B mesons and form factors

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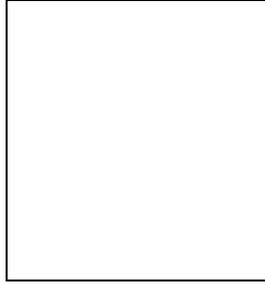
In order to extract some information on the CKM matrix element $|V_{cb}|$, we have to determine form factors in B decays. For this, we show that it is relevant to consider the non-forward amplitude between the heavy-light B and D mesons within the Heavy Quark Effective Theory (HQET). This method provides us crucial information on the shape of the elastic Isgur-Wise (IW) function $\xi(w)$. As examples, one gets that the n-th derivative of $\xi(w)$ at $w = 1$ can be bounded by the $(n-1)$-th one and one obtains an absolute lower bound for the n-th derivative $(-1)^n \xi^{(n)}(1) \geq \frac{(2n+1)!!}{2^{2n}}$. These bounds should be taken into account in the parametrizations of $\xi(w)$ used to extract $|V_{cb}|$. Besides, we have also obtained new information on subleading functions at the order $O(1/m_Q)$ where m_Q is the heavy quark mass, namely $\xi_3(w)$ and $\overline{\Lambda}\xi(w)$.

1 Introduction

In the leading order of the heavy quark expansion of QCD, Bjorken sum rule (SR) [1] relates the slope of the elastic IW function $\xi(w)$ to the IW functions of transition $\tau_{1/2}^{(n)}(w)$ and $\tau_{3/2}^{(n)}(w)$ between the ground state $j^P = \frac{1}{2}^-$ and the $j^P = \frac{1}{2}^+$, $\frac{3}{2}^+$ excited states at zero recoil where $w = 1$ (j is the total angular momentum of the light cloud and n is a radial quantum number). This SR leads to the lower bound $-\xi'(1) = \rho^2 \geq \frac{1}{4}$. Recently, a new SR [2] was formulated by Uraltsev in the heavy quark limit involving also $\tau_{1/2}^{(n)}(1)$ and $\tau_{3/2}^{(n)}(1)$ that implies, combined with Bjorken SR, the much stronger lower bound $\rho^2 \geq \frac{3}{4}$. A basic ingredient in deriving this bound is the consideration of the non-forward amplitude $B(v_i) \to D^{(n)}(v') \to B(v_f)$, allowing for general v_i, v_f, v' and where B is a ground state meson. In order to make a systematic study in the heavy quark limit of QCD, a manifestly covariant formalism within Operator Product

Expansion (OPE) has been developed[3], using the matrix representation for the whole tower of heavy meson states[4]. In this formalism, Uraltsev SR has been recovered plus a new class of SR that allows to bound also higher derivatives of the IW function. Making a natural physical assumption, this new class of SR implies the bound $\sigma^2 \geq \frac{5}{4}\rho^2$ where σ^2 is the curvature of the IW function. Using this formalism including the whole tower of excited states j^P, the rigorous bound $\sigma^2 \geq \frac{5}{4}\rho^2$ has been recovered plus generalizations that extend it to all the derivatives of the IW function $\xi(w)$ at zero recoil, that is shown to be an alternate series in power of $(w-1)$. After reviewing the corresponding sum rules in the heavy quark limit of QCD, the object of the present paper is to extend the formalism to IW functions at subleading order in $1/m_Q$.

2   Sum rules in the heavy quark limit of QCD

Using the OPE and the trace formalism in the heavy quark limit, different initial and final four-velocities v_i and v_f, and heavy quark currents $J_1 = \overline{h}_{v'}^{(c)}\Gamma_1 h_{v_i}^{(b)}$, $J_2 = \overline{h}_{v_f}^{(b)}\Gamma_2 h_{v'}^{(c)}$ where Γ_1 and Γ_2 are arbitrary Dirac matrices, the following sum rule can be written[3]:

$$\{\Sigma_{D=P,V}\Sigma_n Tr[\overline{\mathcal{B}}_f(v_f)\overline{\Gamma}_2\mathcal{D}^{(n)}(v')]$$

$$Tr[\overline{\mathcal{D}}^{(n)}(v')\Gamma_1\mathcal{B}_i(v_i)]\xi^{(n)}(w_i)\xi^{(n)}(w_f)$$

$$+ \; Other \; excited \; states\}$$

$$= -2\xi(w_{if})Tr[\overline{\mathcal{B}}_f(v_f)\overline{\Gamma}_2 P'_+\Gamma_1\mathcal{B}_i(v_i)].$$

(1)

In this formula, v' is the intermediate charmed meson four-velocity, $P'_+ = \frac{1}{2}(1 + v')$ is the positive energy projector on the intermediate c quark, $\xi(w_{if})$ is the elastic IW function that appears because one assumes $v_i \neq v_f$. $\mathcal{B}_i$ and $\mathcal{B}_f$ are the 4×4 matrices of the ground state B or B^* mesons, respectively the pseudoscalar and the vector of the doublet $\frac{1}{2}^-$, while $\mathcal{D}^{(n)}$ stands for all possible ground state or excited state D mesons coupled to B_i and B_f through the currents. In (1) we have made explicit the $j = \frac{1}{2}^-$ D and D^* mesons and their radial excitations of quantum number n. The variables w_i, w_f and w_{if} are defined as $w_i = v_i.v'$, $w_f = v_f.v'$, $w_{if} = v_i.v_f$ and vary within the domain[3]:

$$w_i \geq 1 \qquad , \qquad w_f \geq 1$$

$$w_i w_f - \sqrt{(w_i^2 - 1)(w_f^2 - 1)} \leq w_{if} \leq w_i w_f + \sqrt{(w_i^2 - 1)(w_f^2 - 1)}.$$

(2)

The general SR (1), obtained from the OPE, can be written in the following compact way :

$$L_{Hadrons}(w_i, w_f, w_{if}) = R_{OPE}(w_i, w_f, w_{if})$$

(3)

where the l.h.s. is the sum over the intermediate charmed D states, while the r.h.s. is the OPE counterpart. Within the domain (2), one can derive relatively to any of the variables w_i, w_f and w_{if} and obtain different SR taking different limits to the frontier of the domain.

On the one hand, we can choose, as in ref.[3], the B pseudoscalar meson $\mathcal{B}(v) = P_+(-\gamma_5)$ as initial and final states and vector currents projected along the v_i and v_f four-velocities :

$$J_1 = \overline{h}_{v'}^{(c)} \; \rlap{/}{v}_i h_{v_i}^{(b)} \qquad , \qquad J_2 = \overline{h}_{v_f}^{(b)} \; \rlap{/}{v}_f h_{v'}^{(c)}.$$

(4)

We then obtain SR (1) with the sum of all excited states j^P in a compact form :

$$(w_i + 1)(w_f + 1)\Sigma_{l\geq 0}\frac{l+1}{2l+1}S_l(w_i, w_f, w_{if})\Sigma_n \tau_{l+1/2}^{(l)(n)}(w_i)\tau_{l+1/2}^{(l)(n)}(w_f)$$

$$+\Sigma_{l\geq 1}S_l(w_i, w_f, w_{if})\Sigma_n \tau_{l-1/2}^{(l)(n)}(w_i)\tau_{l-1/2}^{(l)(n)}(w_f) \tag{5}$$

$$= (1 + w_i + w_f + w_{if})\xi(w_{if}).$$

On the other hand, choosing instead the axial currents,

$$J_1 = \overline{h}_{v'}^{(c)} \not{b}_i\gamma_5 h_{v_i}^{(b)} \quad , \quad J_2 = \overline{h}_{v_f}^{(b)} \not{b}_f\gamma_5 h_{v'}^{(c)} \tag{6}$$

we get the following SR :

$$\Sigma_{l\geq 0}S_{l+1}(w_i, w_f, w_{if})\Sigma_n \tau_{l+1/2}^{(l)(n)}(w_i)\tau_{l+1/2}^{(l)(n)}(w_f)$$

$$+(w_i - 1)(w_f - 1)\sum_{l\geq 1}\frac{l}{2l-1}S_{l-1}(w_i, w_f, w_{if})\Sigma_n \tau_{l-1/2}^{(l)(n)}(w_i)\tau_{l-1/2}^{(l)(n)}(w_f) \tag{7}$$

$$= -(1 - w_i - w_f + w_{if})\xi(w_{if}).$$

In (5) and (7), the IW functions $\tau_{l+1/2}^{(l)(n)}(w)$ and $\tau_{l-1/2}^{(l)(n)}(w)$ have been defined in ref. [3] where l and $j = l \pm \frac{1}{2}$ are the orbital and the total angular momentum of the light cloud and S_n is given by

$$S_n(w_i, w_f, w_{if}) = v_{i\nu_1} \ldots v_{i\nu_n} v_{f\mu_1} \ldots v_{f\mu_n} \Sigma_\lambda \epsilon'^{(\lambda)*\nu_1\ldots\nu_n} \epsilon'^{(\lambda)\mu_1\ldots\mu_n} \tag{8}$$

which can take the following expression [3]

$$S_n(w_i, w_f, w_{if}) = \Sigma_{0\leq k\leq \frac{n}{2}}C_{n,k}(w_i^2 - 1)^k(w_f^2 - 1)^k(w_iw_f - w_{if})^{n-2k} \tag{9}$$

with

$$C_{n,k} = (-1)^k\frac{(n!)^2}{(2n)!}\frac{(2n - 2k)!}{k!(n - k)!(n - 2k)!}. \tag{10}$$

On the one hand, making the sum of both equations (5) and (7), one obtains, differentiating relatively to w_{if} :

$$\xi^{(l)}(1) = \frac{1}{4}(-1)^l l!\{\frac{l + 1}{2l + 1}4\Sigma_n[\tau_{l+1/2}^{(l)(n)}(1)]^2 + \Sigma_n[\tau_{l-1/2}^{(l-1)(n)}(1)]^2 + \Sigma_n[\tau_{l-1/2}^{(l)(n)}(1)]^2\} \quad (l \geq 0). \tag{11}$$

This relation shows that $\xi(w)$ is an alternate series in power of $(w - 1)$ because of the factor $(-1)^l$ and reduces to Bjorken SR [1] for $l = 1$.

On the other hand, differentiating (7) relatively to w_{if} and making $w_i = w_f = w_{if} = 1$, one has the simple formula :

$$\xi^{(l)}(1) = (-1)^l l!\Sigma_n[\tau_{l+1/2}^{(l)(n)}(1)]^2 \quad (l \geq 0). \tag{12}$$

Combining (11) and (12), one obtains the relation among the IW functions :

$$\frac{l}{2l + 1}\Sigma_n[\tau_{l+1/2}^{(l)(n)}]^2 - \frac{1}{4}\Sigma_n[\tau_{l-1/2}^{(l)(n)}]^2 = \frac{1}{4}\Sigma_n[\tau_{l-1/2}^{(l-1)(n)}]^2 \tag{13}$$

that reduces to Uraltsev SR [2] for $l = 1$ and generalizes it for all l. Replacing now $\Sigma_n[\tau_{l+1/2}^{(l)(n)}]^2$ from the expression (12) into the generalization of Bjorken SR (11), one obtains :

$$(-1)^l\xi^{(l)}(1) = \frac{1}{4}\frac{2l + 1}{l}l!\{\Sigma_n[\tau_{l-1/2}^{(l-1)(n)}(1)]^2 + \Sigma_n[\tau_{l-1/2}^{(l)(n)}(1)]^2\} \tag{14}$$

which implies the lower bounds :

$$(-1)^l \xi^{(l)}(1) \geq \frac{2l+1}{4}(-1)^{l-1}\xi^{(l-1)}(1)$$

$$\geq \frac{(2l+1)!!}{2^{2l}} \tag{15}$$

that give, in particular, for the lower cases,

$$-\xi'(1) = \rho^2 \geq \frac{3}{4} \quad , \quad \xi''(1) \geq \frac{15}{16}. \tag{16}$$

Considering systematically the derivatives of the SR (5) and (7) relatively to w_i, w_f and w_{if} with the boundary conditions $w_i = w_f = w_{if} = 1$, one obtains a new SR :

$$\frac{4}{3}\rho^2 + (\rho^2)^2 - \frac{5}{3}\sigma^2 + \sum_{n \neq 0} |\xi^{(n)'}(1)|^2 = 0 \tag{17}$$

that in turn implies :

$$\sigma^2 \geq \frac{1}{5}[4\rho^2 + 3(\rho^2)^2] \tag{18}$$

There is a simple intuitive argument to understand the last term in the bound (18), namely the non-relativistic light quark q interacting with a heavy quark Q through a potential. One can indeed prove that [3]

$$\sigma_{NR}^2 \geq \frac{3}{5}(\rho_{NR}^2)^2. \tag{19}$$

Thus, the non-relativistic limit is a good guide-line to study the shape of the IW function $\xi(w)$. We have recently generalized the bound (19) to all the derivatives of $\xi_{NR}(w)$[5]. The method uses the positivity of matrices of moments of the ground state wave function. We have also shown that the method can be generalized to the real function $\xi(w)$ of QCD.

An interesting phenomenological remark is that the simple parametrization for the IW function [6]

$$\xi(w) = (\frac{2}{w+1})^{2\rho^2} \tag{20}$$

satisfies the inequalities (15) and (18) if $\rho^2 \geq \frac{3}{4}$. The result (15), that shows that all derivatives at zero recoil are large, should have important phenomenological implications for the empirical fit needed for the extraction of $|V_{cb}|$ in the semileptonic exclusive decay mode $B \to D^* l \nu$.

3   Sum rules for subleading form factors

We wish now to obtain new sum rules involving subleading quantities in $1/m_Q$ where m_Q is the heavy quark mass [7]. For this, we can perturb the generic SR (3) by $1/m_c$ and $1/m_b$ terms. The perturbation of the r.h.s, the OPE side, is parametrized by six new subleading IW functions concerning the ground state $\frac{1}{2}^-$ and denoted by $L_i(w)$ $(i = 1, \ldots, 6)$ [8]. As for the l.h.s., and considering as intermediate D states the multiplets $\frac{1}{2}^-$, $\frac{1}{2}^+$, $\frac{3}{2}^+$ (higher j^P intermediate states do not in fact contribute [7]), we have three types of matrix elements coresponding to the hadronic transitions $B \to D(\frac{1}{2}^-)$, $B \to D(\frac{1}{2}^+)$ and $B \to D(\frac{3}{2}^+)$. The corrections in $1/m_b$ or $1/m_c$ to the first matrix element are given by the same ground state subleading IW functions $L_i(w)$, while the $O(1/m_b)$ and $O(1/m_c)$ corrections to the matrix elements $B \to D(\frac{1}{2}^+)$, $D(\frac{3}{2}^+)$ have been carefully studied [9] and result in a number of new subleading IW functions. All these corrections are of two types, perturbations of the heavy quark current and perturbations of the Lagrangian.

Beginning with the general SR in the heavy quark limit and perturbing the heavy quark limit matrix elements with $1/m_b$ and $1/m_c$ corrections, the general expression could then be written, making explicit the leading and the $1/m_c$ and $1/m_b$ parts, as :

$$G_0(w_i, w_f, w_{if}) + E_0(w_i, w_f, w_{if}) + \frac{1}{2m_b}[G_b(w_i, w_f, w_{if}) + E_b(w_i, w_f, w_{if})]$$

$$+ \frac{1}{2m_c}[G_c(w_i, w_f, w_{if}) + E_c(w_i, w_f, w_{if})] \tag{21}$$

$$= R_0(w_i, w_f, w_{if}) + \frac{1}{2m_b}R_b(w_i, w_f, w_{if}) + \frac{1}{2m_c}R_c(w_i, w_f, w_{if})$$

where the subindex 0 means the heavy quark limit, while the subindex b or c correspond to the subleading corrections in $1/m_b$ or $1/m_c$, and G or E mean, respectively, ground state or excited state contributions.

In the heavy quark limit, on has :

$$G_0(w_i, w_f, w_{if}) + E_0(w_i, w_f, w_{if}) = R_0(w_i, w_f, w_{if}) \tag{22}$$

that leads to the results quoted in the section 2.

In expression (21), we can vary m_b and m_c as independent parameters and obtain new SR for the subleading quantities. To obtain information on the $1/m_b$ corrections, it is relatively simple to proceed as follows by assuming the formal limit :

$$m_c >> m_b >> \Lambda_{QCD} \tag{23}$$

and perturb both sides of the SR (22) by $1/m_b$ terms. In this limit, one obtains the relation :

$$G_b(w_i, w_f, w_{if}) + E_b(w_i, w_f, w_{if}) = R_b(w_i, w_f, w_{if}). \tag{24}$$

One can then compute $G_b(w_i, w_f, w_{if})$ and $E_b(w_i, w_f, w_{if})$ using respectively the formalism of Falk and Neubert [8] and the the one of Leibovich et al. [9], and obtain SR for the different subleading IW functions $L_i(w)$.

Choosing the axial currents $\Gamma_1 = \not{v}_i \gamma_5$ and $\Gamma_2 = \not{v}_f \gamma_5$ and taking as initial and final states respectively the ground state pseudoscalar and vector mesons at different four-velocities, one obtains the two sum rules [7] :

$$L_4(w) = -6\Sigma_n \Delta E_{1/2}^{(n)} \tau_{1/2}^{(n)}(1) \tau_{1/2}^{(n)}(w)$$

$$-L_5(w) + (w+1)L_6(w) = 2\Sigma_n \Delta E_{1/2}^{(n)} \tau_{1/2}^{(n)}(1) \tau_{1/2}^{(n)}(w) - 4(w+1)\Sigma_n \Delta E_{3/2}^{(n)} \tau_{3/2}^{(n)}(1) \tau_{3/2}^{(n)}(w). \tag{25}$$

The functions $L_i(w)$ ($i = 4, 5, 6$) are not independent but are given in terms of two functions [8], namely the elastic IW function $\xi(w)$, a subleading function $\xi_3(w)$ and the fundamental parameter of HQET $\overline{\Lambda}$ ($\overline{\Lambda} = m_B - m_b$, which stands for the energy of the light cloud). Therefore, we finally obtain the relations, valid for all w :

$$\overline{\Lambda}\xi(w) = 2(w+1)\Sigma_n \Delta E_{3/2}^{(n)} \tau_{3/2}^{(n)}(1) \tau_{3/2}^{(n)}(w) + 2\Sigma_n \Delta E_{1/2}^{(n)} \tau_{1/2}^{(n)}(1) \tau_{1/2}^{(n)}(w) \tag{26}$$

$$\xi_3(w) = (w+1)\Sigma_n \Delta E_{3/2}^{(n)} \tau_{3/2}^{(n)}(1) \tau_{3/2}^{(n)}(w) - 2\Sigma_n \Delta E_{1/2}^{(n)} \tau_{1/2}^{(n)}(1) \tau_{1/2}^{(n)}(w). \tag{27}$$

These new SR reduce to known SR for $w = 1$, for $\overline{\Lambda}$ obtained by Voloshin [10], and for $\xi_3(1)$ obtained by Le Yaouanc et al. [11] and by Uraltsev [2], and generalize them to all values of w.

Therefore, we have shown that the subleading quantities $\overline{\Lambda}\xi(w)$ and $\xi_3(w)$, that are functions of w, can be expressed in terms of leading quantities, namely the transition IW functions $\tau_j^{(n)}$

and the corresponding level spacings $\Delta E_j^{(n)}$ ($j = \frac{1}{2}, \frac{3}{2}$). We can now discuss phenomenological applications of these results, in particular the check of Bakamjian-Thomas quark models. Within this scheme, $\xi(w)$ is given by (20) with $\rho^2 = 1.02$, while one gets, for the $n = 0$ states [6] :

$$\tau_j^{(0)}(w) = \tau_j^{(0)}(1)(\frac{2}{w+1})^{2\sigma_j^2} \tag{28}$$

with $\tau_{1/2}^{(0)}(1) = 0.22$, $\sigma_{1/2}^2 = 0.83$, $\tau_{3/2}^{(0)}(1) = 0.54$ and $\sigma_{3/2}^2 = 1.50$. Assuming the reasonable saturation of the SR with the lowest $n = 0$ states, one gets, from the first relation (26), a sensibly constant value for $\overline{\Lambda} = 0.513 \pm 0.015$.

4 Conclusion

Using sum rules in the heavy quark limit of QCD, as formulated in ref. [3], lower bounds for the derivatives of the elastic IW function $\xi(w)$ have been found. Any phenomenological parametrization of $\xi(w)$ intending to fit the CKM matrix element $|V_{cb}|$ in $B \to D^{(*)}l\nu$ should satisfy these bounds. Moreover, we have found non-trivial new information on subleading contributions in $1/m_Q$. As a result, the fundamental quantity of HQET $\overline{\Lambda}$ appears to be a ratio of leading functions and $\xi_3(w)$ is also given in terms of leading quantities. To proceed further phenomenologically, we use as an Ansatz for these functions the results of the Bakamjian-Thomas quark models, that gives covariant form factors in the heavy quark limit, satisfies IW scaling and also Bjorken and Uraltsev sum rules. One obtains in this way for a very wide range of w the expected constancy for $\overline{\Lambda}$, with numerical value of the order of 0.5 and which is in agreement with other methods as, for example, the QCDSR approach.

Acknowledgments

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