave functions turn out to be suppressed by  $1/m_b^2$  in this special mode. The constructive long-distance contribution almost cancels the destructive NLO radiative correction for both the form factors  $F_V$  and  $F_A$ . The *B* meson wave function  $\phi_+$  can then be extracted from the observed  $B \to \gamma l\nu$  decay spectrum using the leading formalism, which suffers only the  $O(1/m_b^2)$  and  $O(\alpha_s^2)$  uncertainly. We conclude that the  $B \to \gamma l\nu$  decay is the cleanest mode for determining this important nonperturbative input for the perturbation theories of exclusive B meson decays. The determination can be refined by including the evolution and resummation effects into the factorization formulas [20, 22, 23, 24, 28].

Measuring the  $B \to \gamma l \nu$  spectrum in the lepton and photon energies [20],

$$\frac{d^2\Gamma}{d\eta dy} = \frac{\alpha G_F^2 |V_{ub}|^2 m_B^3}{64\pi^2} (1-\eta) \left\{ [F_V^2(q^2) + F_A^2(q^2)] [2(1-y)(1-y-\eta) + \eta^2] -2F_V(q^2)F_A(q^2)\eta(2-2y-\eta) \right\},$$
(40)

with the lepton energy fraction  $y = 2E_l/m_B$ ,  $1 - \eta \le y \le 1$ , we can extract the information of the form factors  $F_V$ and  $F_A$  separately. It is then possible to fix the two two-parton B meson wave functions  $\phi_{\pm}$  simultaneously from Eq. (25). At this NLP level, the three-parton wave functions are still absent following the reasoning in Sec. II.B. The long-distance contribution and the NLO corrections also cancel each other as indicated in Eq. (28). With the  $B \to \gamma l \nu$  branching ratio around  $10^{-6}$ , the above experimental determination is possible.

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## APPENDIX A: THREE-PARTON CONTRIBUTION

We start with Eq. (1.3) in Ref. [51]:

$$G^{(1)}(z) = \int d^4 w \frac{i(\not z - \not w)}{2\pi^2 (z - w)^4} ig \ \mathcal{A}(w) \frac{i \ \not w}{2\pi^2 w^4} , \qquad (A1)$$

which describes the interaction of a quark with a gluon. In momentum space the above expression becomes

where  $l(k_2)$  is the momentum carried by the incoming quark (gluon). The Feynman parametrization gives

$$G^{(1)}(z) = -\int du \int \frac{d^4l}{(2\pi)^4} e^{il \cdot z} \int \frac{d^4k_2}{(2\pi)^4} e^{iuk_2 \cdot z} \frac{(\not\!\!\!/ + u \not\!\!\!/ k_2)\gamma^{\alpha}(\not\!\!/ - \bar{u} \not\!\!\!/ k_2)}{(l^2)^2} ig\tilde{A}_{\alpha}(k_2) , \qquad (A3)$$

where the variable change  $l + \bar{u}k_2 \rightarrow l$ ,  $\bar{u} \equiv 1 - u$ , has been applied.

In the case we are considering, the gluon momentum  $k_2$  is of  $O(\overline{\Lambda})$ , since the *B* meson is dominated by soft dynamics. We expand the above expression up to  $O(k_2)$ :

$$G^{(1)}(x) = -\int du \int \frac{d^4l}{(2\pi)^4} e^{il \cdot z} \int \frac{d^4k_2}{(2\pi)^4} e^{iuk_2 \cdot z} \left\{ \frac{l\!\!/\gamma^{\alpha} l\!\!/}{(l^2)^2} + \frac{u k_2 \gamma^{\alpha} l\!\!/}{(l^2)^2} - \frac{\bar{u} l\!\!/\gamma^{\alpha} k_2}{(l^2)^2} \right\} ig\tilde{A}_{\alpha}(k_2) ,$$
  
$$= -\int du \int \frac{d^4l}{(2\pi)^4} e^{il \cdot z} \left\{ \frac{l\!\!/\gamma^{\alpha} l\!\!/}{(l^2)^2} igA_{\alpha}(uz) + \frac{u n\!\!/+\gamma^{\alpha} l\!\!/-\bar{u} l\!\!/\gamma^{\alpha} n\!\!/}{(l^2)^2} ig\partial_{\beta}A_{\alpha}(uz) n_{-}^{\beta} \right\} .$$
(A4)

The first term on the right-hand side of Eq. (A4), contributing to a phase factor [51], will be dropped. For convenience, we work in the light-cone gauge  $A^+ = 0$ , in which the second and third terms are rewritten as

$$G^{(1)}(z) = i \int dug G_{\alpha\beta}(uz) n_{-}^{\beta} \int \frac{d^4l}{(2\pi)^4} e^{il \cdot z} \frac{u \not n_{+} \gamma^{\alpha} \not l - \bar{u} \not l \gamma^{\alpha} \not n_{+}}{(l^2)^2} .$$
(A5)

It is clear that the field strength  $gG_{\alpha\beta}(uz)n_{-}^{\beta}$  can be factored together with the rescaled *b* quark field *h* and the light quark field  $\bar{u}$  into the nonlocal matrix element in Eq. (15). The integrand depending on *l* is then identified as the hard kernel in momentum space for the three-parton contribution. Employing Eq. (A5) for Fig. 2, and substituting  $P_2 - k_1 - uk_2$  for *l*, we obtain Eq. (16).

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