Excited $[\mathbf{70},\ell^+]$ baryons in large N_c QCD

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The masses of the positive parity $[\mathbf{70},0^+]$ and $[\mathbf{70},2^+]$ non-strange baryons are calculated in large N_c QCD by considering the most dominant operators in an $1/N_c$ expansion. The approach is based on the introduction of an excited core, obtained after the last particle (an excited quark) has been removed. Configuration mixing is neglected, for simplicity. Although being a sub-leading $1/N_c$ order, we find that the spin-spin interaction plays a dominant role in describing the data. The role of N_c^0 operators is also pointed out. We show how the contribution of the linear term in N_c, of the spin-spin and of the spin-orbit terms vary with the excitation energy.

1. Introduction

The large N_c QCD approach suggested by 't Hooft [1] and considered in detail by Witten [2] has become a powerful tool in baryon spectroscopy. The method is based on the result that, for N_f flavors, the ground state baryons display an exact SU($2N_f$) spin-flavor symmetry in the large N_c limit of QCD [3]. It has been applied with great success to the ground state baryons ($N = 0$ band), described by the symmetric representation **56** of SU(6), where $N_f = 3$ [3, 4, 5, 6, 7, 8, 9]. For excited baryons this symmetry is broken. However, based on the observation that excited states can be approximately classified as SU($2N_f$) multiplets, considerable efforts have previously been made to analyze the excited states belonging to the $[\mathbf{70},1^-]$ multiplet ($N = 1$ band) in the large N_c limit [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20], with obvious success.

The $N = 2$ band contains the multiplets $[\mathbf{56}',0^+]$, $[\mathbf{56},2^+]$, $[\mathbf{70},0^+]$, $[\mathbf{70},2^+]$ and $[\mathbf{20},1^+]$. Among them, the baryons supposed to belong to $[\mathbf{56},2^+]$ or $[\mathbf{56}',0^+]$ have been considered so far in Refs. [19] and [21]. The method of Ref. [19] has recently been extended to higher excitations belonging to the $[\mathbf{56},4^+]$ multiplet ($N = 4$ band) [22]. For simplicity, in these studies configuration mixing has been neglected.

The symmetric representation **56** requires a much simpler treatment than the mixed representation **70**. The main reason is that for the symmetric representation it is not necessary to distinguish between excited and core quarks. This is possible because of the structure of the wave function. Let us consider the case $[\mathbf{56},2^+]$. For large N_c this multiplet becomes $[[\mathbf{N_c}],2^+]$ [3]. Its intrinsic wave function (center of mass coordinate

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[3]This is a partition-type notation. It is consistent with the label **56** of the irrep [3] of SU(6). The dimension of the symmetric representation of SU($2N_f$) containing N_c particles and labelled by the partition

removed), written in a harmonic oscillator single particle basis, takes the form

$$|[\mathbf{N_c}], 2^+\rangle = \sqrt{\frac{N_c-1}{N_c}}|[N_c](0s)^{N_c-1}(0d)\rangle + \sqrt{\frac{1}{N_c-1}}|[N_c](0s)^{N_c-2}(0p)^2\rangle, \tag{1}$$

which shows that for $N_c \to \infty$ one can neglect the second term. This cumbersome term, involves an excited core after the removal of the last particle, while in the first term the entire excitation is carried by one quark, excited to the $\ell = 2$ shell. The remaining symmetric core of $N_c - 1$ unexcited quarks is simpler than an excited core. Now, if we consider the spin-orbit 1-body interaction

$$H_{SO} = w(r)\vec{\ell} \cdot \vec{s}\,, \tag{2}$$

it is easy to show that

$$\langle\Psi|\ell^i s^i|\Psi\rangle = \begin{cases} \mathcal{O}(N_c^0) & \text{if } \chi \text{ is mixed symmetric (MS)} \\ \mathcal{O}(N_c^{-1}) & \text{if } \chi \text{ is symmetric (S)} \end{cases}, \tag{3}$$

as proved in Ref. [23] and, in a different manner, in the Appendix below. In this equation Ψ and χ are the total and the spin-flavor wave functions of the excited baryon respectively. (Note that χ and the spatial wave function of the Appendix have the same permutation symmetry, thus the same partition.) With this result, the spin-orbit operator takes the simple form used in Ref. [19].

The situation with the multiplets $[\mathbf{70}, 0^+]$ and $[\mathbf{70}, 2^+]$ is different. This can be illustrated by writing the orbital part of the intrinsic wave functions following the same prescription as above. For $\ell = 0$ one obtains

$$|[\mathbf{N_c-1,1}], 0^+\rangle_{\rho,\lambda} = \sqrt{\frac{1}{3}}|[N_c-1,1]_{\rho,\lambda}(0s)^{N_c-1}(1s)\rangle + \sqrt{\frac{2}{3}}|[N_c-1,1]_{\rho,\lambda}(0s)^{N_c-2}(0p)^2\rangle. \tag{4}$$

In the first term $1s$ is the first single particle radially excited state with $n = 1$, $\ell = 0$ ($N = 2n + \ell$). In the second term the two quarks are excited to the p-shell to get $N = 2$. They are coupled to $\ell = 0$. The lower indices ρ and λ are constituent quark model notations. They distinguish between states which are antisymmetric and symmetric under permutation of the first two particles, respectively. By analogy, for $\ell = 2$ one has

$$|[\mathbf{N_c-1,1}], 2^+\rangle_{\rho,\lambda} = \sqrt{\frac{1}{3}}|[N_c-1,1]_{\rho,\lambda}(0s)^{N_c-1}(0d)\rangle + \sqrt{\frac{2}{3}}|[N_c-1,1]_{\rho,\lambda}(0s)^{N_c-2}(0p)^2\rangle, \tag{5}$$

where the two quarks in the p-shell are coupled to $\ell = 2$. We have obtained the expressions (4) and (5) by using the procedure developed in Ref. [24] based on generalized Jacobi coordinates[4]. The $N_c = 3$ case is demonstrated in Ref. [25].

One can see that the coefficients of the linear combinations (4) and (5) are independent of N_c which means that both terms have to be considered in the large N_c limit. In Eqs. (4) and (5) the first term can be treated as in the $[\mathbf{70}, 1^-]$ sector, *i.e.* as an excited quark

$[N_c]$ is $d_{[N_c]} = \frac{(2N_f+N_c-1)!}{N_c!\,(2N_f-1)!}$. With $N_f = 3$ and $N_c = 3$ one recovers **56**.

[4]For consistency with Ref. [24] we use the harmonic oscillator notation $|n\ell\rangle$ for single particle states everywhere in the text.

coupled to a ground state core [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. The second term will be treated here as an excited quark coupled to an excited core[5].

To our knowledge this is the first attempt to incorporate core excitations in the system and treat the mass operator accordingly. As there are similarities with the $[\mathbf{70}, 1^-]$ multiplet, we view this study as an extension of Refs. [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. In practice we shall combine the techniques of Refs. [13] and [16].

Before ending this section we should recall that in reality the excited states are resonances, so they have a finite width. This is not taken into account in the present approach, similarly to constituent quark models where resonances are treated as bound states. However, it is important to notice, that the bound state picture turned out to describe the baryon phenomenology satisfactorily, thus being a rather realistic picture of baryon resonances [26]. The decay widths have been considered, for example, in Refs. [20, 26]. The conclusion was that a general large N_c analysis does not predict narrow widths, which would vanish in the large N_c limit. Contrary, generic large N_c counting gives widths of order N_c^0. According to Ref. [20] the narrowness of the excited states is an artifact of simple quarks model assumptions used in the calculations, as it is the case here.

2. The wave function

We shall separately discuss the two distinct configurations entering the wave functions (4) and (5).

2.1. The configurations $(0s)^{N_c-1}(1s)$ and $(0s)^{N_c-1}(0d)$

In Eqs. (4) and (5) the first term, containing the configurations $(0s)^{N_c-1}(1s)$ and $(0s)^{N_c-1}(0d)$ respectively, is similar in structure with the wave functions of the $[\mathbf{70}, 1^-]$ multiplet ($N_c = 3$), the difference being that the quark is now excited to the $N = 2n+\ell = 2$ band ($\ell = 0$ or $\ell = 2$) instead of $N = \ell = 1$.

The configurations $(0s)^{N_c-1}(1s)$ and $(0s)^{N_c-1}(0d)$ are then described by a wave function written in the notation of Ref. [13] as

$$
\begin{aligned}
&|JJ_3; II_3; (\ell = \ell_q; S = I + \rho)\rangle = \\
&\sum_{m_\ell, m_S, m_1, m_2, \alpha_1, \alpha_2} \left(\begin{array}{cc|c} \ell_q & S & J \\ m_{\ell_q} & m_S & J_3 \end{array} \right) \left(\begin{array}{cc|c} S_c & \frac{1}{2} & S \\ m_1 & m_2 & m_S \end{array} \right) \left(\begin{array}{cc|c} I_c & \frac{1}{2} & I \\ \alpha_1 & \alpha_2 & I_3 \end{array} \right) \\
&\sum_{\eta = \pm 1} c_{\rho,\eta} |S_c = I_c = I + \tfrac{\eta}{2}; m_1, \alpha_1\rangle \otimes |\tfrac{1}{2}; m_2, \alpha_2\rangle \otimes |\ell_q, m_{\ell_q}\rangle,
\end{aligned}
\tag{6}
$$

where S and I are the total spin and isospin of the baryon, $\ell = \ell_q$ is the angular momentum of the excited quark, S_c and I_c are the core spin and isospin respectively, $\rho \equiv S - I = \pm 1, 0$ and $\eta/2 \equiv I_c - I = \pm 1/2$. The coefficients $c_{\rho,\eta}$ will be presented below. The coupling coefficients are SU(2) coefficients, as we consider here only non-strange baryons ($N_f = 2$).

2.2. The configurations $(0s)^{N_c-2}(0p)^2$

In order to better understand the $1/N_c$ counting it is useful to rewrite the orbital part of the wave function using the fractional parentage technique [27] by which the last particle

[5]One can also consider two excited quarks and leave the core in the ground state but this situation is more complicated.

is decoupled. Here we consider the only case treated in the literature, where the mixed symmetric state belonging to the representation $[N_c-1,1]$ has the last particle in the second row of the corresponding Young diagram. Then, the second term of (4) or (5) takes the form

$$|[N_c-1,1]_{\rho,\lambda}(0s)^{N_c-2}(0p)^2,\ell^+\rangle \;=\; \sqrt{\frac{N_c-2}{N_c}}\Psi_{[N_c-1]}\left((0s)^{N_c-2}\;(0p)\right)\phi_{[1]}(0p) -\sqrt{\frac{2}{N_c}}\Psi_{[N_c-1]}\left((0s)^{N_c-3}\;(0p)^2\right)\phi_{[1]}(0s)\;, \tag{7}$$

the decoupling being valid both for $\ell=0$ and 2. Here all states are normalized. The first factor in each term in the right-hand side is a symmetric (N_c-1)-particle wave function and $\phi_{[1]}$ is a one particle wave function associated to the N_c-th particle. One can see that for large N_c the coefficient of the first term is $\mathcal{O}(1)$ and of the second $\mathcal{O}(N_c^{-1/2})$. Then, in the large N_c limit, one can neglect the second term and take into account only the first term in the wave function, where the N_c-th particle has an $\ell=1$ excitation. A similar decomposition can be made for a symmetric state $[N_c]$ or a mixed symmetric state $[N_c-1,1]$ containing one excited quark only (see Appendix). In that case one can immediately recover the considerations developed in Ref. [23] for the spin-orbit or other operators.

When both the decoupled quark and the core are excited, the wave function takes the form

$$\begin{aligned}
&|JJ_3;II_3;(\ell;S=I+\rho)\rangle = \\
&\quad\sum_{m_q,m_c,m_\ell,m_S,m_1,m_2,\alpha_1,\alpha_2}\left(\begin{array}{cc|c}\ell & S & J\\ m_\ell & m_S & J_3\end{array}\right)\left(\begin{array}{cc|c}\ell_q & \ell_c & \ell\\ m_q & m_c & m_\ell\end{array}\right)\\
&\quad\left(\begin{array}{cc|c}S_c & \frac{1}{2} & S\\ m_1 & m_2 & m_S\end{array}\right)\left(\begin{array}{cc|c}I_c & \frac{1}{2} & I\\ \alpha_1 & \alpha_2 & I_3\end{array}\right)\\
&\quad\sum_{\eta=\pm1}c_{\rho,\eta}|S_c=I_c=I+\frac{\eta}{2};m_1,\alpha_1\rangle\otimes|\frac{1}{2};m_2,\alpha_2\rangle\otimes|\ell_q,m_q\rangle\otimes|\ell_c,m_c\rangle,
\end{aligned}\tag{8}$$

which is a generalization of (6) still for non-strange baryons. It contains an extra SU(2) Clebsch-Gordan coefficient which couples the angular momentum ℓ_q of the excited quark to the angular momentum ℓ_c of the excited core. The coefficients $c_{\rho,\eta}$ are the same in (6) and (8). They are defined in Refs. [13] and [16]. We recall that for the symmetric (SYM) $[N_c]$ and the mixed (MS) $[N_c-1,1]$ representations of the permutation group S_{N_c}, the non-vanishing coefficients $c_{\rho,\eta}$ are given by

$$c^{\rm MS}_{\pm,\pm}=1, \tag{9}$$

$$c^{\rm MS}_{0+}=\sqrt{\frac{S[N_c+2(S+1)]}{N_c(2S+1)}}, \qquad c^{\rm MS}_{0-}=-\sqrt{\frac{(S+1)[N_c-2S]}{N_c(2S+1)}}, \tag{10}$$

$$c^{\rm SYM}_{0+}=-c^{\rm MS}_{0-}, \qquad c^{\rm SYM}_{0-}=c^{\rm MS}_{0+}. \tag{11}$$

In Ref. [13] they were defined as elements of an orthogonal basis rotation. In terms of group theory language these coefficients can be identified with isoscalar factors of the permutation group [27, 28]. They are factors of the Clebsch-Gordan coefficients needed in the flavor-space inner product and are related to the position of the last particle in the corresponding Young tableaux.

3. The Mass Operator

For the $[\mathbf{70}, \ell^+]$ sector the building blocks which form the operators entering in the mass formula consist of the excited core operators ℓ_c^i, S_c^i, T_c^a and G_c^{ia} and the excited quark operators ℓ_q^i, s^i, t^a and g^{ia}. We also introduce the tensor operator[6]

$$\ell_{ab}^{(2)ij} = \frac{1}{2}\left\{\ell_a^i, \ell_b^j\right\} - \frac{1}{3}\delta_{i,-j}\vec{\ell}_a \cdot \vec{\ell}_b \ , \tag{12}$$

with $a = c$, $b = q$ or vice versa or $a = b = c$ or $a = b = q$. For simplicity when $a = b$, we shall use a single index c, for the core, and q for the excited quark so that the operators are $\ell_c^{(2)ij}$ and $\ell_q^{(2)ij}$ respectively. The latter case represents the tensor operator used in the analysis of the $[\mathbf{70}, 1^-]$ multiplet (see *e.g.* Ref. [13]).

Table 1
List of operators and the coefficients resulting from the fit with $\chi^2_{\rm dof} \simeq 0.83$.

Operator	Fitted coef. (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 = 555 \pm 11$
$O_2 = \ell_q^i s^i$	$c_2 = 47 \pm 100$
$O_3 = \frac{3}{N_c}\ell_q^{(2)ij} g^{ia} G_c^{ja}$	$c_3 = -191 \pm 132$
$O_4 = \frac{1}{N_c}(S_c^i S_c^i + s^i S_c^i)$	$c_4 = 261 \pm 47$

Table 2
Matrix elements of N.

	O_1	O_2	O_3	O_4
$^4N[\mathbf{70}, 2^+]\frac{7}{2}^+$	N_c	$\frac{2}{3}$	$-\frac{1}{6N_c}(N_c+1)$	$\frac{5}{2N_c}$
$^2N[\mathbf{70}, 2^+]\frac{5}{2}^+$	N_c	$\frac{2}{9N_c}(2N_c-3)$	0	$\frac{1}{4N_c^2}(N_c+3)$
$^4N[\mathbf{70}, 2^+]\frac{5}{2}^+$	N_c	$-\frac{1}{9}$	$\frac{5}{12N_c}(N_c+1)$	$\frac{5}{2N_c}$
$^4N[\mathbf{70}, 0^+]\frac{3}{2}^+$	N_c	0	0	$\frac{5}{2N_c}$
$^2N[\mathbf{70}, 2^+]\frac{3}{2}^+$	N_c	$-\frac{1}{3N_c}(2N_c-3)$	0	$\frac{1}{4N_c^2}(N_c+3)$
$^4N[\mathbf{70}, 2^+]\frac{3}{2}^+$	N_c	$-\frac{2}{3}$	0	$\frac{5}{2N_c}$
$^2N[\mathbf{70}, 0^+]\frac{1}{2}^+$	N_c	0	0	$\frac{1}{4N_c^2}(N_c+3)$
$^4N[\mathbf{70}, 2^+]\frac{1}{2}^+$	N_c	-1	$-\frac{7}{12N_c}(N_c+1)$	$\frac{5}{2N_c}$

[6] The irreducible spherical tensors are defined according to Ref. [29].

Table 3
Matrix elements of Δ.

	O_1	O_2	O_3	O_4
$^2\Delta[\mathbf{70},2^+]\frac{5}{2}^+$	N_c	$-\frac{2}{9}$	0	$\frac{1}{N_c}$
$^2\Delta[\mathbf{70},2^+]\frac{3}{2}^+$	N_c	$\frac{1}{3}$	0	$\frac{1}{N_c}$
$^2\Delta[\mathbf{70},0^+]\frac{1}{2}^+$	N_c	0	0	$\frac{1}{N_c}$

We apply the $1/N_c$ counting rules presented in Refs. [13] and [16] and use their conclusions in selecting the most dominant operators in the practical analysis. For non-strange baryons, Table I of Ref. [13] gives a list of 18 linearly independent operators. If the core is excited the number of operators appearing in the mass formula is much larger. However due to lack of data, here we have to consider a restricted list. The selection is suggested by the conclusion of Ref. [13], ($N_f = 2$) and of Ref. [16] ($N_f = 3$), that only a few operators, of some specific structure, bring a dominant contribution to the mass. Following the notation of Ref. [16] these are O_1, O_2, O_3 and O_4 exhibited here in Table I. The first is the trivial operator of order $\mathcal{O}(N_c)$. The second is the 1-body part of the spin-orbit operator of order $\mathcal{O}(1)$ which acts on the excited quark. The third is a composite 2-body operator formally of order $\mathcal{O}(1)$ as well. It involves the tensor operator (12) acting on the excited quark and the SU(6) generators g^{ia} acting on the excited quark and G_c^{ja} acting on the core. The latter is a coherent operator which introduces an extra power N_c so that the order of O_3 is $\mathcal{O}(1)$, as it can be seen from Table 2. In order to take into account its contribution we have applied the rescaling introduced in Ref. [16] which consists in introducing a multiplicative factor of 3. Without this factor the coefficient c_3 becomes too large, as noticed in Ref. [16] [7]. The dynamics of the operator O_3 is less understood. Previous studies [13, 16] speculate about its connection to a flavor exchange mechanism [30, 31] related to long distance meson exchange interactions. Finally, the last operator is the spin-spin interaction, the only one of order $\mathcal{O}(1/N_c)$ which we consider here. Higher order operators are neglected. Accordingly the mass operator of the $[\mathbf{70},\ell^+]$ multiplet is approximated by

$$M_{[\mathbf{70},\ell^+]} = \sum_{i=1}^{4} c_i O_i \ , \tag{13}$$

where the coefficients c_i have to be found in a numerical fit to the available data, as described below. The diagonal matrix elements of the operators O_i ($i = 1, ..., 4$) are given in Tables 2 and 3. They have been obtained from the wave functions (4) and (5). Each matrix element contains the additional contribution of the two terms of the wave function. We derived general analytic formulae for each case in terms of N_c. For the first term which has a ground state core, we used Eq. (6) to recover the matrix elements of Appendix A of Ref. [13]. For the second term, using Eq. (8), we have derived analytic expressions which generalize those of Ref. [13] to an excited core with angular momentum $\ell_c \neq 0$.

[7] Alternatively the factor 3 could be included in the definition (12) of the tensor operator, as sometimes done in the literature. In practice what it matters is the product $c_i O_i$.

Taking $\ell_c = 0$ we reobtained the corresponding expressions of Ref. [13]. Details will be given elsewhere.

4. Fit and conclusions

In Table 4 we present the masses of the resonances which we have interpreted as belonging to the $[\mathbf{70}, 0^+]$ or to the $[\mathbf{70}, 2^+]$ multiplet. For simplicity, mixing of multiplets is neglected in this first attempt. The resonances shown in column 8, correspond to either three stars ("very likely") or to two stars ("fair") or to one star ("poor") status, according to Particle Data Group (PDG) [32]. Therefore we used the full listings to determine a mass average in each case. The experimental error to the mass was calculated as the quadrature of two uncorrelated errors, one being the average error from the same references and the other was the difference between the average mass and the farthest observed mass. For the $P_{11}(2100)$* resonance we report results from fitting the experimental value of Ref. [33], as being more recent than the average over the PDG values. Note that the observed mass of Ref. [33] is in agreement with the recent coupled channel analysis of Manley and Saleski [34].

Several remarks are in order. Due to its large error in the mass, the resonance $F_{15}(2000)$ could be either described by the $|^2N[\mathbf{70}, 2^+]5/2^+\rangle$ state or by the $|^4N[\mathbf{70}, 2^+]5/2^+\rangle$ state (inasmuch as they appear separated by about 60-70 MeV only, in quark model studies, see, *e.g.*, [35, 36]). Here we identified $F_{15}(2000)$ with the $|^4N[\mathbf{70}, 2^+]5/2^+\rangle$ state because it gives a better fit. Regarding the $F_{35}(1905)$ resonance there is also some ambiguity. In Ref. [19] it was identified as a $|^4\Delta[\mathbf{56}, 2^+]5/2^+\rangle$ state following Ref. [35], but in Ref. [22] the interpretation $|^4\Delta[\mathbf{70}, 2^+]5/2^+\rangle$ was preferred due to a better χ^2 fit and other considerations related to the decay width. Here we return to the identification made in Ref. [19] and assign the $|^4\Delta[\mathbf{70}, 2^+]5/2^+\rangle$ state to the second resonance from this sector, namely $F_{35}(2000)$, as indicated in Table 4. We hope that an analysis based on configuration mixing and improved data could better clarify the resonance assignment in this sector in the future. Presently the resulting $\chi^2_{\rm dof}$ is about 0.83 and the fitted values of c_i are given in Table 1. Besides the seven fitted masses Table 4 also contains few predictions.

We found that the contributions of $S_c^i S_c^i$ and $s^i S_c^i$ are nearly equal when treated as independent operators. Therefore, for simplicity, we assumed that they have the same coefficient in the mass operator. One can see that the spin-spin interaction given by O_4 is the dominant interaction, as in the $[\mathbf{56}, 2^+]$ multiplet [19] or in the $[\mathbf{56}, 4^+]$ multiplet [22]. Thus the main contributions to the mass come from O_1 and O_4. It is remarkable that c_1 and c_4 of the multiplets $[\mathbf{56}, 2^+]$ and $[\mathbf{70}, \ell^+]$, both located in the $N = 2$ band, are very close to each other. In terms of the present notation the result of Ref. [19] for $[\mathbf{56}, 2^+]$ is $c_1 = 541 \pm 4$ MeV and $c_4 = 241 \pm 14$ MeV as compared to $c_1 = 555 \pm 11$ MeV and $c_4 = 261 \pm 47$ MeV here. Such similarity gives confidence in the large N_c approach and in the present fit. In Fig. 1 the presently known values of c_1, c_2 and c_4 with error bars are represented for the excited bands studied within the large N_c expansion: $N = 1$ is from Ref. [16], $N = 2$ from Ref. [19] and from the present work, $N = 4$ from Ref. [22]. Extrapolation to higher energies, $N > 4$, suggests that the contribution of the spin dependent operators would vanish, while the linear term in N_c, which in a quark model

Table 4: The partial contribution and the total mass (MeV) predicted by the $1/N_c$ expansion as compared with the empirically known masses.

	$1/N_c$ expansion results						
	Partial contribution (MeV)				Total (MeV)	Empirical (MeV)	Name, status
	c_1O_1	c_2O_2	c_3O_3	c_4O_4			
$^4N[\mathbf{70},2^+]\frac{7}{2}^+$	1665	31	42	217	1956 ± 95	2016 ± 104	$F_{17}(1990)$**
$^2N[\mathbf{70},2^+]\frac{5}{2}^+$	1665	10	0	43	1719 ± 34		
$^4N[\mathbf{70},2^+]\frac{5}{2}^+$	1665	-5	-106	217	1771 ± 88	1981 ± 200	$F_{15}(2000)$**
$^4N[\mathbf{70},0^+]\frac{3}{2}^+$	1665	0	0	217	1883 ± 17	1879 ± 17	$P_{13}(1900)$**
$^2N[\mathbf{70},2^+]\frac{3}{2}^+$	1665	-16	0	43	1693 ± 42		
$^4N[\mathbf{70},2^+]\frac{3}{2}^+$	1665	-31	0	217	1851 ± 69		
$^2N[\mathbf{70},0^+]\frac{1}{2}^+$	1665	0	0	43	1709 ± 25	1710 ± 30	$P_{11}(1710)$***
$^4N[\mathbf{70},2^+]\frac{1}{2}^+$	1665	-47	149	217	1985 ± 26	1986 ± 26	$P_{11}(2100)$*
$^2\Delta[\mathbf{70},2^+]\frac{5}{2}^+$	1665	-10	0	87	1742 ± 29	1976 ± 237	$P_{35}(2000)$**
$^2\Delta[\mathbf{70},2^+]\frac{3}{2}^+$	1665	16	0	87	1768 ± 38		
$^2\Delta[\mathbf{70},0^+]\frac{1}{2}^+$	1665	0	0	87	1752 ± 19	1744 ± 36	$P_{31}(1750)$*

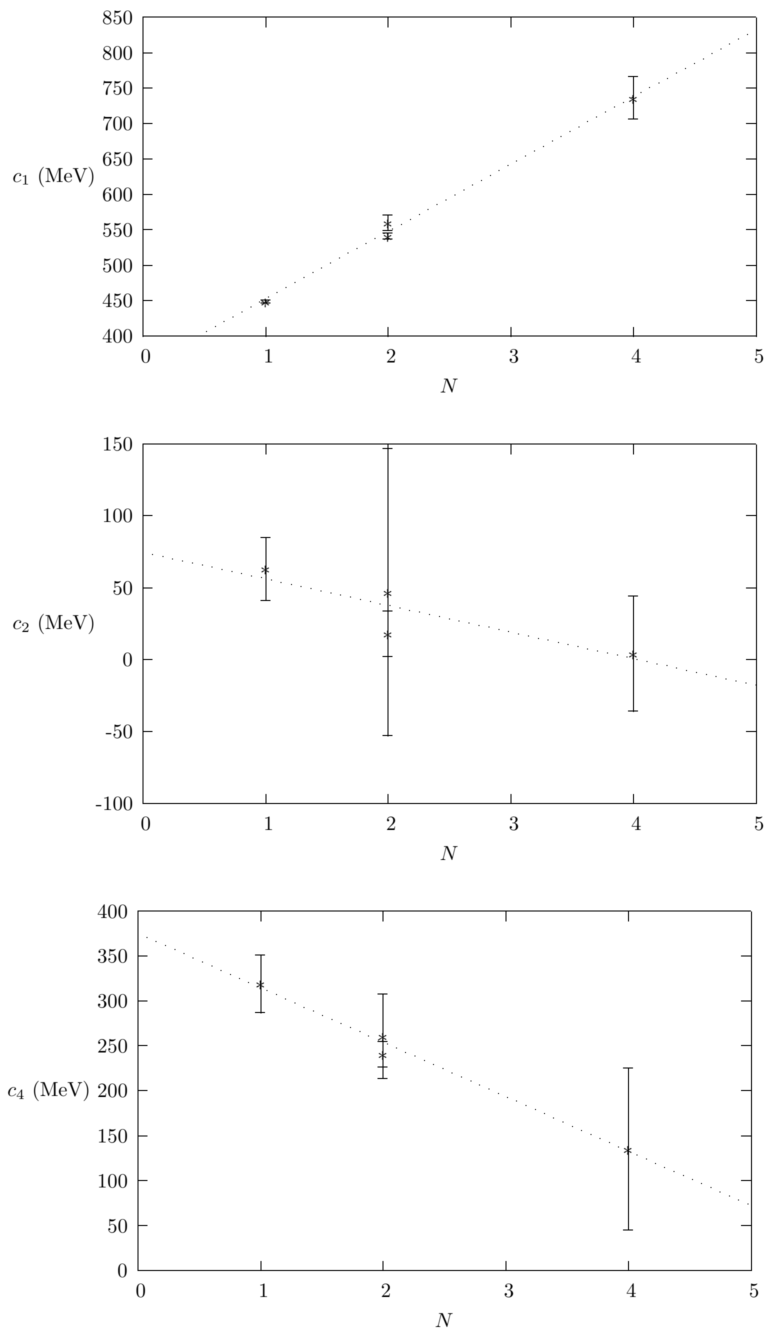

Figure 1. The coefficients c_i vs N from various sources: for $N = 1$ from Ref. [16] for $N = 2$ from Ref. [19] (lower values) and present work (upper values), for $N = 4$ from Ref. [22]. The straight lines are to guide the eye.

picture would contain the free mass term, the kinetic and the confinement energy, would carry the entire excitation. Such a behaviour gives a deeper insight into the large N_c mass operator and is consistent with the intuitive picture developed in Ref. [37] where at high energies the spin dependent interactions vanish as a consequence of the chiral symmetry restoration.

Finally note that the contributions of O_2 and O_3 lead to large errors in the coefficients c_i obtained in the χ^2 fit, which could possibly be removed with better data. The operator O_3 containing the tensor term plays an important role in the reduction of $\chi^2_{\rm dof}$ and it should be further investigated.

In conclusion we believe that the mass fit of the $[\mathbf{70}, \ell^+]$ baryons is encouraging in the present form. It is consistent with the conclusions of previous studies that the dominant interaction is of a spin-spin nature. It would be interesting to extend this analysis such as to incorporate configuration mixing with the $[\mathbf{56}, 2^+]$ states. In this way one could obtain a complete description of the $N = 2$ band baryons, leaving alone the $[\mathbf{20}, 1^+]$ multiplet for which no candidate has been found so far. On the other hand improved experimental data is highly desirable.

Acknowledgments

We are most grateful to Aneesh Manohar and Carlos Schat for a careful reading of the manuscript and good advise for the numerical fit. Useful correspondence with José Goity and Norberto Scoccola is gratefully acknowledged. We thank Pierre Stassart for useful comments. Both of us benefited of hospitality at ECT* Trento where we could have fruitful discussions on this subject. The work of one of us (N. M.) was supported by the Institut Interuniversitaire des Sciences Nucléaires (Belgium).

Appendix

Here we express a wave function of a given symmetry described by the partition $[f]$ by using fractional parentage coefficients. They are related to the isoscalar factors of the permutation group [27]. For one-body operators we need one-body fractional parentage coefficients. In this way one can decouple the last particle from the rest. In the simple case where the spatial wave function contains only one excited quark, for example having the structure $(0s)^{N_c-1}(0d)$ (two units of orbital excitation), and symmetry $[N_c - 1, 1]$ one can show that

$$
\begin{aligned}
|[N_c-1,1](0s)^{N_c-1}(0d), 2^+\rangle &= \sqrt{\frac{N_c-1}{N_c}}\Psi_{[N_c-1]}(0s)^{N_c-1}\phi_{[1]}(0d) \\
&-\sqrt{\frac{1}{N_c}}\Psi_{[N_c-1]}\left((0s)^{N_c-2}(0d)\right)\phi_{[1]}(0s)\ . \qquad (14)
\end{aligned}
$$

In the case of the spin-orbit operator one can see that only the first term contributes (the operator acts on the last particle only). Its matrix element is proportional to the square of the coefficient of the first term, *i.e.* with $\frac{N_c-1}{N_c}$ which for large N_c gives to the spin-orbit the order $\mathcal{O}(1)$ in powers of $1/N_c$.

On the other hand for a symmetric state $[N_c]$ with the same structure one obtains

$$|[N_c](0s)^{N_c-1}(0d),2^+\rangle = \sqrt{\frac{1}{N_c}}\Psi_{[N_c-1]}(0s)^{N_c-1}\phi_{[1]}(0d) + \sqrt{\frac{N_c-1}{N_c}}\Psi_{[N_c-1]}\left((0s)^{N_c-2}(0d)\right)\phi_{[1]}(0s)\ , \quad (15)$$

which implies that the spin-orbit operator is of order $\mathcal{O}(1/N_c)$. Both results are in agreement with Eq. (2).

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