

Upper Bounds on Rare K and B Decays from Minimal Flavour Violation

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Abstract

We study the branching ratios of rare K and B decays in models with minimal flavour violation, using the presently available information from the universal unitarity triangle analysis and from the measurements of $Br(B \rightarrow X_s \gamma)$, $Br(B \rightarrow X_s l^+ l^-)$ and $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. We find the following upper bounds: $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) < 11.9 \times 10^{-11}$, $Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 4.6 \times 10^{-11}$, $Br(K_L \rightarrow \mu^+ \mu^-)_{SD} < 1.4 \times 10^{-9}$, $Br(B \rightarrow X_s \nu \bar{\nu}) < 5.2 \times 10^{-5}$, $Br(B \rightarrow X_d \nu \bar{\nu}) < 2.2 \times 10^{-6}$, $Br(B_s \rightarrow \mu^+ \mu^-) < 7.4 \times 10^{-9}$, $Br(B_d \rightarrow \mu^+ \mu^-) < 2.2 \times 10^{-10}$ at 95% probability. We analyze in detail various possible scenarios with positive or negative interference of Standard Model and New Physics contributions, and show how an improvement of experimental data corresponding to the projected 2010 B factory integrated luminosities will allow to disentangle and test these different possibilities. Finally, anticipating that subsequently the leading role in constraining this kind of new physics will be taken over by the rare decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $B_{s,d} \rightarrow \mu^+ \mu^-$, that are dominated by the Z^0 -penguin function C , we also present plots for several branching ratios as functions of C . We point out an interesting triple correlation between $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $B \rightarrow X_s \gamma$ and $B \rightarrow X_s l^+ l^-$ present in MFV models.

1 Introduction

Recently, great experimental progress has been made in the study of Flavour Changing Neutral Current (FCNC) decays, leading not only to an impressive accuracy in the extraction of CKM parameters from the Unitarity Triangle (UT) analysis [1, 2], but also to stringent constraints on models with extra sources of flavour and CP violation, although an accidental agreement of the UT analysis with the Standard Model (SM) cannot yet be excluded [3, 4].

One is then naturally led to consider models with Minimal Flavour Violation (MFV) [5], in which flavour and CP violation is governed entirely by the CKM matrix [6, 7] and the relevant operators in effective Hamiltonians for weak decays are the same as in the SM.

As pointed out in [5], there exists a universal unitarity triangle (UUT) valid in all these models, that can be constructed independently of the parameters specific to a given model. Moreover, there exist several relations between various branching ratios that allow straightforward tests of these models. A review has been given in [8].

This formulation of MFV agrees with the one of [9, 10] except for the case of models with two Higgs doublets at large $\tan\beta$, where also additional operators, strongly suppressed in the SM, can contribute significantly and the relations in question are not necessarily satisfied. In the present paper, MFV will be defined as in [5, 8].

As reviewed in [8], this class of models can be formulated to a very good approximation in terms of eleven parameters: four parameters of the CKM matrix and seven values of the universal master functions $F_i(v)$ that parametrize the short distance contributions to rare decays with v denoting symbolically the parameters of a given MFV model. However, as argued in [8], the new physics contributions to the functions

$$S(v), \quad C(v), \quad D'(v), \tag{1.1}$$

representing respectively $\Delta F = 2$ box diagrams, Z^0 -penguin diagrams and the magnetic photon penguin diagrams, are the most relevant ones for phenomenology, with the remaining functions producing only minor deviations from the SM in low-energy processes. Several explicit calculations within models with MFV confirm this conjecture. We have checked the impact of these additional functions on our analysis, and we will comment on it in Section 3.

Now, the existence of a UUT implies that the four CKM parameters can be determined independently of the values of the functions in (1.1). Moreover, only $C(v)$ and $D'(v)$ enter the branching ratios for radiative and rare decays so that constraining their values by (at least) two specific branching ratios allows to obtain straightforwardly the ranges for all branching ratios within the class of MFV models. Analyses of that type can be found in [8, 9, 11].¹

The unique decay to determine the function $D'(v)$ is $B \rightarrow X_s \gamma$, whereas a number of decays such as $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $B_{s,d} \rightarrow \mu^+ \mu^-$, $B \rightarrow X_{s,d} \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 l^+ l^-$ can be used

¹An alternative approach is to extract from rare decays the relevant Wilson coefficients [12, 13, 14]. However, since in MFV models these coefficients have nontrivial correlations among themselves, we find it more transparent to express the physical quantities in terms of the functions in eq. (1.1).

to determine $C(v)$. The decays $B \rightarrow X_{s,d}l^+l^-$ depend on both $C(v)$ and $D'(v)$ and can be used together with $B \rightarrow X_s\gamma$ and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ to determine $C(v)$.

Eventually the decays $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$, being the theoretically cleanest ones [15, 16], will be used to determine $C(v)$. However, so far only three events of $K^+ \rightarrow \pi^+\nu\bar{\nu}$ have been observed [17, 18, 19] and no event of $K_L \rightarrow \pi^0\nu\bar{\nu}$, with the same comment applying to $B_{s,d} \rightarrow \mu^+\mu^-$, $B \rightarrow X_{s,d}\nu\bar{\nu}$ and $K_L \rightarrow \pi^0l^+l^-$. On the other hand the branching ratio for $B \rightarrow X_s\gamma$ has been known for some time and the branching ratio for $B \rightarrow X_sl^+l^-$ has been recently measured by Belle [20] and BaBar [21] collaborations. The latter, combined with $K^+ \rightarrow \pi^+\nu\bar{\nu}$, provide presently the best estimate of the range for $C(v)$ within MFV models.

The main goals of the present paper are

- to calculate various branching ratios as functions of $C(v)$ within MFV models,
- to determine the allowed range for $C(v)$ from presently available data,
- to find the upper bounds for the branching ratios of $K^+ \rightarrow \pi^+\nu\bar{\nu}$, $K_L \rightarrow \pi^0\nu\bar{\nu}$, $B_{s,d} \rightarrow \mu^+\mu^-$, $B \rightarrow X_{s,d}\nu\bar{\nu}$ and $K_L \rightarrow \pi^0l^+l^-$ within MFV models as defined here,
- to assess the impact of future measurements on MFV models.

Our paper is organized as follows. Section 2 can be considered as a guide to the literature, where the formulae for the branching ratios in question can be found. In this Section we also give the list of the input parameters. In Section 3 we present our numerical analysis of various branching ratios as functions of $C(v)$ and their expectation values and upper bounds. A brief summary of our results is given in Section 4.

2 Basic Formulae

In the MFV models considered here there are no new complex phases and flavour changing transitions are governed by the CKM matrix. Moreover, the only relevant operators are those already present in the SM. Consequently, new physics enters only through the Wilson coefficients of the SM operators that can receive additional contributions due to the exchange of new virtual particles beyond the SM ones.

Any weak decay amplitude can be then cast in the simple form

$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i F_i(v), \quad F_i(v) = F_{\text{SM}}^i + F_{\text{New}}^i \quad (\text{real}), \quad (2.1)$$

where $F_i(v)$ are the *master functions* of MFV models [8]

$$S(v), X(v), Y(v), Z(v), E(v), D'(v), E'(v) \quad (2.2)$$

with v denoting collectively the parameters of a given MFV model. Examples of models in this class are the Two Higgs Doublet Model II and the Minimal Supersymmetric Standard Model (MSSM) without new sources of flavour violation and for small or moderate $\tan\beta$. Also models

with one universal extra dimension [22, 23] and the simplest little Higgs models are of MFV type [24].

In order to find the functions $F_i(v)$ in (2.2), one first looks at various functions resulting from penguin diagrams: C (Z^0 penguin), D (γ penguin), E (gluon penguin), D' (γ -magnetic penguin) and E' (chromomagnetic penguin). Subsequently box diagrams have to be considered. Here we have the box function S ($\Delta F = 2$ transitions), as well as the $\Delta F = 1$ box functions $B^{\nu\bar{\nu}}$ and $B^{\bar{l}l}$ relevant for decays with $\nu\bar{\nu}$ and $\bar{l}l$ in the final state, respectively.

While the $\Delta F = 2$ box function S and the penguin functions E , D' and E' are gauge independent, this is not the case for C , D and the $\Delta F = 1$ box diagram functions $B^{\nu\bar{\nu}}$ and $B^{\bar{l}l}$. In phenomenological applications it is more convenient to work with gauge independent functions [25]

$$X(v) = C(v) + B^{\nu\bar{\nu}}(v), \quad Y(v) = C(v) + B^{\bar{l}l}(v), \quad Z(v) = C(v) + \frac{1}{4}D(v). \quad (2.3)$$

We have the following correspondence between the most interesting FCNC processes and the master functions in the MFV models [8, 26]:

$K^0 - \bar{K}^0$ -mixing (ε_K)	$S(v)$
$B_{d,s}^0 - \bar{B}_{d,s}^0$ -mixing ($\Delta M_{s,d}$)	$S(v)$
$K \rightarrow \pi\nu\bar{\nu}$, $B \rightarrow X_{d,s}\nu\bar{\nu}$	$X(v)$
$K_L \rightarrow \mu\bar{\mu}$, $B_{d,s} \rightarrow \bar{l}l$	$Y(v)$
$K_L \rightarrow \pi^0 l^+ l^-$	$Y(v), Z(v), E(v)$
ε' , $\Delta S = 1$	$X(v), Y(v), Z(v), E(v)$
Nonleptonic $\Delta B = 1$	$X(v), Y(v), Z(v), E(v), E'(v)$
$B \rightarrow X_s \gamma$	$D'(v), E'(v)$
$B \rightarrow X_s$ gluon	$E'(v)$
$B \rightarrow X_s l^+ l^-$	$Y(v), Z(v), E(v), D'(v), E'(v)$

This table means that the observables like branching ratios, mass differences $\Delta M_{d,s}$ in $B_{d,s}^0 - \bar{B}_{d,s}^0$ -mixing and the CP violation parameters ε and ε' , can all be to a very good approximation expressed in terms of the corresponding master functions and the relevant CKM factors. The remaining entries in the formulae for these observables are low-energy quantities such as the parameters B_i , that can be calculated within the SM and the QCD factors η_{QCD}^i describing the renormalization group evolution of operators for scales $\mu \leq M_W$. These factors being universal can be calculated, similarly to B_i , in the SM. The remaining, model-specific QCD corrections can be absorbed in the functions F_i .

The formulae for the processes listed above in the SM, given in terms of the master functions and CKM factors can be found in many papers. The full list using the same notation is given in [27]. An update of these formulae with additional references is given in two papers on universal extra dimensions [22, 23], where one has to replace $F_i(v, 1/R)$ by $F_i(v)$ to obtain the formulae in a general MFV model. In what follows we will use the formulae of [22, 23] except that:

- We will set the functions

$$B^{\nu\bar{\nu}}(v), \quad B^{\bar{l}l}(v), \quad E(v) \quad (2.4)$$

to their SM values and we will trade the functions $D'(v)$ and $E'(v)$ for the low-energy coefficient $C_7^{\text{eff}}(\mu_b)$ which enters both $b \rightarrow s\gamma$ and $b \rightarrow sl^+l^-$. In this manner the only free variables are the functions listed in (1.1), plus the $D(v)$ function. As remarked below, this latter function has only a minor impact on our analysis. We have also explored the possible impact of NP contributions to $B^{\nu\bar{\nu}}(v)$ and $B^{\bar{l}l}(v)$, as will be discussed at the end of Section 3.

- In obtaining $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$ we have included the recently calculated long distance contributions [28] that enhance the branching ratio by roughly 6%. This amounts effectively to a charm parameter of $P_c = 0.43 \pm 0.07$.
- We will use the formula for $(K_L \rightarrow \pi^0 l^+ l^-)_{\text{CPV}}$ from [29, 30].
- We will use the complete NLO formulae for $B \rightarrow X_s \gamma$ from [31].
- We will use the complete NNLO formulae for $B \rightarrow X_s l^+ l^-$ from [32, 13].

Branching Ratios	Formula	Reference	Parameters
$Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$	(4.24)	[22]	$Br(K^+ \rightarrow \pi^0 e^+\nu), m_c$
$Br(K_L \rightarrow \pi^0\nu\bar{\nu})$	(4.27)	[22]	$Br(K^+ \rightarrow \pi^0 e^+\nu)$
$Br(K_L \rightarrow \mu^+\mu^-)_{\text{SD}}$	(4.32)	[22]	m_c
$Br(K_L \rightarrow \pi^0 l^+ l^-)_{\text{CPV}}$	(43)	[30]	see [30]
$Br(B \rightarrow X_s \nu\bar{\nu})$	(4.29)	[22]	$Br(B \rightarrow X_c l\bar{\nu})$
$Br(B \rightarrow X_d \nu\bar{\nu})$	(4.29)	[22]	$Br(B \rightarrow X_c l\bar{\nu})$
$Br(B_s \rightarrow \mu^+\mu^-)$	(4.30)	[22]	F_{B_s}
$Br(B_d \rightarrow \mu^+\mu^-)$	(4.30)	[22]	F_{B_d}

Table 1: *Guide to the formulae. See text for explanations. The dependence of all branching ratios on CKM parameters and the top quark mass is not explicitly reported.*

In Table 1 we indicate where the formulae in question can be found and which additional input parameters are involved in them. In Table 2 we give the numerical values of all the parameters involved in the analysis.

Finally, for the reader's convenience, and in order to show the relative importance of NP contributions to the processes we consider, we report below numerical formulae for the branching ratios in terms of F_{New}^i in eq. (2.1). These numerical expressions have been obtained for central values of the parameters in Table 2, as functions of $\Delta C \equiv C(v) - C_{\text{SM}}$, $\Delta C_7^{\text{eff}} \equiv C_7^{\text{eff}} - C_{7\text{SM}}^{\text{eff}}$, $\Delta D \equiv D(v) - D_{\text{SM}}$, $\Delta B^{\bar{l}l} \equiv B^{\bar{l}l}(v) - B_{\text{SM}}^{\bar{l}l}$ and $\Delta B^{\nu\bar{\nu}} \equiv B^{\nu\bar{\nu}}(v) - B_{\text{SM}}^{\nu\bar{\nu}}$. With the aid of eq. (2.5), it is possible to quickly check the impact of NP contributions in any given MFV model. As

Parameter	Value	Gaussian (σ)
λ	0.2255	0.0014
$ V_{cb} $	0.0415	0.0007
$\bar{\rho}$	0.191	0.046
$\bar{\eta}$	0.353	0.028
F_{B_s}	230 MeV	30 MeV
F_{B_d}	189 MeV	27 MeV
$Br(B \rightarrow X_c l \bar{\nu})$	0.1045	0.0021
$Br(K^+ \rightarrow \pi^0 e^+ \nu)$	0.0487	0.0006
m_t^{pole}	178.0 GeV	4.3 GeV
\bar{m}_b	4.21 GeV	0.08 GeV
\bar{m}_c	1.3 GeV	0.1 GeV
$\alpha_s(M_Z)$	0.119	0.003

Table 2: Values of the relevant parameters used in the analysis.

a first insight, we see that the dependence of $Br(B \rightarrow X_s l^+ l^-)$ on ΔD is relatively weak, as can be read off from the small prefactors in the formulae below. From eq. (2.5) one can also check whether the NP contribution to box diagrams in any given model is large enough as to modify significantly our results obtained for $\Delta B^{\bar{l}} = \Delta B^{\nu\bar{\nu}} = 0$ in the next Section. Finally, these formulae allow to understand the structure of our numerical results. We have ²:

$$\begin{aligned}
Br(B \rightarrow X_s l^+ l^-, 0.04 < q^2(\text{GeV}) < 1) &= 1.16 \cdot 10^{-6} \left(1 + 0.38 (\Delta B^{\bar{l}})^2 + 0.46 \Delta C_7^{\text{eff}} \Delta B^{\bar{l}} \right. \\
&\quad + 0.41 \Delta C \Delta B^{\bar{l}} - 3.47 \Delta C_7^{\text{eff}} + 0.56 \Delta B^{\bar{l}} + 4.31 (\Delta C_7^{\text{eff}})^2 \\
&\quad \left. + 0.19 (\Delta C)^2 + 0.38 \Delta C - 0.11 \Delta C_7^{\text{eff}} \Delta D \right),
\end{aligned}$$

$$\begin{aligned}
Br(B \rightarrow X_s l^+ l^-, 1 < q^2(\text{GeV}) < 6) &= 1.61 \cdot 10^{-6} \left(1 + 1.33 (\Delta B^{\bar{l}})^2 + 1.26 \Delta C_7^{\text{eff}} \Delta B^{\bar{l}} \right. \\
&\quad + 1.43 \Delta C \Delta B^{\bar{l}} - 0.31 \Delta D \Delta B^{\bar{l}} + 2.08 \Delta B^{\bar{l}} + 1.42 (\Delta C_7^{\text{eff}})^2 \\
&\quad \left. + 0.67 (\Delta C)^2 + 1.36 \Delta C - 0.29 \Delta C_7^{\text{eff}} \Delta D - 0.18 \Delta D \right),
\end{aligned}$$

$$\begin{aligned}
Br(B \rightarrow X_s l^+ l^-, 14.4 < q^2(\text{GeV}) < 25) &= 3.70 \cdot 10^{-7} \left(1 + 1.18 (\Delta B^{\bar{l}})^2 + 0.70 \Delta C_7^{\text{eff}} \Delta B^{\bar{l}} + 0.60 \Delta C_7^{\text{eff}} \right. \\
&\quad + 1.27 \Delta C \Delta B^{\bar{l}} - 0.27 \Delta D \Delta B^{\bar{l}} + 2.18 \Delta B^{\bar{l}} + 0.21 (\Delta C_7^{\text{eff}})^2 \\
&\quad \left. + 0.60 (\Delta C)^2 + 1.24 \Delta C - 0.16 \Delta C_7^{\text{eff}} \Delta D - 0.24 \Delta D \right),
\end{aligned}$$

$$Br(B_d \rightarrow \mu^+ \mu^-) = 1.08 \cdot 10^{-10} \left(1 + \Delta B^{\bar{l}} + \Delta C \right)^2,$$

$$Br(B_s \rightarrow \mu^+ \mu^-) = 3.76 \cdot 10^{-9} \left(1 + \Delta B^{\bar{l}} + \Delta C \right)^2,$$

²Notice that we have discarded terms with coefficients smaller than 0.1 in $Br(B \rightarrow X_s l^+ l^-)$.

$$\begin{aligned}
Br(B \rightarrow X_d \nu \bar{\nu}) &= 1.50 \cdot 10^{-6} (1 + 0.65 (\Delta C + \Delta B^{\nu \bar{\nu}}))^2, \\
Br(B \rightarrow X_s \nu \bar{\nu}) &= 3.67 \cdot 10^{-5} (1 + 0.65 (\Delta C + \Delta B^{\nu \bar{\nu}}))^2, \\
Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) &= 8.30 \cdot 10^{-11} \left(1 + 0.20 (\Delta C + \Delta B^{\nu \bar{\nu}})^2 + 0.89 (\Delta C + \Delta B^{\nu \bar{\nu}}) \right), \\
Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) &= 3.10 \cdot 10^{-11} (1 + 0.65 (\Delta C + \Delta B^{\nu \bar{\nu}}))^2, \\
Br(K_L \rightarrow \mu^+ \mu^-) &= 8.58 \cdot 10^{-10} (1 + 0.82 (\Delta C + \Delta B^{\bar{l} l}))^2.
\end{aligned} \tag{2.5}$$

3 Numerical Analysis

Our numerical analysis consists of three steps:

1. Extracting CKM parameters using the UUT analysis;
2. Determining the allowed range for ΔC and ΔC_7^{eff} from presently available data;
3. Computing the expectation values of rare decays based on these allowed ranges.

For the first step, we use the very recent results of the **Ufit** collaboration on the UUT analysis [4]:

$$\bar{\rho} = 0.191 \pm 0.046, \quad \bar{\eta} = 0.353 \pm 0.028. \tag{3.1}$$

Since the UUT analysis is independent of loop functions, the above results are in particular independent of the top quark mass.

In the second step, to minimize the theoretical input, we have traded $D'(v)$ and $E'(v)$ for C_7^{eff} , which is the relevant low-energy quantity entering $Br(B \rightarrow X_s \gamma)$ and $Br(B \rightarrow X_s l^+ l^-)$. Concerning $Br(B \rightarrow X_s \gamma)$, we compare the theoretical value with the experimental results of CLEO [33], Belle [34] and BaBar [35] in the corresponding kinematic ranges, adding a conservative 10% flat theoretical error to the theoretical prediction. This error contains both the uncertainties due to the cutoff in the photon spectrum [36] and the ones related to higher order effects, which are particularly large since we are omitting here model-specific NLO terms for the NP contribution. For $Br(B \rightarrow X_s l^+ l^-)$, we use the experimental data in the q^2 regions $0.04 < q^2(\text{GeV}) < 1$, $1 < q^2(\text{GeV}) < 6$ and $14.4 < q^2(\text{GeV}) < 25$ to avoid the theoretical uncertainty due to the presence of $c\bar{c}$ resonances.

The second and third steps are carried out using the approach of ref. [40]: taking $C(v)$, $C_7^{\text{eff}}(\mu_b)$ and $D(v)$ to have a flat *a-priori* distribution and using the available experimental data and theoretical inputs, we determine the *a-posteriori* probability density function (p.d.f.) for $C(v)$, $C_7^{\text{eff}}(\mu_b)$ and all the rare decays listed in Table 3. Concerning $D(v)$, it plays only a marginal role in these decays and therefore it is not well determined by the analysis. We varied ΔD in the conservative range $\pm 4D_{\text{SM}}$. Even this rather large variation has little impact on the extraction of the allowed range for $C(v)$.

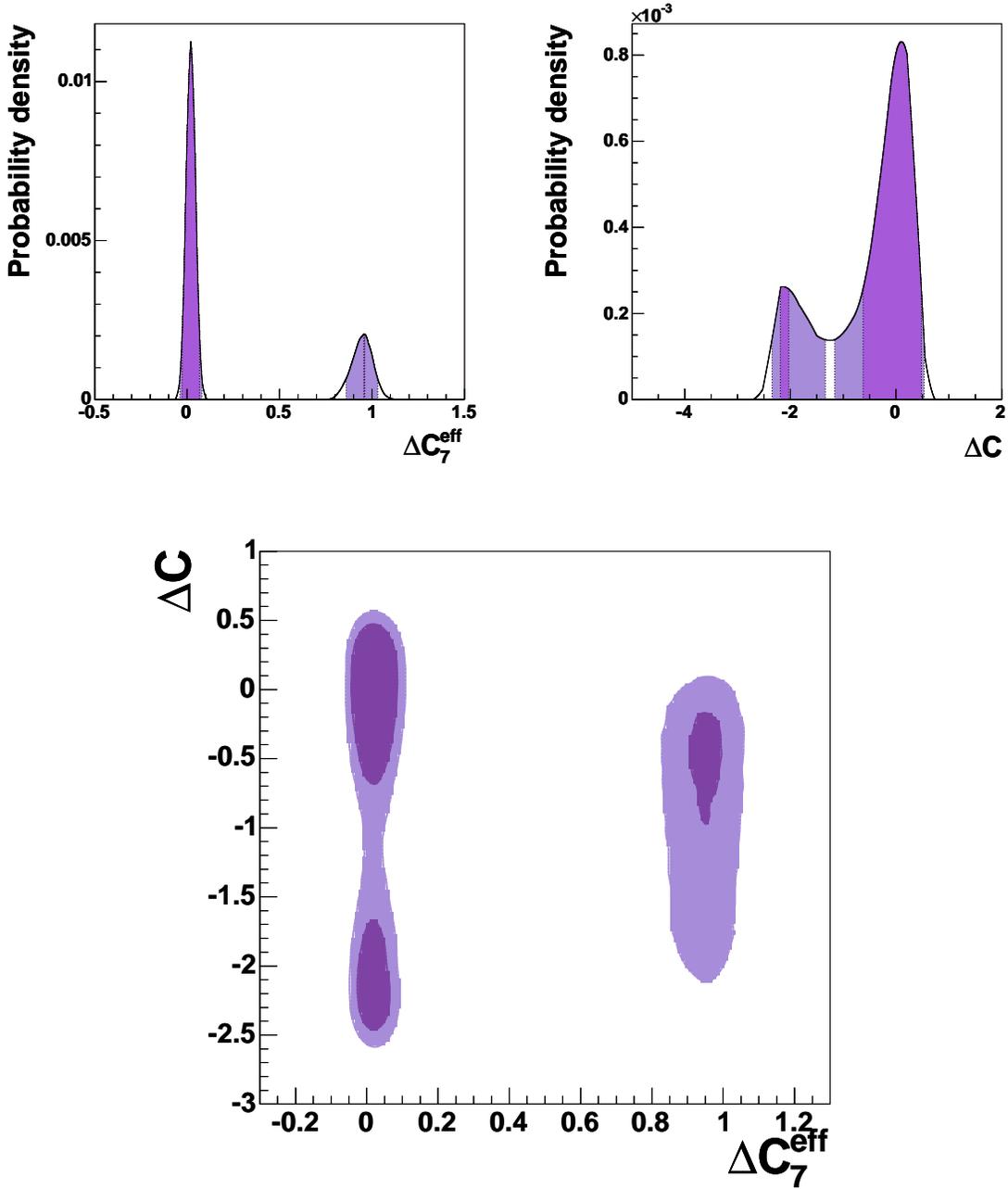


Figure 1: *P.d.f.* 's for ΔC_7^{eff} (top-left), ΔC (top-right) and ΔC vs. ΔC_7^{eff} (bottom). Dark (light) areas correspond to the 68% (95%) probability region.

In Figure 1 we plot the p.d.f. for $\Delta C(v)$ and ΔC_7^{eff} , that represent F_{New}^i in (2.1) and enter eq. (2.5). In Figure 2 we plot the p.d.f. for the branching ratios. The corresponding upper bounds at 95% probability are reported in Table 3, where, for comparison, we also report the results obtained within the SM, using the same CKM parameters obtained from the UUT

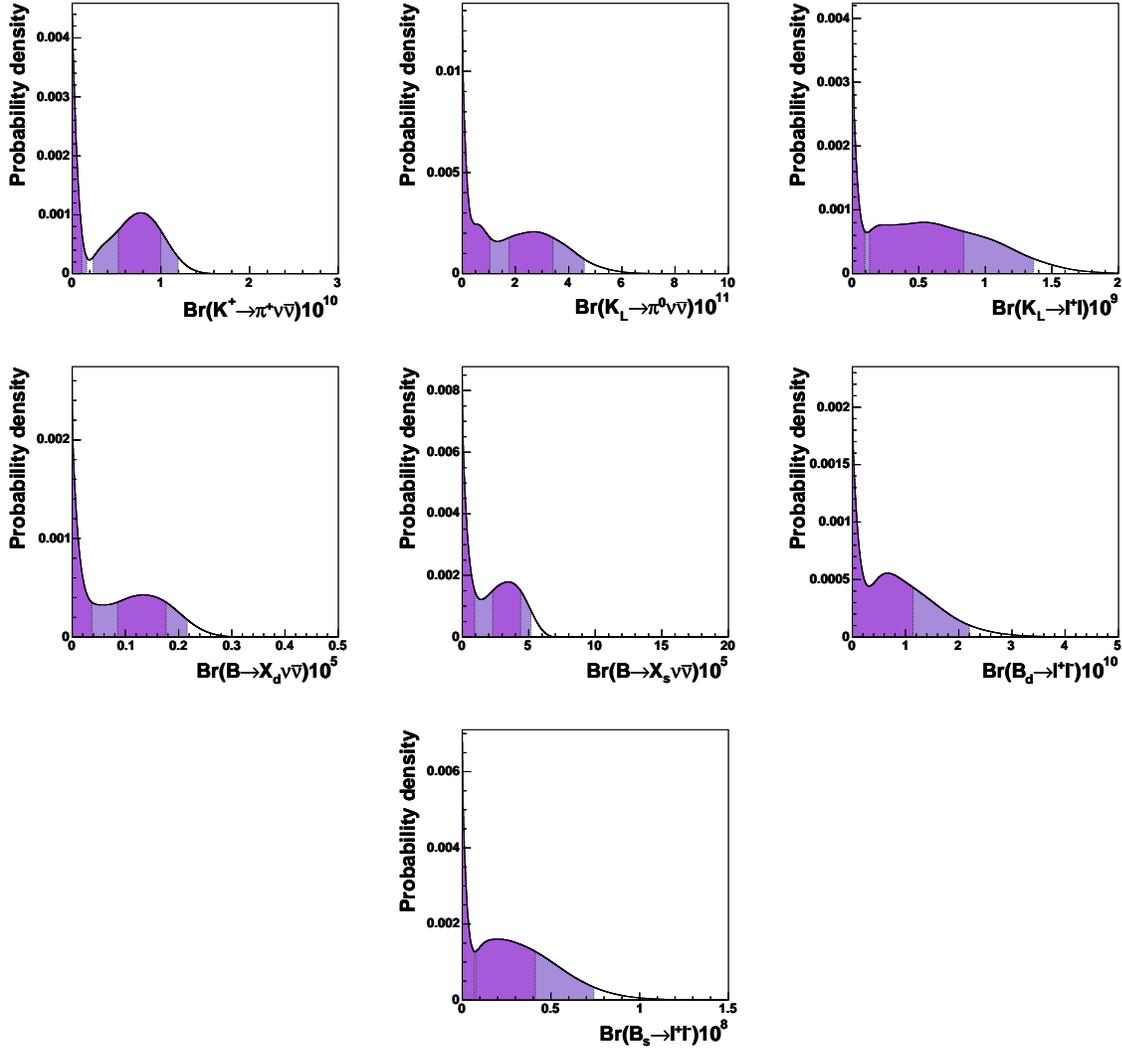


Figure 2: *P.d.f.*'s for the branching ratios of the rare decays $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$, $Br(K_L \rightarrow \mu^+ \mu^-)_{SD}$, $Br(B \rightarrow X_{d,s} \nu \bar{\nu})$, and $Br(B_{d,s} \rightarrow \mu^+ \mu^-)$. Dark (light) areas correspond to the 68% (95%) probability region.

analysis. Finally, in Figures 3 and 4 we plot the branching ratios of the rare decays vs. $C(v)$, to make the impact of future measurements on the determination of $C(v)$ more transparent.

Let us now comment on our results. As can be seen from Figure 1, we have two possible solutions for ΔC_7^{eff} , one very close to the SM, and the other corresponding to reversing the sign of $C_7^{\text{eff}}(\mu_b)$ (recall that $C_7^{\text{eff}}(\mu_b)$ is negative in the SM and equal to $C_7^{\text{eff}}(\mu_b) \approx -0.33$). The second solution is disfavoured: it is barely accessible at 68% probability, in accordance with the results of [14]. This result is easy to understand. In the case of the second solution for ΔC_7^{eff} , the branching ratio $Br(B \rightarrow X_s l^+ l^-)$ becomes larger than the experimental value. The

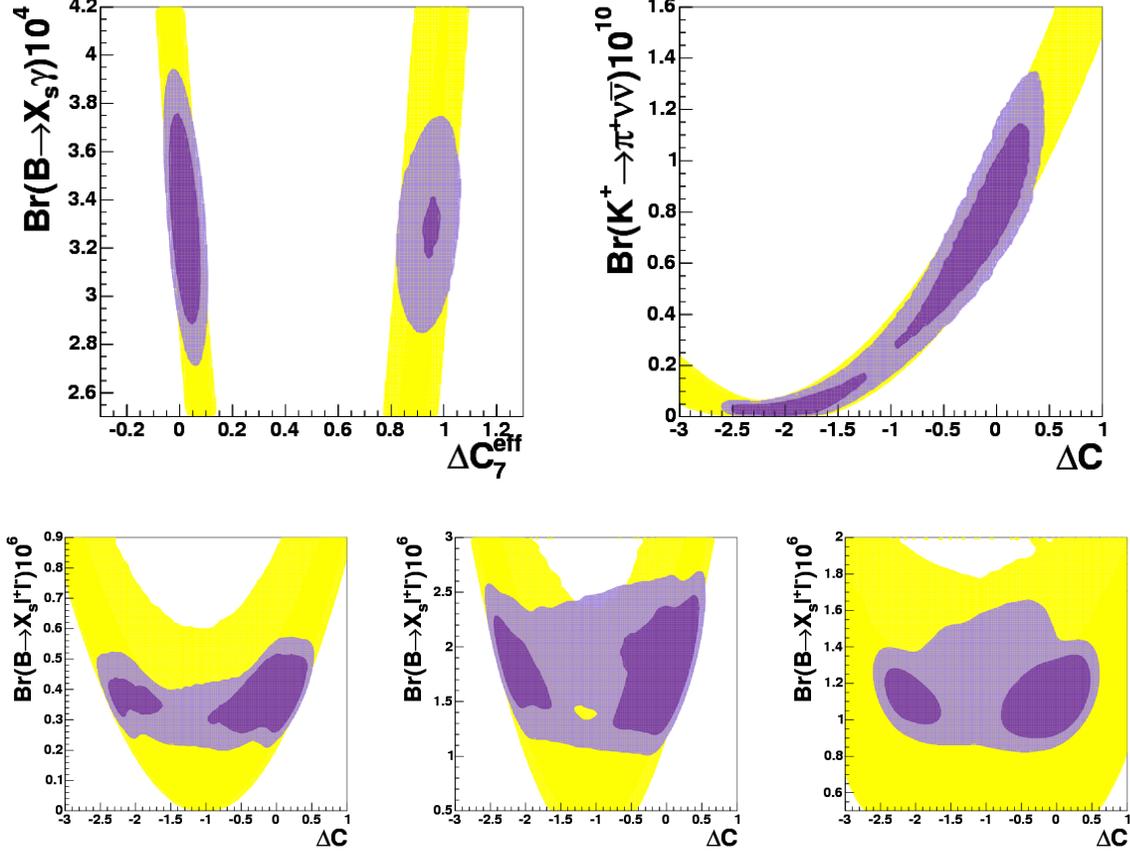


Figure 3: *P.d.f.'s* for the branching ratios of the rare decays used to constrain ΔC and ΔC_7^{eff} as a function of these parameters: $Br(B \rightarrow X_s \gamma)_{E_\gamma > 1.8 \text{ GeV}}$ (top-left), $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ (top-right), $Br(B \rightarrow X_s l^+ l^-)_{14.4 < q^2 (\text{GeV}) < 25}$ (bottom-left), $Br(B \rightarrow X_s l^+ l^-)_{1 < q^2 (\text{GeV}) < 6}$ (bottom-center), $Br(B \rightarrow X_s l^+ l^-)_{0.04 < q^2 (\text{GeV}) < 1}$ (bottom-right). Dark (light) areas correspond to the 68% (95%) probability region. Very light areas correspond to the range obtained without using the experimental information.

full results are:

$$\begin{aligned} \Delta C_7^{\text{eff}} &= (0.02 \pm 0.047) \cup (0.958 \pm 0.002) \text{ at } 68\% \text{ probability,} \\ \Delta C_7^{\text{eff}} &= [-0.039, 0.08] \cup [0.859, 1.031] \text{ at } 95\% \text{ probability.} \end{aligned} \quad (3.2)$$

Since we have two separate ranges for ΔC_7^{eff} , in the following we will also present separately the results corresponding to the “LOW” or “HI” solution for ΔC_7^{eff} (see Figures 5 and 6).

As can be seen in Figure 1 we have two solutions for ΔC , one close to the SM and the other corresponding to reversing the sign of C . We recall that $C \approx 0.81$ in the SM. The ranges obtained are

$$\Delta C = (-0.16 \pm 0.53) \cup (-2.15 \pm 0.08) \text{ at } 68\% \text{ probability,}$$

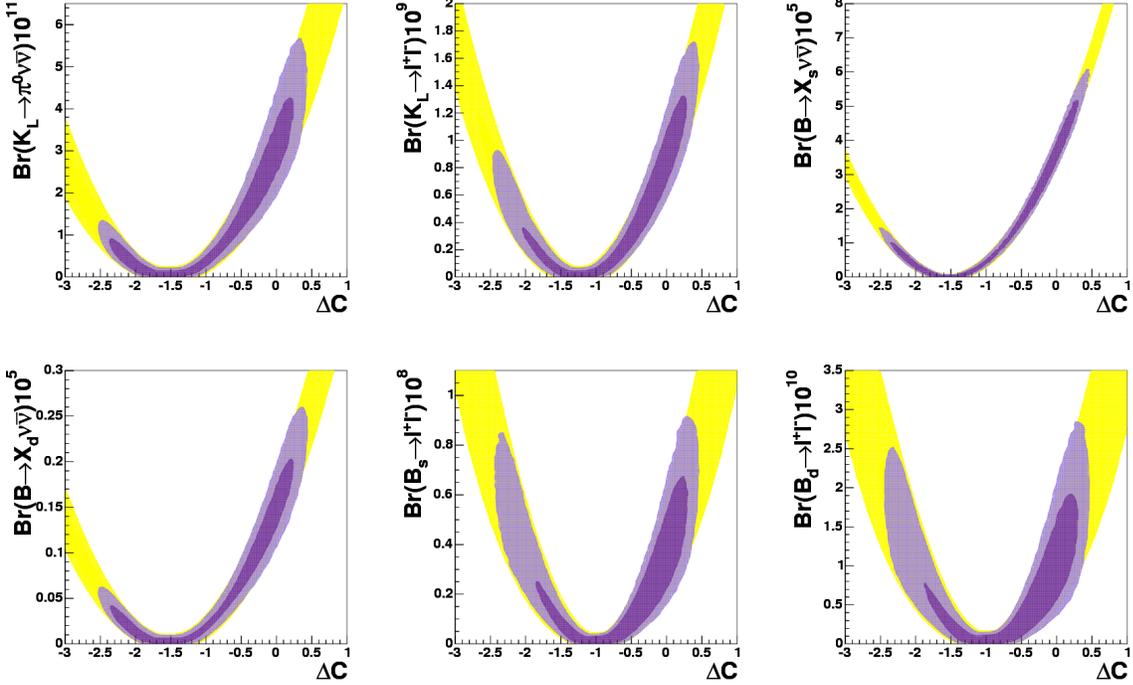


Figure 4: *P.d.f.'s for the branching ratios of the rare decays $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$, $Br(K_L \rightarrow \mu^+ \mu^-)_{SD}$, $Br(B \rightarrow X_{d,s} \nu \bar{\nu})$, and $Br(B_{d,s} \rightarrow \mu^+ \mu^-)$ as a function of ΔC . Dark (light) areas correspond to the 68% (95%) probability region. Very light areas correspond to the range obtained without using the experimental information.*

$$\Delta C = [-1.25, 0.44] \cup [-2.39, -1.45] \text{ at 95\% probability.} \quad (3.3)$$

From the plot of ΔC vs ΔC_7^{eff} in Figure 1, it is evident that the situation is different for the HI and LOW solutions for ΔC_7^{eff} . Indeed, the two solutions correspond to the following ranges for ΔC :

$$\text{LOW : } \Delta C = (-0.03 \pm 0.41) \cup (-2.18 \pm 0.02) \text{ at 68\% probability,}$$

$$\text{LOW : } \Delta C = [-0.75, 0.50] \cup [-2.49, -1.60] \text{ at 95\% probability,}$$

$$\text{HI : } \Delta C = (-0.68 \pm 0.58) \text{ at 68\% probability,}$$

$$\text{HI : } \Delta C = [-1.98, 0.04] \text{ at 95\% probability.} \quad (3.4)$$

These results are easy to understand. For the LOW solution the solutions with ΔC being positive and negative are consistent with the data on $B \rightarrow X_s l^+ l^-$. On the other hand for the HI solution, $\Delta C < 0$ is favoured as with $\Delta C > 0$ the difficulty with a too high $Br(B \rightarrow X_s l^+ l^-)$ becomes more acute.

For the reader's convenience, we report in Table 4 the values of the X , Y and Z functions obtained by summing SM and NP contributions and by applying all the available experimental

Branching Ratios	MFV (95%)	SM (68%)	SM (95%)	exp
$Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \times 10^{11}$	< 11.9	8.3 ± 1.2	$[6.1, 10.9]$	$(14.7^{+13.0}_{-8.9})$ [19]
$Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) \times 10^{11}$	< 4.59	3.08 ± 0.56	$[2.03, 4.26]$	$< 5.9 \cdot 10^4$ [37]
$Br(K_L \rightarrow \mu^+ \mu^-)_{SD} \times 10^9$	< 1.36	0.87 ± 0.13	$[0.63, 1.15]$	-
$Br(B \rightarrow X_s \nu \bar{\nu}) \times 10^5$	< 5.17	3.66 ± 0.21	$[3.25, 4.09]$	< 64 [38]
$Br(B \rightarrow X_d \nu \bar{\nu}) \times 10^6$	< 2.17	1.50 ± 0.19	$[1.12, 1.91]$	-
$Br(B_s \rightarrow \mu^+ \mu^-) \times 10^9$	< 7.42	3.67 ± 1.01	$[1.91, 5.91]$	$< 2.7 \cdot 10^2$ [39]
$Br(B_d \rightarrow \mu^+ \mu^-) \times 10^{10}$	< 2.20	1.04 ± 0.34	$[0.47, 1.81]$	$< 1.5 \cdot 10^3$ [39]

Table 3: Upper bounds for rare decays in MFV models at 95% probability, the corresponding values in the SM (using inputs from the UUT analysis) and the available experimental information. See the text for details.

Function	MFV (68%)	MFV (95%)
X	$[-0.71, -0.55] \cup [0.86, 1.90]$	$[-0.86, 0.10] \cup [0.30, 1.95]$
Y	$[-1.23, -1.06] \cup [0.33, 1.37]$	$[-1.38, -0.44] \cup [-0.24, 1.43]$
Z	$[-1.51, -1.40] \cup [-0.25, 1.31]$	$[-1.74, -1.05] \cup [-0.92, 1.46]$

Table 4: Values at 68% and 95% probability for the functions X , Y and Z . See the text for details.

constraints.

The impact of $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ on the bounds on NP contributions can be seen by comparing Figure 1 with Figure 7, where $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ was not used as a constraint.³ As can be seen from Figure 7, the role of $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ is to suppress the solution with $\Delta C \sim -2$, which corresponds to destructive interference with the SM in $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and in the other rare decays. In this respect, a further improvement of the experimental error on $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ will be extremely useful in further reducing the importance of this negative-interference solution for ΔC , which is responsible for the peaks around zero for all the rare decays in Figure 2.

We also note that eliminating $\Delta C < 0$ by means of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ would basically also eliminate the HI solution for ΔC_7^{eff} . We therefore conclude that finding $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ larger than the SM value would help in eliminating the positive sign of C_7^{eff} . To our knowledge this triple correlation between $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $B \rightarrow X_s \gamma$ and $B \rightarrow X_s l^+ l^-$ has not been discussed in the literature so far. It is very peculiar to MFV and is generally not present in models with new flavour violating contributions.

The upper bound on $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ in Table 3 has been obtained using the experimental information on this decay. It corresponds to the following 95% probability ranges:

$$Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = [0, 0.17] \cup [0.24, 1.19]$$

³In order to fully exploit the experimental information on $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, we use directly the likelihood function obtained by deriving the experimental CL [41].

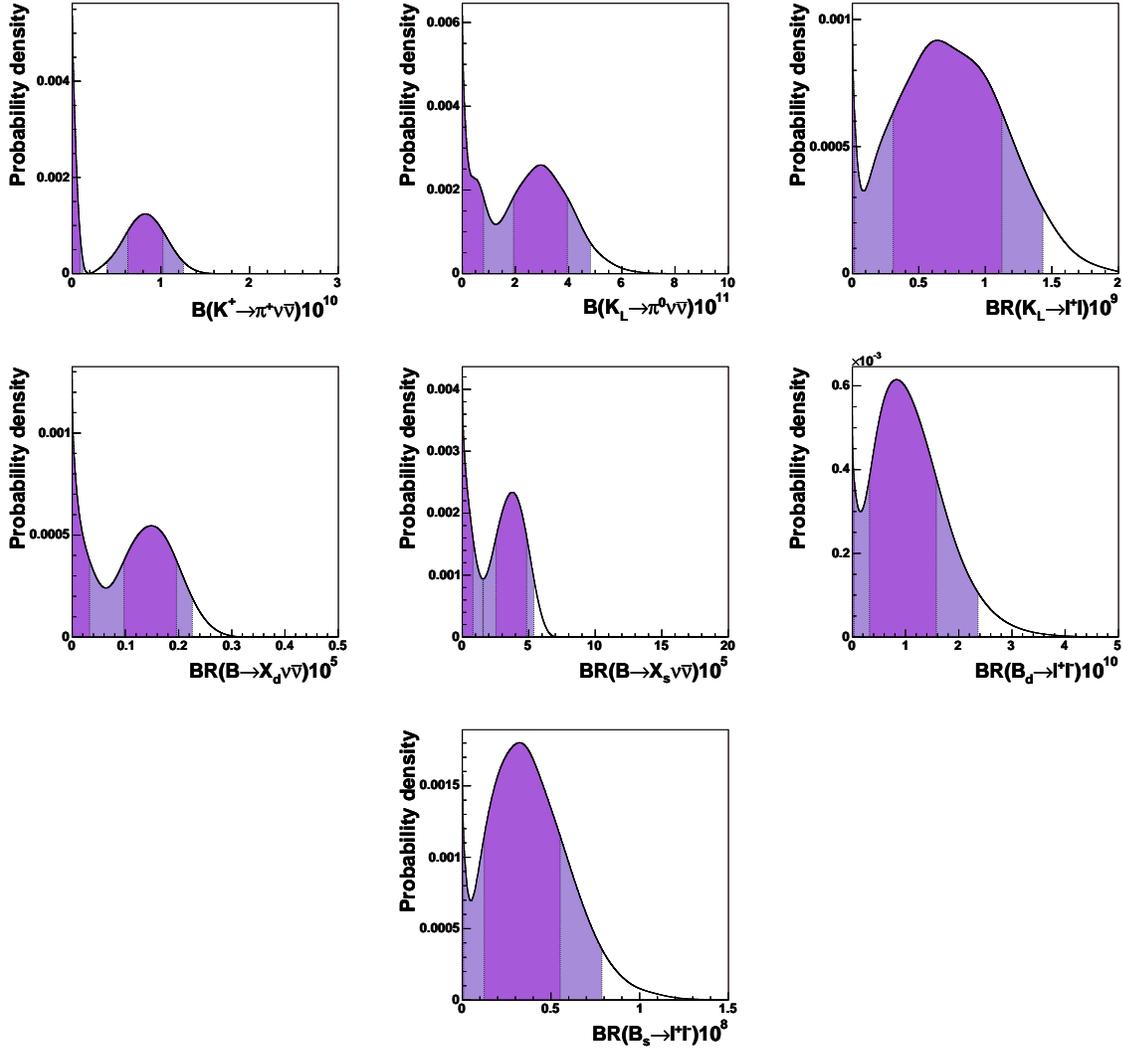


Figure 5: *P.d.f.*'s for the branching ratios of the rare decays $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$, $Br(K_L \rightarrow \mu^+ \mu^-)_{SD}$, $Br(B \rightarrow X_{d,s} \nu \bar{\nu})$, and $Br(B_{d,s} \rightarrow \mu^+ \mu^-)$ considering only the LOW solution for ΔC_7^{eff} . Dark (light) areas correspond to the 68% (95%) probability region.

$$(\text{LOW} : [0, 0.12] \cup [0.39, 1.26], \text{HI} : [0, 0.81]) \times 10^{-10}. \quad (3.5)$$

If we do not use the experimental result on $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, we obtain instead:

$$Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = [0, 0.15] \cup [0.28, 1.12] \times 10^{-10}, \quad (3.6)$$

corresponding to an upper bound of 11.2×10^{-11} at 95% probability.

We have also analyzed the decays $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$ using the formulae of [29, 30]. In the models with MFV these decays are dominated by the contribution from the indirect CP violation that is basically fixed by the measured values of ϵ_K and $K_S \rightarrow \pi^0 l^+ l^-$. The dependence on $C(v)$ enters only in the subdominant direct CP-violating component and the

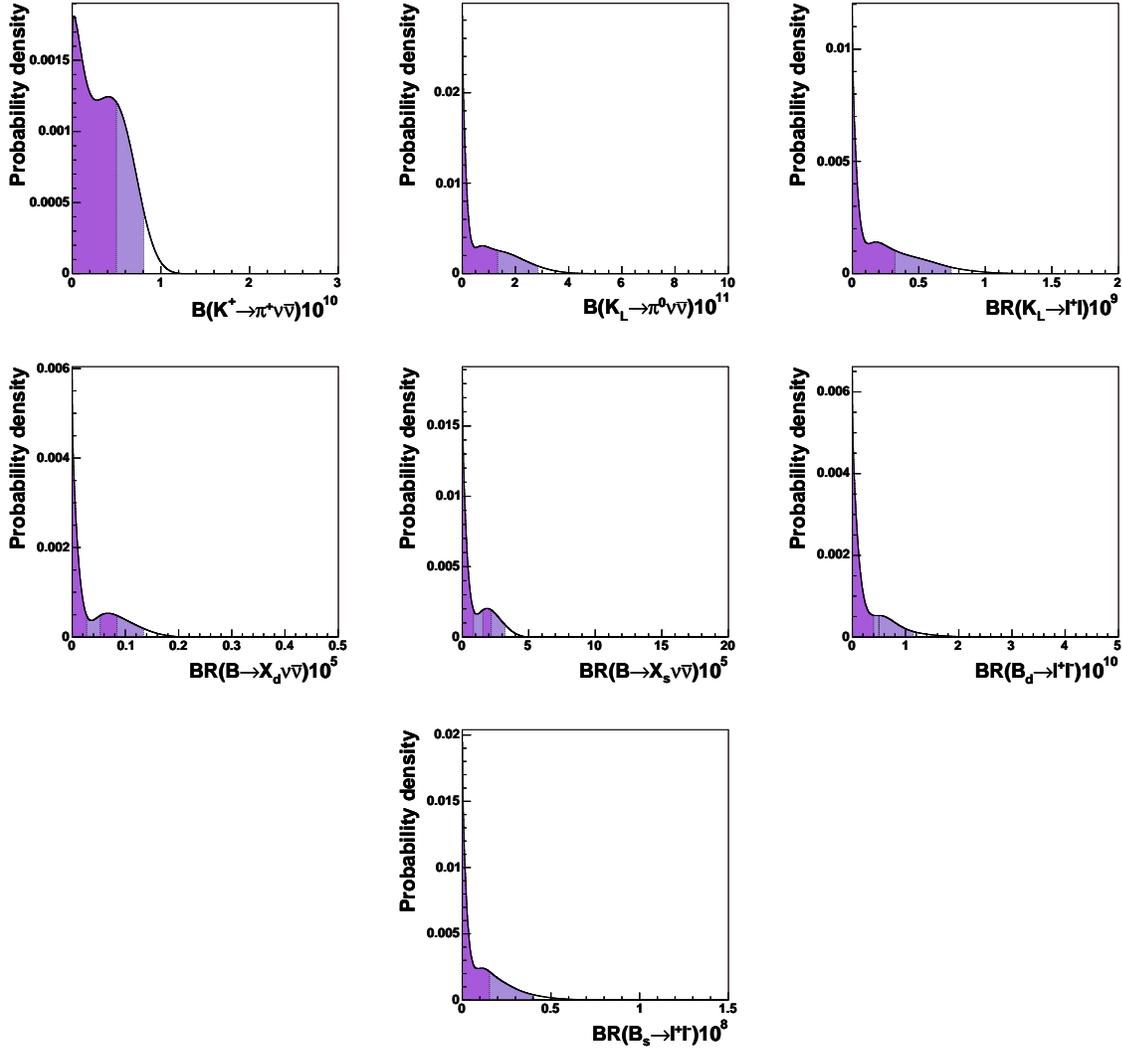


Figure 6: *P.d.f.*'s for the branching ratios of the rare decays $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$, $Br(K_L \rightarrow \mu^+ \mu^-)_{SD}$, $Br(B \rightarrow X_{d,s} \nu \bar{\nu})$, and $Br(B_{d,s} \rightarrow \mu^+ \mu^-)$ considering only the HI solution for ΔC_7^{eff} . Dark (light) areas correspond to the 68% (95%) probability region.

interference of indirect and direct CP-violating contributions. We find that $Br(K_L \rightarrow \pi^0 e^+ e^-)$ and $Br(K_L \rightarrow \pi^0 \mu^+ \mu^-)$ can be enhanced with respect to the SM value by at most 8% and 10%, respectively. In view of theoretical uncertainties in these decays that are larger than these enhancements, a clear signal of new physics within the MFV scenario is rather unlikely from the present perspective. Therefore we do not show the corresponding p.d.f.s.

Concerning $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$, its 95% probability ranges are given by

$$Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) = [0, 4.59] \text{ (LOW : } [0, 4.83], \text{ HI : } [0, 2.84]) \times 10^{-11} \quad (3.7)$$

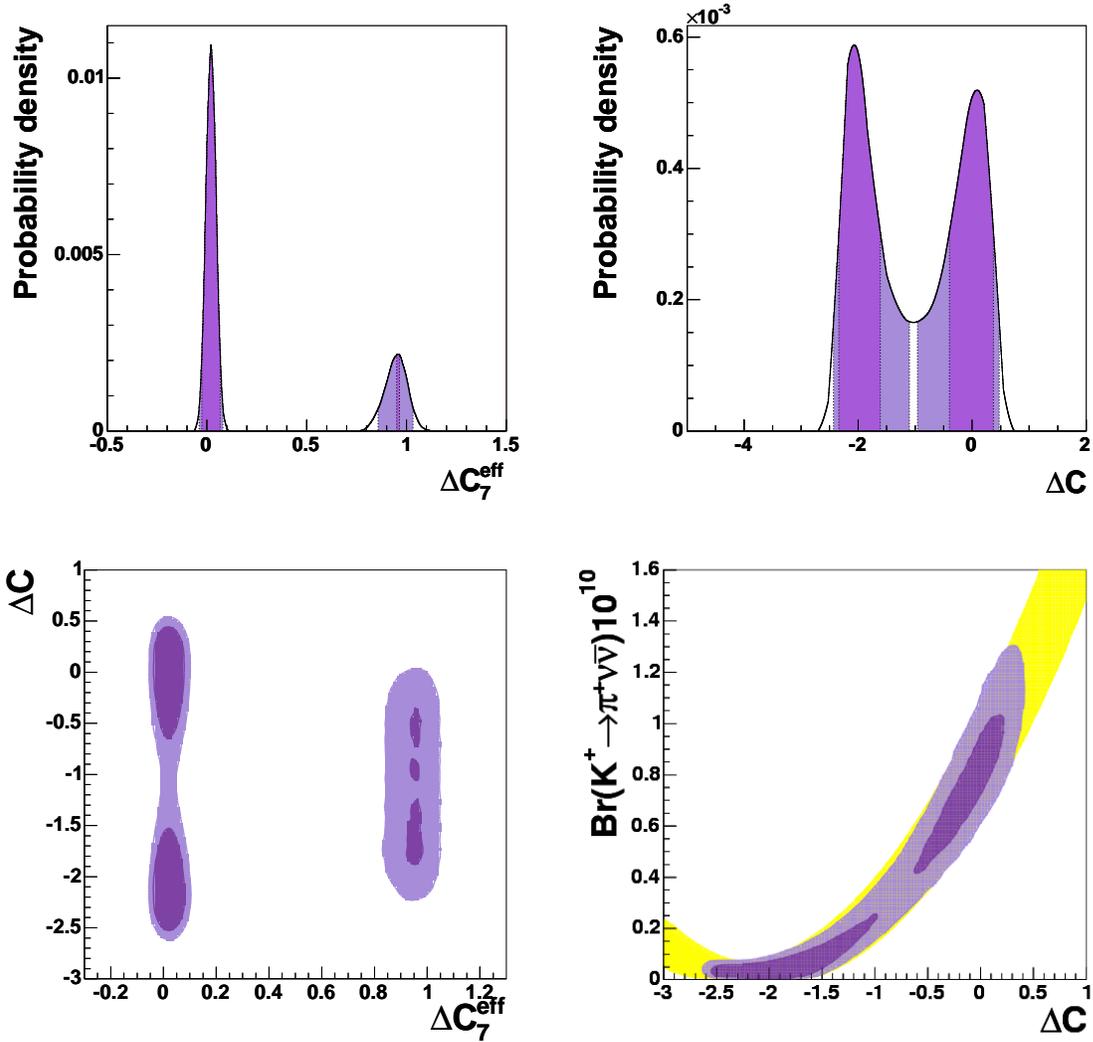


Figure 7: *P.d.f.'s for ΔC_7^{eff} (top-left), ΔC (top-right), ΔC vs. ΔC_7^{eff} (bottom-left) and $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ vs ΔC (bottom-right) obtained without using $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ as a constraint. Dark (light) areas correspond to the 68% (95%) probability region.*

(see Figures 2, 5 and 6). In Figure 8 we see explicitly the correlation between the charged and neutral Kaon decay modes. We observe a very strong correlation, a peculiarity of models with MFV [42]. In particular, a large enhancement of $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ characteristic of models with new complex phases is not possible [43]. An observation of $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ larger than $6 \cdot 10^{-11}$ would be a clear signal of new complex phases or new flavour changing contributions that violate the correlations between B and K decays.

The 95% probability ranges for $Br(K_L \rightarrow \mu^+ \mu^-)_{\text{SD}}$ are

$$Br(K_L \rightarrow \mu^+ \mu^-)_{\text{SD}} = [0, 1.36] \text{ (LOW : } [0, 1.44], \text{ HI : } [0, 0.74]) \times 10^{-9}. \quad (3.8)$$

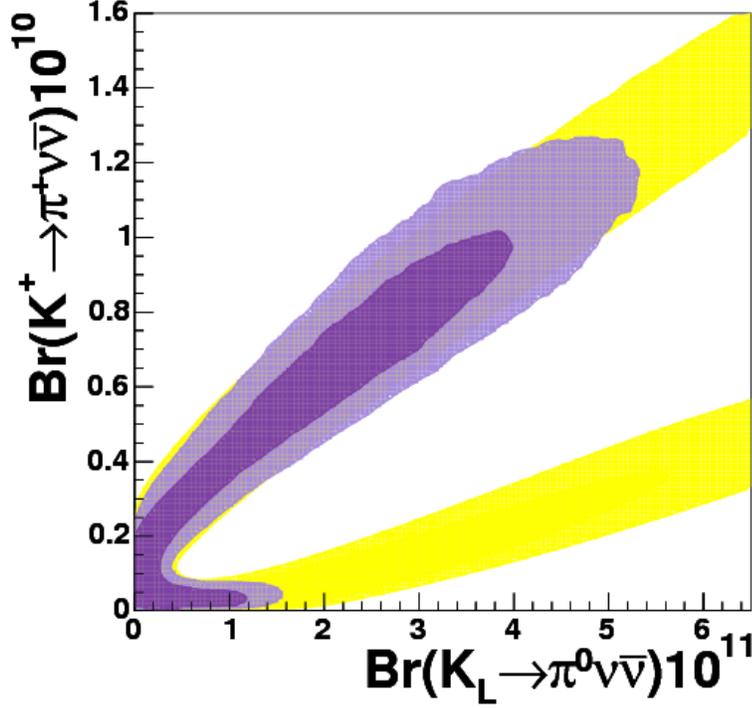


Figure 8: *P.d.f.* for the branching ratios of the rare decays $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ vs $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. Dark (light) areas correspond to the 68% (95%) probability region. Very light areas correspond to the range obtained without using the experimental information.

As in the previous cases, the HI solution corresponds to a much lower upper bound.

Let us now consider B decays:

$$\begin{aligned}
Br(B \rightarrow X_s \nu \bar{\nu}) &= [0, 5.17] \text{ (LOW : } [0, 1.56] \cup [1.59, 5.4], \text{ HI : } [0, 3.22]) \times 10^{-5}, \\
Br(B \rightarrow X_d \nu \bar{\nu}) &= [0, 2.17] \text{ (LOW : } [0, 2.26], \text{ HI : } [0, 1.34]) \times 10^{-6}, \\
Br(B_s \rightarrow \mu \bar{\mu}) &= [0, 7.42] \text{ (LOW : } [0, 7.91], \text{ HI : } [0, 3.94]) \times 10^{-9}, \\
Br(B_d \rightarrow \mu \bar{\mu}) &= [0, 2.20] \text{ (LOW : } [0, 2.37], \text{ HI : } [0, 1.15]) \times 10^{-10}. \tag{3.9}
\end{aligned}$$

The reader may wonder whether other observables could help improving the constraints on ΔC and testing MFV models. In particular, the Forward-Backward asymmetry in $B \rightarrow X_s l^+ l^-$ is known to be a very sensitive probe of C_7^{eff} and of C [44]. Indeed, the HI and LOW solutions for ΔC_7^{eff} and corresponding possible values of ΔC give rise to different profiles of the normalized \bar{A}_{FB} , defined as

$$\bar{A}_{\text{FB}}(\hat{s}) = \frac{\int_{-1}^1 d \cos \theta_l \frac{d^2 \Gamma(b \rightarrow s \mu^+ \mu^-)}{d \hat{s} d \cos \theta_l} \text{sgn}(\cos \theta_l)}{\int_{-1}^1 d \cos \theta_l \frac{d^2 \Gamma(b \rightarrow s \mu^+ \mu^-)}{d \hat{s} d \cos \theta_l}}. \tag{3.10}$$

This can be seen explicitly in Figure 9. Therefore, a measurement of $\bar{A}_{\text{FB}}(\hat{s})$ at a Super B factory will be extremely helpful in distinguishing the various scenarios discussed above [45]. On the other hand, concerning the CP asymmetry in $B \rightarrow X_s \gamma$ decays [46], it turns out that in MFV models its value is reduced with respect to the SM, once the constraint on the branching ratio is taken into account, so that it is not expected to play a significant role in present and future analyses [47].

In Figure 5 we show the p.d.f.'s for the branching ratios of rare decays for the LOW solution. The corresponding result for the HI solution is given in Figure 6. Clearly the branching ratios of various decays are larger in the case of the LOW solution.

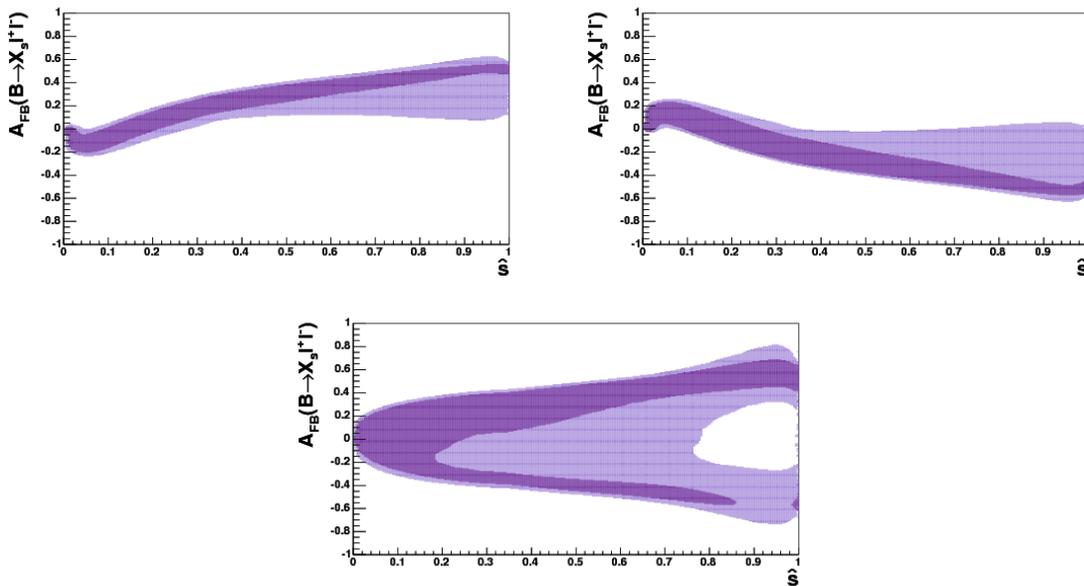


Figure 9: *P.d.f.'s for the normalized forward-backward asymmetry in $B \rightarrow X_s l^+ l^-$ for the LOW solution for ΔC_7^{eff} with $\Delta C > -1$ (left), for the LOW solution with $\Delta C < -1$ (center) and for the HI solution for ΔC_7^{eff} (right). Dark (light) areas correspond to the 68% (95%) probability region.*

Before concluding this section, let us make a few steps towards the future and consider a realistic scenario for the projected integrated luminosities of Belle and BaBar, plus a 10% measurement of $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. For concreteness, let us assume the following 2010 experimental data:

$$\begin{aligned}
 Br(B \rightarrow X_s l^+ l^-)_{0.04 < q^2 (\text{GeV}^2) < 1} &= (1.13 \pm 0.25) \times 10^{-6}, \\
 Br(B \rightarrow X_s l^+ l^-)_{1 < q^2 (\text{GeV}^2) < 6} &= \begin{cases} (1.49 \pm 0.21) \times 10^{-6} (\text{Belle}) \\ (1.80 \pm 0.18) \times 10^{-6} (\text{BaBar}) \end{cases} \\
 Br(B \rightarrow X_s l^+ l^-)_{14.4 < q^2 (\text{GeV}^2) < 25} &= \begin{cases} (4.18 \pm 0.48) \times 10^{-7} (\text{Belle}) \\ (5.00 \pm 0.93) \times 10^{-7} (\text{BaBar}) \end{cases}
 \end{aligned}$$

$$Br(B \rightarrow X_s \gamma) = \begin{cases} (3.51 \pm 0.16) \times 10^{-4} (\text{Belle}) \\ (3.67 \pm 0.16) \times 10^{-4} (\text{BaBar incl.}) \\ (3.29 \pm 0.16) \times 10^{-4} (\text{BaBar semincl.}) \end{cases} \quad (3.11)$$

corresponding to an integrated luminosity of 1 ab^{-1} and 700 fb^{-1} for Belle and BaBar, respectively. Additionally a reduction to 5% of the theoretical uncertainty in $Br(B \rightarrow X_s \gamma)$ thanks to the ongoing NNLO computation is assumed [48].⁴

We can see the dramatic effect of these improvements in Figures 10-12. B -factory data will completely eliminate the non-standard solution for ΔC_7^{eff} , while they cannot distinguish the two solutions for ΔC (considering only branching ratio measurements), see Figure 12. However, this ambiguity is perfectly resolved by $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, leading to the impressive results in Figures 10 and 11.

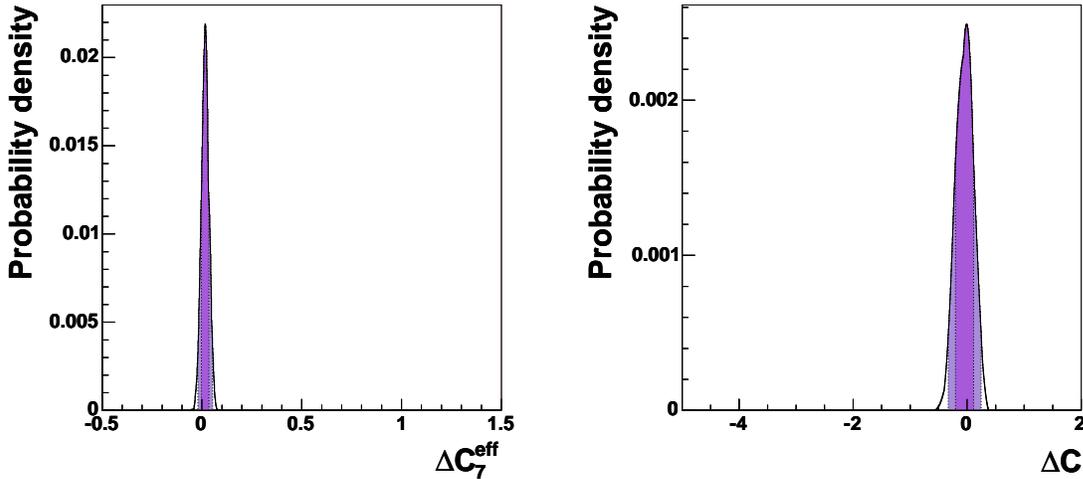


Figure 10: $P.d.f.$'s for ΔC_7^{eff} (left) and ΔC (right) in the future scenario specified by eq. (3.11). Dark (light) areas correspond to the 68% (95%) probability region.

With so powerful experimental data, one can even think of generalizing our analysis by allowing for substantial deviations from the SM in box diagrams. If the size of new physics contributions to box diagrams is comparable to the SM ones, the results of our “future” analysis would not change sizably. On the other hand, a dramatic modification could occur for contributions to box diagrams much larger than the SM ones; however, it is very difficult to conceive new-physics models in which this possibility can be realized.

4 Messages

The main message of our paper is the following one:

⁴The future results for $Br(B \rightarrow X_s \gamma)$ are referred to the same kinematic ranges as the present results.

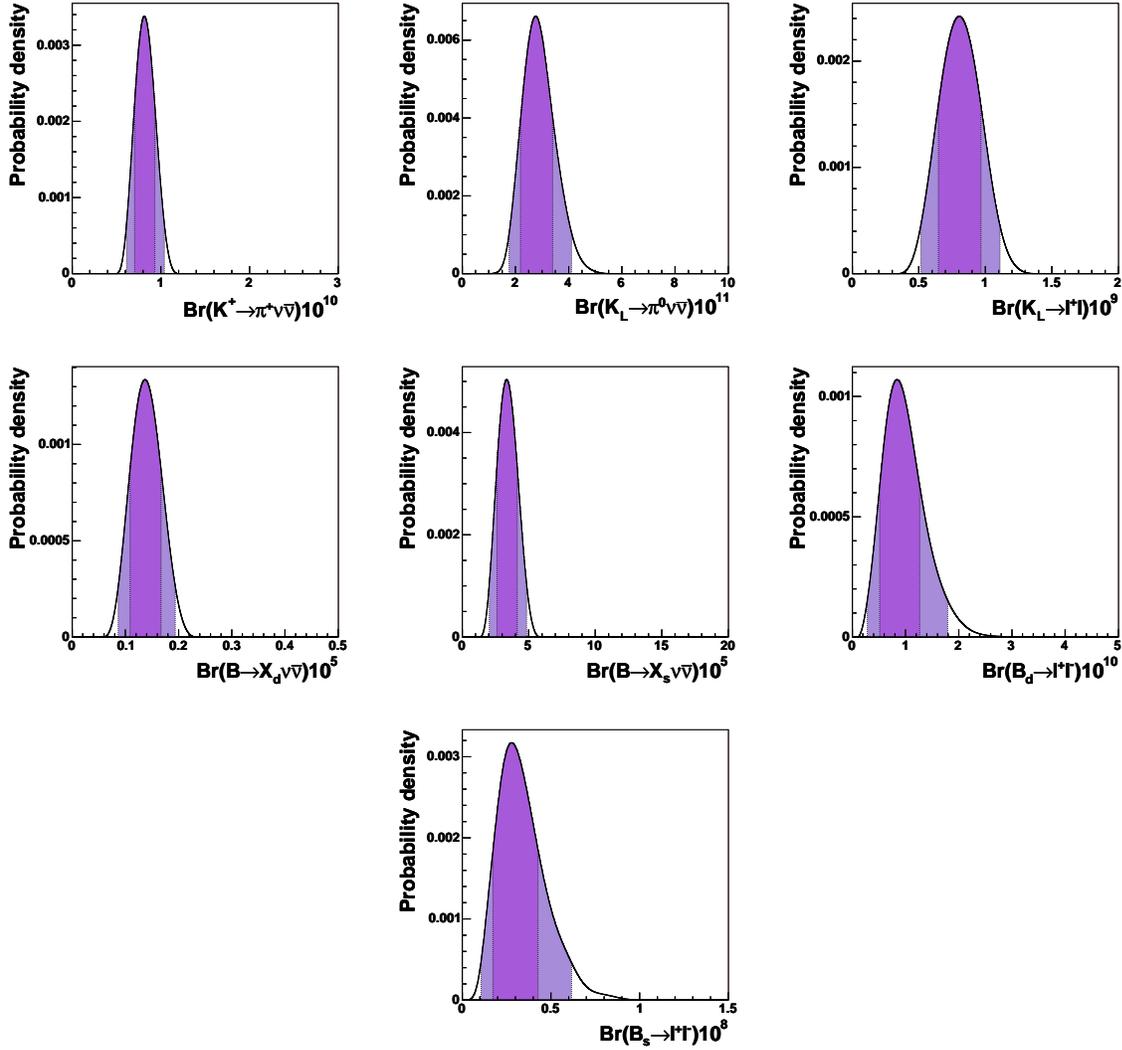


Figure 11: *P.d.f.'s for the branching ratios of the rare decays $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$, $Br(K_L \rightarrow \mu^+ \mu^-)_{SD}$, $Br(B \rightarrow X_{d,s} \nu \bar{\nu})$, and $Br(B_{d,s} \rightarrow \mu^+ \mu^-)$ in the future scenario specified by eq. (3.11). Dark (light) areas correspond to the 68% (95%) probability region.*

The existing constraints coming from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $B \rightarrow X_s \gamma$ and $B \rightarrow X_s l^+ l^-$ do not allow within the MFV scenario of [5] for substantial departures of the branching ratios for all rare K and B decays from the SM estimates. This is evident from Table 3.

There are other messages signalled by our analysis. These are:

- The decays $B \rightarrow X_{s,d} l^+ l^-$ will not offer a precise value for the function C even in the presence of precise measurements of their branching ratios, unless the theoretical errors in these decays and $B \rightarrow X_s \gamma$ and the experimental error on the branching ratio of the latter decay are reduced substantially. This is clearly seen in Figure 3.

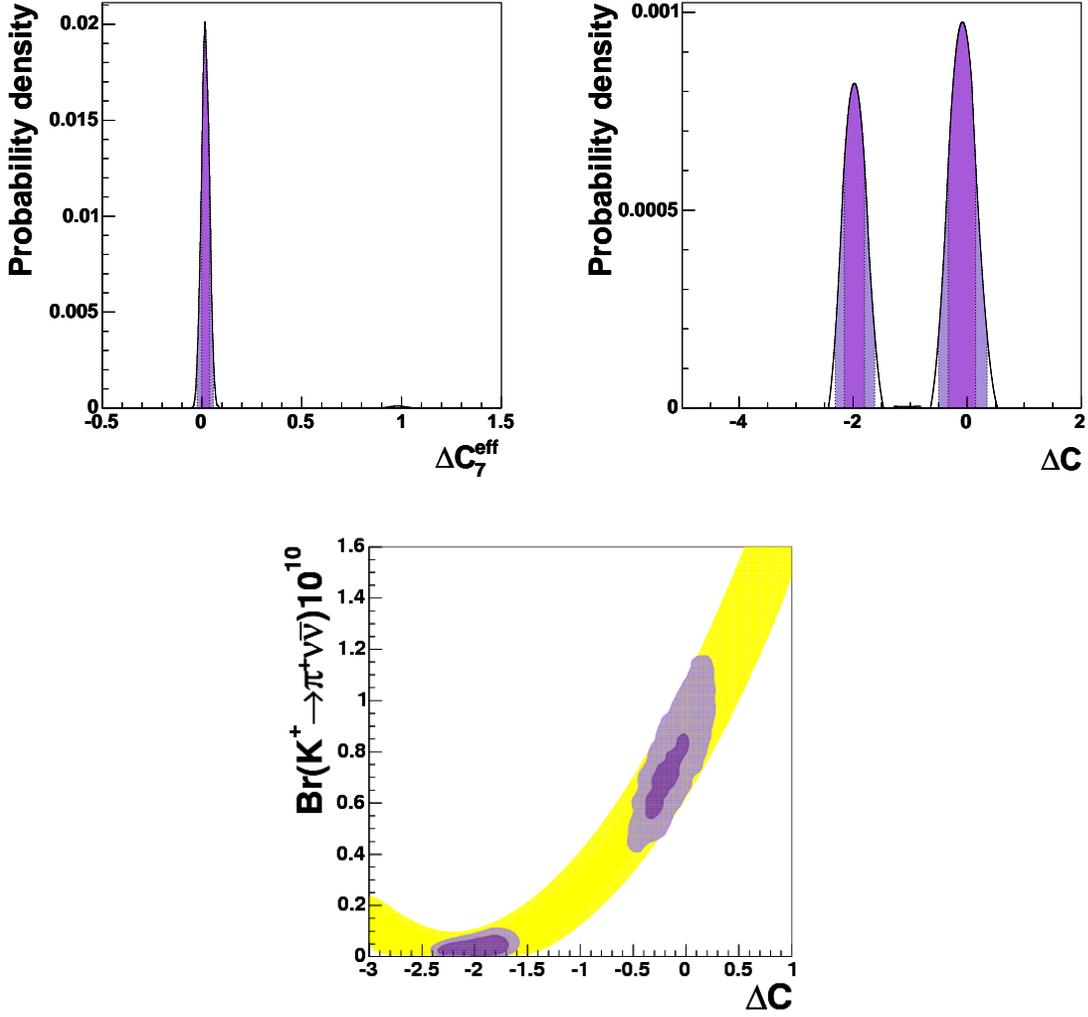


Figure 12: *P.d.f.'s for ΔC_7^{eff} (top-left), ΔC (top-right) and $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ vs ΔC (bottom) obtained without using $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ as a constraint, in the future scenario specified by eq. (3.11). Dark (light) areas correspond to the 68% (95%) probability region.*

- The situation is considerably better in the case of $B_{d,s} \rightarrow \mu^+ \mu^-$ but as seen in Figure 4, for a given value of $Br(B_{d,s} \rightarrow \mu^+ \mu^-)$ there are generally two solutions for ΔC and C , that cannot be disentangled on the basis of these decays alone.
- The great potential of the decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in measuring the function C is clearly visible in Figures 3 and 4, with the unique value obtained in the case of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in the full allowed range of C . In the case of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ the two solutions are only present for $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ significant smaller than the SM value. Similar comment applies to $B \rightarrow X_{s,d} \nu \bar{\nu}$.

- Assuming that future more precise measurements of the $K \rightarrow \pi\nu\bar{\nu}$ branching ratios will be consistent with the MFV upper bounds presented here, the determination of C through these decays will imply much sharper predictions for various branching ratios that could confirm or rule out the MFV scenario. In this context the correlations between various branching ratios discussed in [8] will play the crucial role.
- One of such correlations predicts that the measurement of $\sin 2\beta$ and of $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$ implies only two values of $Br(K_L \rightarrow \pi^0\nu\bar{\nu})$ in the full class of MFV models that correspond to two signs of the function X [42]. Figure 8 demonstrates that the solution with $X < 0$, corresponding to the values in the left lower corner, is practically ruled out so that a unique prediction for $Br(K_L \rightarrow \pi^0\nu\bar{\nu})$ can in the future be obtained.
- A strong violation of any of the 95% probability upper bounds on the branching ratios considered here by future measurements will imply a failure of MFV as defined in [5], unless an explicit MFV scenario can be found in which the contributions of box diagrams are significantly larger than assumed here. Dimensional arguments [43] and explicit calculations indicate that such a possibility is rather remote.
- If the only violation of the upper bounds in Table 3 occurs in $B_s \rightarrow \mu^+\mu^-$ and $B_d \rightarrow \mu^+\mu^-$, it will be most likely due to new operators beyond the SM ones. For example, the scalar operators which arise in MFV SUSY models at large $\tan\beta$ can enhance $Br(B_s \rightarrow \mu^+\mu^-)$ up to the present experimental upper bound [9, 13, 49].
- Conversely, a violation of the upper bounds for the other channels in Table 3 would signal the presence of new sources of flavour and in particular of CP violation. This can be confirmed observing a violation of the correlations between K and B decays discussed above.
- In particular, recalling that in most extensions of the SM the decays $K \rightarrow \pi\nu\bar{\nu}$ are governed by the single $(V - A) \otimes (V - A)$ operator, the violation of the upper bounds on at least one of the $K \rightarrow \pi\nu\bar{\nu}$ branching ratios, will either signal the presence of new complex weak phases at work or new contributions that violate the correlations between the B decays and K decays.

Assuming that the MFV scenario will survive future tests, the next step will be to identify the correct model in this class. Clearly, direct searches at high energy colliders can rule out or identify specific extensions of the SM. But also FCNC processes can play an important role in this context, provided the theoretical and experimental uncertainties in some of them will be sufficiently decreased. In this case, by studying simultaneously several branching ratios it should be in principle possible to select the correct MFV models by just identifying the pattern of enhancements and suppressions relative to the SM that is specific to a given model. If this pattern is independent of the values of the parameters defining the model, no detailed quantitative analysis of the enhancements and suppressions is required in order to rule it out.

As an example the distinction between the MSSM with MFV and the models with one universal extra dimension should be straightforward:

- In the MSSM with MFV the branching ratios for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $B \rightarrow X_d \nu \bar{\nu}$ and $B_d \rightarrow \mu^+ \mu^-$ are generally suppressed relative to the SM expectations, while those governed by V_{ts} like $B \rightarrow X_s \nu \bar{\nu}$, $B_s \rightarrow \mu^+ \mu^-$ and $B \rightarrow X_s \gamma$ can be enhanced or suppressed depending on the values of parameters involved [50].
- In the model with one universal extra dimension analyzed in [22, 23], branching ratios for essentially all rare decays are enhanced, the enhancement being stronger for the decays governed by V_{ts} than for those where V_{td} is involved. A prominent exception is the suppression of $B \rightarrow X_{s,d} \gamma$ [23, 51].

Finally, if MFV will be confirmed, and some new particles will be observed, the rare processes discussed in this work will constitute a most powerful tool to probe the spectrum of the NP model, which might not be entirely accessible via direct studies at the LHC.

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References

- [1] M. Bona *et al.* [UTfit Collaboration], arXiv:hep-ph/0501199.
- [2] J. Charles *et al.* [CKMfitter Group], arXiv:hep-ph/0406184.
- [3] M. Ciuchini, E. Franco, F. Parodi, V. Lubicz, L. Silvestrini and A. Stocchi, eConf **C0304052**, WG306 (2003) [arXiv:hep-ph/0307195].
- [4] M. Bona *et al.* [UTfit Collaboration], in preparation; Talk given by M. Pierini at the CKM 2005 Workshop, <http://ckm2005.ucsd.edu/speakers/WG6/Pierini-WG6-fri2.pdf>.
- [5] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, Phys. Lett. B **500**, 161 (2001) [arXiv:hep-ph/0007085].
- [6] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).

- [7] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
- [8] A. J. Buras, *Acta Phys. Polon. B* **34**, 5615 (2003) [arXiv:hep-ph/0310208].
- [9] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, *Nucl. Phys. B* **645**, 155 (2002) [arXiv:hep-ph/0207036].
- [10] C. Bobeth, T. Ewerth, F. Krüger and J. Urban, *Phys. Rev. D* **66**, 074021 (2002) [arXiv:hep-ph/0204225].
- [11] A. J. Buras, *Phys. Lett. B* **566**, 115 (2003) [arXiv:hep-ph/0303060].
- [12] A. Ali, E. Lunghi, C. Greub and G. Hiller, *Phys. Rev. D* **66**, 034002 (2002) [arXiv:hep-ph/0112300].
- [13] G. Hiller and F. Krüger, *Phys. Rev. D* **69**, 074020 (2004) [arXiv:hep-ph/0310219].
- [14] P. Gambino, U. Haisch and M. Misiak, *Phys. Rev. Lett.* **94**, 061803 (2005) [arXiv:hep-ph/0410155].
- [15] A. J. Buras, F. Schwab and S. Uhlig, arXiv:hep-ph/0405132.
- [16] G. Isidori, *Annales Henri Poincare* **4**, S97 (2003) [arXiv:hep-ph/0301159]; G. Isidori, eConf **C0304052**, WG304 (2003) [arXiv:hep-ph/0307014].
- [17] S. C. Adler *et al.* [E787 Collaboration], *Phys. Rev. Lett.* **79**, 2204 (1997) [arXiv:hep-ex/9708031]; S. C. Adler *et al.* [E787 Collaboration], *Phys. Rev. Lett.* **84**, 3768 (2000) [arXiv:hep-ex/0002015].
- [18] S. Adler *et al.* [E787 Collaboration], *Phys. Rev. Lett.* **88**, 041803 (2002) [arXiv:hep-ex/0111091]; S. Adler *et al.* [E787 Collaboration], *Phys. Rev. D* **70**, 037102 (2004) [arXiv:hep-ex/0403034].
- [19] V. V. Anisimovsky *et al.* [E949 Collaboration], *Phys. Rev. Lett.* **93**, 031801 (2004) [arXiv:hep-ex/0403036].
- [20] K. Abe *et al.* [Belle Collaboration], arXiv:hep-ex/0408119.
- [21] B. Aubert *et al.* [BABAR Collaboration], *Phys. Rev. Lett.* **93**, 081802 (2004) [arXiv:hep-ex/0404006].
- [22] A. J. Buras, M. Spranger and A. Weiler, *Nucl. Phys. B* **660**, 225 (2003) [arXiv:hep-ph/0212143].
- [23] A. J. Buras, A. Poschenrieder, M. Spranger and A. Weiler, *Nucl. Phys. B* **678**, 455 (2004) [arXiv:hep-ph/0306158].

- [24] S. R. Choudhury, N. Gaur, A. Goyal and N. Mahajan, *Phys. Lett. B* **601**, 164 (2004) [arXiv:hep-ph/0407050]; A. J. Buras, A. Poschenrieder and S. Uhlig, arXiv:hep-ph/0410309; A. J. Buras, A. Poschenrieder and S. Uhlig, arXiv:hep-ph/0501230.
- [25] G. Buchalla, A. J. Buras and M. K. Harlander, *Nucl. Phys. B* **349**, 1 (1991).
- [26] A. J. Buras and M. K. Harlander, *Adv. Ser. Direct. High Energy Phys.* **10**, 58 (1992).
- [27] G. Buchalla, A. J. Buras and M. E. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996) [arXiv:hep-ph/9512380].
- [28] G. Isidori, F. Mescia and C. Smith, [arXiv:hep-ph/0308008].
- [29] G. Buchalla, G. D'Ambrosio and G. Isidori, *Nucl. Phys. B* **672**, 387 (2003) [arXiv:hep-ph/0308008].
- [30] G. Isidori, C. Smith and R. Unterdorfer, *Eur. Phys. J. C* **36**, 57 (2004) [arXiv:hep-ph/0404127].
- [31] A. Ali and C. Greub, *Z. Phys. C* **49**, 431 (1991); A. Ali and C. Greub, *Phys. Lett. B* **259**, 182 (1991); A. Ali and C. Greub, *Phys. Lett. B* **361**, 146 (1995) [arXiv:hep-ph/9506374]; A. J. Buras, A. Czarnecki, M. Misiak and J. Urban, *Nucl. Phys. B* **631**, 219 (2002) [arXiv:hep-ph/0203135]; K. Adel and Y. P. Yao, *Phys. Rev. D* **49**, 4945 (1994) [arXiv:hep-ph/9308349]; C. Greub and T. Hurth, *Phys. Rev. D* **56**, 2934 (1997) [arXiv:hep-ph/9703349]; A. J. Buras, A. Kwiatkowski and N. Pott, *Nucl. Phys. B* **517**, 353 (1998) [arXiv:hep-ph/9710336]; M. Ciuchini, G. Degrossi, P. Gambino and G. F. Giudice, *Nucl. Phys. B* **527**, 21 (1998) [arXiv:hep-ph/9710335]; C. Bobeth, M. Misiak and J. Urban, *Nucl. Phys. B* **567**, 153 (2000) [arXiv:hep-ph/9904413]; K. G. Chetyrkin, M. Misiak and M. Munz, *Nucl. Phys. B* **520**, 279 (1998) [arXiv:hep-ph/9711280]; M. Misiak and M. Munz, *Phys. Lett. B* **344**, 308 (1995) [arXiv:hep-ph/9409454]; N. Pott, *Phys. Rev. D* **54**, 938 (1996) [arXiv:hep-ph/9512252]; Z. Ligeti, M. E. Luke, A. V. Manohar and M. B. Wise, *Phys. Rev. D* **60**, 034019 (1999) [arXiv:hep-ph/9903305]; C. Greub, T. Hurth and D. Wyler, *Phys. Lett. B* **380**, 385 (1996) [arXiv:hep-ph/9602281]; C. Greub, T. Hurth and D. Wyler, *Phys. Rev. D* **54**, 3350 (1996) [arXiv:hep-ph/9603404]; A. J. Buras, A. Czarnecki, M. Misiak and J. Urban, *Nucl. Phys. B* **611**, 488 (2001) [arXiv:hep-ph/0105160].
- [32] A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, *Nucl. Phys. B* **685**, 351 (2004) [arXiv:hep-ph/0312128]; C. Bobeth, P. Gambino, M. Gorbahn and U. Haisch, *JHEP* **0404**, 071 (2004) [arXiv:hep-ph/0312090]; C. Bobeth, M. Misiak and J. Urban, *Nucl. Phys. B* **574**, 291 (2000) [arXiv:hep-ph/9910220]; H. H. Asatryan, H. M. Asatrian, C. Greub and M. Walker, *Phys. Rev. D* **65**, 074004 (2002) [arXiv:hep-ph/0109140]; H. H. Asatryan, H. M. Asatrian, C. Greub and M. Walker, *Phys. Rev. D* **66**, 034009 (2002) [arXiv:hep-ph/0204341]; A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, *Nucl. Phys. B* **648**, 254 (2003) [arXiv:hep-ph/0208088]; H. M. Asatrian, K. Bieri, C. Greub and

- A. Hovhannisyanyan, Phys. Rev. D **66**, 094013 (2002) [arXiv:hep-ph/0209006]; H. M. Asatrian, H. H. Asatryan, A. Hovhannisyanyan and V. Poghosyan, Mod. Phys. Lett. A **19**, 603 (2004) [arXiv:hep-ph/0311187]; A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, Eur. Phys. J. C **33**, S288 (2004) [arXiv:hep-ph/0310187].
- [33] S. Chen *et al.* [CLEO Collaboration], Phys. Rev. Lett. **87**, 251807 (2001) [arXiv:hep-ex/0108032].
- [34] P. Koppenburg *et al.* [Belle Collaboration], Phys. Rev. Lett. **93**, 061803 (2004) [arXiv:hep-ex/0403004];
- [35] J. Walsh for the Babar Collaboration, talk presented at Moriond QCD, 2005, <http://moriond.in2p3.fr/QCD/2005/SundayAfternoon/Walsh.ppt>.
- [36] M. Neubert, Eur. Phys. J. C **40**, 165 (2005) [arXiv:hep-ph/0408179]; D. Benson, I. I. Bigi and N. Uraltsev, Nucl. Phys. B **710**, 371 (2005) [arXiv:hep-ph/0410080].
- [37] A. Alavi-Harati *et al.* [The E799-II/KTeV Collaboration], Phys. Rev. D **61**, 072006 (2000) [arXiv:hep-ex/9907014].
- [38] R. Barate *et al.* [ALEPH Collaboration], Eur. Phys. J. C **19**, 213 (2001) [arXiv:hep-ex/0010022].
- [39] M. Herndon [CDF and D0 Collaborations], FERMILAB-CONF-04-391-E SPIRES entry *To appear in the proceedings of 32nd International Conference on High-Energy Physics (ICHEP 04), Beijing, China, 16-22 Aug 2004*
- [40] M. Ciuchini *et al.*, JHEP **0107**, 013 (2001) [arXiv:hep-ph/0012308].
- [41] The experimental CL can be found at http://www.phy.bnl.gov/e949/E949Archive/br_cls.dat, and the likelihood we use can be found at <http://www.utfit.org/kpinunubar/ckm-kpinunubar.html>.
- [42] A. J. Buras and R. Fleischer, Phys. Rev. D **64** (2001) 115010 [arXiv:hep-ph/0104238].
- [43] A. J. Buras, A. Romanino and L. Silvestrini, Nucl. Phys. B **520**, 3 (1998) [arXiv:hep-ph/9712398]; G. Colangelo and G. Isidori, JHEP **9809**, 009 (1998) [arXiv:hep-ph/9808487]; A. J. Buras, G. Colangelo, G. Isidori, A. Romanino and L. Silvestrini, Nucl. Phys. B **566**, 3 (2000) [arXiv:hep-ph/9908371]; G. Buchalla, G. Hiller and G. Isidori, Phys. Rev. D **63**, 014015 (2001) [arXiv:hep-ph/0006136]; D. Atwood and G. Hiller, arXiv:hep-ph/0307251; A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Nucl. Phys. B **697**, 133 (2004) [arXiv:hep-ph/0402112]; A. J. Buras, T. Ewerth, S. Jager and J. Rosiek, Nucl. Phys. B **714**, 103 (2005) [arXiv:hep-ph/0408142].
- [44] G. Burdman, Phys. Rev. D **57**, 4254 (1998) [arXiv:hep-ph/9710550].
- [45] A. G. Akeroyd *et al.* [SuperKEKB Physics Working Group], arXiv:hep-ex/0406071; J. L. Hewett *et al.*, arXiv:hep-ph/0503261.

- [46] A. L. Kagan and M. Neubert, Phys. Rev. D **58**, 094012 (1998) [arXiv:hep-ph/9803368].
- [47] T. Hurth, E. Lunghi and W. Porod, Nucl. Phys. B **704**, 56 (2005) [arXiv:hep-ph/0312260].
- [48] K. Bieri, C. Greub and M. Steinhauser, Phys. Rev. D **67** (2003) 114019 [arXiv:hep-ph/0302051]; M. Misiak and M. Steinhauser, Nucl. Phys. B **683**, 277 (2004) [arXiv:hep-ph/0401041]; M. Gorbahn and U. Haisch, Nucl. Phys. B **713** (2005) 291 [arXiv:hep-ph/0411071]; M. Gorbahn, U. Haisch and M. Misiak, arXiv:hep-ph/0504194; H. M. Asatrian, C. Greub, A. Hovhannisyanyan, T. Hurth and V. Poghosyan, arXiv:hep-ph/0505068.
- [49] K. S. Babu and C. F. Kolda, Phys. Rev. Lett. **84**, 228 (2000) [arXiv:hep-ph/9909476]; C. S. Huang, W. Liao and Q. S. Yan, Phys. Rev. D **59**, 011701 (1999) [arXiv:hep-ph/9803460]; S. R. Choudhury and N. Gaur, Phys. Lett. B **451**, 86 (1999) [arXiv:hep-ph/9810307]; C. S. Huang, W. Liao, Q. S. Yan and S. H. Zhu, Phys. Rev. D **63**, 114021 (2001) [Erratum-ibid. D **64**, 059902 (2001)] [arXiv:hep-ph/0006250]; C. Bobeth, T. Ewerth, F. Kruger and J. Urban, Phys. Rev. D **64** (2001) 074014 [arXiv:hep-ph/0104284]; A. Dedes, Mod. Phys. Lett. A **18**, 2627 (2003) [arXiv:hep-ph/0309233]; A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, Phys. Lett. B **546**, 96 (2002) [arXiv:hep-ph/0207241]; A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, Nucl. Phys. B **659**, 3 (2003) [arXiv:hep-ph/0210145]; S. Baek, P. Ko and W. Y. Song, JHEP **0303**, 054 (2003) [arXiv:hep-ph/0208112]; G. L. Kane, C. Kolda and J. E. Lennon, arXiv:hep-ph/0310042; A. Dedes and B. T. Huffman, Phys. Lett. B **600**, 261 (2004) [arXiv:hep-ph/0407285].
- [50] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, Nucl. Phys. B **592**, 55 (2001) [arXiv:hep-ph/0007313].
- [51] K. Agashe, N. G. Deshpande and G. H. Wu, Phys. Lett. B **514**, 309 (2001) [arXiv:hep-ph/0105084].