

# What we (don't) know about black hole formation in high-energy collisions

Vitor Cardoso\*

McDonnell Center for the Space Sciences, Department of Physics,  
Washington University, St. Louis, Missouri 63130, USA †

Emanuele Berti‡

McDonnell Center for the Space Sciences, Department of Physics,  
Washington University, St. Louis, Missouri 63130, USA

Marco Cavaglia§

Department of Physics and Astronomy, University of Mississippi, University, MS 38677-1848, USA  
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Higher-dimensional scenarios allow for the formation of mini-black holes from TeV-scale particle collisions. The purpose of this paper is to review and compare different methods for the estimate of the total gravitational energy emitted in this process. To date, black hole formation has mainly been studied using an apparent horizon search technique. This approach yields only an upper bound on the gravitational energy emitted during black hole formation. Alternative calculations based on instantaneous collisions of point particles and black hole perturbation theory suggest that the emitted gravitational energy may be smaller. New and more refined methods may be necessary to accurately describe black hole formation in high-energy particle collisions.

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## I. INTRODUCTION

The standard model of particle physics has been successfully tested up to energies of  $\sim 1$  TeV. However, its foundations are still mysterious. Much effort has been spent to explain the hierarchy problem, i.e. the huge difference between the electroweak scale,  $m_{\text{EW}} \sim 300$  GeV, and the Planck scale,  $M_{\text{Pl}} \sim 10^{19}$  GeV. While electroweak interactions have been probed at distances  $m_{\text{EW}}^{-1} \sim 10^{-16}$  cm, gravitational forces have not been probed at distances  $M_{\text{Pl}}^{-1} \sim 10^{-33}$  cm. Gravity has only been accurately measured in the  $\sim 0.01$  cm range [1].

If gravity is modified at scales smaller than 1 mm, the hierarchy problem can be solved by assuming the existence of  $n$  compact extra dimensions of length  $\sim R$  [2]. Gauss's law in  $D = 4+n$  dimensions implies that two test masses  $m_1, m_2$  at a distance  $r \ll R$  feel a gravitational potential

$$V(r) = G_D \frac{m_1 m_2}{r^n}, \quad r \ll R, \quad (1)$$

where  $G_D$  is the  $D$ -dimensional Newton constant. The four-dimensional Newtonian potential is recovered at distances  $r \gg R$ , where  $r^n \sim R^n$  and  $G_4 = G_D/R^n$ . The  $D$ -dimensional Planck mass,  $M_{\text{Pl},D}$ , is obtained by equating the Schwarzschild radius of an object of mass  $m$  to

its Compton wavelength,  $\lambda = 1/m$ :

$$(M_{\text{Pl},D})^{D-2} \sim \frac{1}{G_D}. \quad (2)$$

(Here and throughout the paper we set  $\hbar = 1$  and  $c = 1$ .)  $M_{\text{Pl},D}$  is related to the effective four-dimensional Planck scale  $M_{\text{Pl}}$  by  $M_{\text{Pl}}^2 \sim (M_{\text{Pl},D})^{D-2} R^n$ . The hierarchy problem is solved by imposing equal scales in the higher-dimensional setting, i.e.,  $m_{\text{EW}} = M_{\text{Pl},D}$ . This condition relates the size  $R$  of the extra dimensions to the number of extra dimensions  $n$ . A single extra dimension implies deviations from Newtonian gravity over solar system distances. It is thus excluded empirically. If  $n = 2$ , the size of the extra dimensions is  $R \sim 0.3$  mm. This value has been recently ruled out by experiments with torsion pendulums [1]. However, any  $n \geq 3$  gives modifications of Newtonian gravity at distances smaller than those currently probed by experiment.

An exciting consequence of TeV-scale gravity is the possibility of production of black holes (BHs) in particle colliders [3, 4] and ultra high energy cosmic ray interactions with the atmosphere [5]. A naive estimate of the cross section for  $M_{\text{Pl},D} \sim 1$  TeV predicts that super-TeV particle colliders will produce BHs at a rate of few per second.

The BH starts to decay after forming. First, it radiates all the excess multipole moments. The BH has initially a highly asymmetric shape. It then settles down to a stationary state. (In four dimensions it must be a Kerr-Newman BH, by the uniqueness theorem.) This phase is commonly called *balding phase*. The endpoint of the balding phase is a spinning BH. This BH starts to lose angular momentum (*spin-down phase*). Page [6] has shown that a spinning BH radiates mainly along the equatorial

†Also at Centro de Física Computacional, Universidade de Coimbra, P-3004-516 Coimbra, Portugal

\*Electronic address: vcardoso@wugrav.wustl.edu

‡Electronic address: berti@wugrav.wustl.edu

§Electronic address: cavaglia@olemiss.edu

plane, and that this radiation carries away most of the BH angular momentum. After completing the spin-down phase, the BH radiates its remaining degrees of freedom through Hawking radiation (*Schwarzschild phase*) [7, 8]. The endpoint of the Schwarzschild phase is either explosive or a BH remnant is left [9].

The focus of this paper is on the formation of a BH in high-energy particle collisions. In particular, we are interested in the total gravitational energy radiated. Different methods have been discussed in the literature. The standard procedure is to describe the incoming particles by two Aichelburg-Sexl shock waves [10], and then find a closed trapped surface in the union of these shock waves [11–18]. The shock wave approach can be used to find lower bounds on the mass of the newly formed BH, i.e., upper bounds on the amount of gravitational radiation emitted during the process. However, there is little discussion in the literature on how well these results approximate the actual gravitational emission. The aim of this paper is to provide a critical discussion of the different methods. In Section II we describe briefly the standard Aichelburg-Sexl shock wave technique, the main results of this approach, and its shortcomings. In Section III we discuss two alternative techniques. Conclusions are presented in Section IV.

## II. BLACK HOLE FORMATION VIA AICHELBURG-SEXL SHOCK WAVES

Consider a  $D$ -dimensional Schwarzschild-Tangherlini solution with mass  $M$  [19]. The metric in spherical coordinates is

$$ds^2 = - \left( 1 - \frac{16\pi G_D M}{(D-2)\Omega_{D-2}} \frac{1}{r^{D-3}} \right) dt^2 + \left( 1 - \frac{16\pi G_D M}{(D-2)\Omega_{D-2}} \frac{1}{r^{D-3}} \right)^{-1} dr^2 + r^2 d\Omega_{D-2}^2 \quad (3)$$

where  $\Omega_{D-2}$  is the volume of the unit  $(D-2)$ -dimensional sphere

$$\Omega_{D-2} = \frac{2\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}, \quad (4)$$

and  $\Gamma[z]$  is Euler's Gamma function. Boosting this solution to very large values of the Lorentz factor  $\gamma$  at fixed total energy  $\mu = M\gamma$ , we obtain the Aichelburg-Sexl solution

$$ds^2 = -d\bar{u}d\bar{v} + (d\bar{x}^i)^2 + \Phi(\bar{x}^i)\delta(\bar{u})d\bar{u}^2, \quad (5)$$

where  $\Phi$  depends only on the transverse radius  $\bar{\rho}^2 = \bar{x}^i\bar{x}_i$ :

$$\begin{aligned} \Phi &= -8G_D\mu \log \bar{\rho}, \quad D = 4 \\ \Phi &= \frac{16\pi G_D\mu}{\Omega_{D-3}(D-4)\bar{\rho}^{D-4}}, \quad D > 4. \end{aligned} \quad (6)$$

Equation (5) describes a particle with energy  $\mu$  moving in the  $+z$ -direction. The spacetime is flat outside the null

TABLE I: Upper limits on the efficiency of gravitational radiation emission for different spacetime dimensions  $D$ , using the trapped surface method.

$D$	$\epsilon$ (%)
4	29.3
6	36.1
8	39.3
10	41.2
$\infty$	50

plane  $\bar{u} = 0$ . If we consider an identical shock wave traveling along  $\bar{v} = 0$  in the  $-z$  direction, the two solutions can be superposed to yield a solution for two colliding shock waves. BH formation can be studied by identifying a future trapped surface in the solution, with no need to calculate the gravitational field [11–18, 20]. If the collision is head-on, it is easy to find a trapped surface in the union of two flat disks with radii

$$\rho_c = \left( \frac{8\pi G_D\mu}{\Omega_{D-3}} \right)^{1/(D-3)}. \quad (7)$$

Since the BH horizon is always in the exterior region, the method gives a lower bound on the final BH mass. If the final BH is non-spinning, its mass is

$$M_{BH} \gtrsim \mu \left( \frac{(D-2)\Omega_{D-2}}{2\Omega_{D-3}} \right)^2. \quad (8)$$

The total gravitational energy radiated is  $2\mu - M_{BH}$ , and the efficiency is  $\epsilon \equiv 1 - M_{BH}/2\mu$ . An efficiency of 0% implies that no gravitational radiation is emitted. An upper limit on the efficiency follows from Eq. (8). Some values are shown in Table I.

The above formalism can be extended to the study of collisions with nonzero impact parameter [13–15, 17]. This makes possible the computation of the BH production cross-section.

### A. Accuracy of the results

The trapped surface method gives only a lower bound on the BH mass. The BH mass can be anything between this bound and the center of mass (c.m.) energy of the collision. D'Eath and Payne studied this problem in more detail for the four-dimensional case [12]. Bondi's news function, which describes the emission of gravitational radiation, can be written as an infinite series around the collision axis. The first term of the series is Eq. (8). Making some assumptions on the angular dependence of the radiation, and extrapolating off the axis, the second term is found to decrease the efficiency for gravitational wave generation in head-on collisions to 16%. Although this derivation relies on various approximations, the reduction in efficiency is significant, and may signal that

the series is slowly converging. The situation seems to get even worse in higher dimensions (see Section III) and for large impact parameters. As an illustration of how the trapped surface method may lead to an inaccurate estimate of the BH mass, let us consider the collision of two non-spinning BHs initially at rest. Using similar arguments to the trapped surface method, Hawking [21] placed an upper limit of 29.3% on the efficiency of gravitational wave generation. The exact result can be obtained through a numerical solution of Einstein's equations [22–28], and is around 0.1%, i.e. two orders of magnitude smaller than the Hawking bound. The results for ultra-relativistic collisions are likely to be more accurate. However, the total energy radiated may still be much smaller than the upper limits of Table I.

### B. Finite size particles

The Aichelburg-Sexl solution describes the metric of a massless pointlike particle with a very large boost. Classically, the point-particle assumption is accurate along the collision axis because of the large Lorentz contraction due to the boost. However, it fails for directions transversal to the motion. If the colliding particles are strings, string size effects can be modeled by considering beam-beam collisions, with beam sizes of order  $\lambda_S$ , where  $\lambda_S \gtrsim l_{Pl}$  [20]. The effects of finite-size transversal dimensions have been studied analytically by Kohlprath and Veneziano [20], and lead to a smaller cross section. (We are not aware of any numerical study.) For a reasonable cross section the c.m. energy should satisfy  $E > M_{Pl}(\lambda_S/l_{Pl})^{D-3}$ .

### C. Spin effects

In general, colliding particles have intrinsic spin and should be modeled by metrics other than the Tangherlini metric. A naive generalization of the Aichelburg-Sexl approach is to boost the rotating Myers-Perry metric [29]. This has been done by several authors [30]. The results are cumbersome enough to make the trapped surface method very difficult to implement. A seemingly better candidate to model spinning high-energy particles was recently proposed in Ref. [31]. This model consists in a spinning radiation beam-pulse which includes dragging effects, in contrast to the boosted Kerr metric.

Another question concerns the angular momentum of the BH, given the intrinsic spin and angular momentum of the colliding particles. (This is also an important point in astrophysics [32].) The cross section and the angular momentum of the final BH can be estimated by assuming the net angular momentum carried by gravitational waves to be negligible [8, 13]. However, a more accurate investigation of angular momentum effects is needed.

### D. Charge effects

String length and spin effects should be important if the incoming particles have energy close to the Planck scale. It is likely that both of these effects are suppressed for super-Planckian energies. Charge effects are expected to dominate at very high energies because gauge fields are confined on the brane and decay more slowly than the gravitational field of a neutral particle [33]. Estimates of charge effects in BH production from high-energy collisions are not yet available in the literature. A first attempt was presented in Ref. [34] using a perturbative method.

## III. OTHER METHODS TO ESTIMATE THE ENERGY RELEASED

Because of the problems listed above, it is desirable to explore different methods. In this section we discuss two possible approaches. Although they are not free from shortcomings, quantitative results on the BH formation process can be obtained. These techniques give consistent results in  $D = 4$ .

The formalism to handle gravitational waves in  $D$ -dimensional flat spacetimes is discussed in Ref. [35]. The nonlinearity of Einstein's equations makes the treatment of the gravitational radiation problem difficult. It is a standard procedure to work only with the weak radiative solution. In this limit, the energy-momentum self-interaction term of the gravitational wave can be neglected. This approach is justified in most problems where the total amount of gravitational radiation released is negligible in comparison to the total energy content of the spacetime. (High-energy collisions of two BHs require in principle the inclusion of nonlinear effects.) Let us assume an asymptotically flat  $D$ -dimensional spacetime with metric  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ . At this linearized level it can be shown that

(i) gravitational waves in a  $D$ -dimensional spacetime have  $D(D-3)/2$  independent polarizations;

(ii) the perturbation amplitude in the wave zone,  $h_{\mu\nu}$ , can be expressed in terms of the energy momentum tensor as

$$h_{\mu\nu}(t, \mathbf{x}) = -8\pi G_D \frac{1}{(2\pi r)^{(D-2)/2}} \partial_t^{(\frac{D-4}{2})} \times \left[ \int d^{D-1} \mathbf{x}' S_{\mu\nu}(t - |\mathbf{x} - \mathbf{x}'|, \mathbf{x}') \right], \quad (9)$$

where  $\partial_t^{(\frac{D-4}{2})}$  stands for the  $\frac{D-4}{2}$ th derivative with respect to time and

$$S_{\mu\nu} = T_{\mu\nu} - \frac{1}{D-2} \eta_{\mu\nu} T^\alpha{}_\alpha. \quad (10)$$

The Fourier transform and the energy spectrum are (see

Ref. [35] for details)

$$h_{\mu\nu}(\omega, \mathbf{x}) = -\frac{8\pi G_D \omega^{(D-4)/2}}{(2\pi r)^{(D-2)/2}} e^{i\omega r} \int d^{D-1} \mathbf{x}' S_{\mu\nu}(\omega, \mathbf{x}'), \quad (11)$$

$$\frac{d^2 E}{d\omega d\Omega} = 2G_D \frac{\omega^{D-2}}{(2\pi)^{D-4}} \times \left( T^{\mu\nu}(\omega, \mathbf{k}) T_{\mu\nu}^*(\omega, \mathbf{k}) - \frac{1}{D-2} |T^\lambda{}_\lambda(\omega, \mathbf{k})|^2 \right), \quad (12)$$

respectively.

### A. Instantaneous Collisions in Even $D$ -Dimensions

In general, two scattering bodies release gravitational energy due to momentum exchange. If the collision is hard, i.e. the incoming and outgoing trajectories have constant velocities, the metric perturbation and the released energy can be computed exactly. (This method was first derived by Weinberg [36, 37] and later explored in Ref. [38].) These calculations assume an instantaneous collision, and are valid for arbitrary velocities and low energies. The resulting spectrum is flat in four dimensions [39], and a cutoff frequency is needed to obtain a finite total energy. A suitable cutoff enables us to estimate the total energy radiated by the collision of two ultra-relativistic BHs. This method has been recently generalized to higher dimensions in Ref. [35]. In what follows we revisit this result and estimate the total energy radiated in the high-energy collision of two  $D$ -dimensional BHs.

Consider a system of freely moving particles with  $D$ -momenta  $P_i^\mu$ , energies  $E_i$  and  $(D-1)$ -velocities  $\mathbf{v}$ . These quantities change abruptly at  $t = 0$  to corresponding primed quantities due to the collision. The energy-momentum tensor is

$$T^{\mu\nu}(t, \mathbf{v}) = \sum_i \left[ \frac{P_i^\mu P_i^\nu}{E_i} \delta^{D-1}(\mathbf{x} - \mathbf{v}t) \Theta(-t) + \frac{P_i'^\mu P_i'^\nu}{E_i'} \delta^{D-1}(\mathbf{x}' - \mathbf{v}'t) \Theta(t) \right]. \quad (13)$$

Substituting Eq. (13) in Eqs. (11) and (12), the metric perturbation  $h_{\mu\nu}$  and the radiation spectrum can be obtained. Let us consider a head-on collision of a particle with mass  $m_1$  and Lorentz factor  $\gamma_1$  with a particle of mass  $m_2$  and Lorentz factor  $\gamma_2$  in the c.m. frame. We assume without loss of generality that the motion is in the  $(x_{D-1}, x_D)$  plane:

$$\begin{aligned} P_1 &= \gamma_1 m_1 (1, 0, 0, \dots, v_1 \sin \theta_1, v_1 \cos \theta_1), \\ P_1' &= (E_1', 0, 0, \dots, 0, 0), \end{aligned} \quad (14)$$

$$\begin{aligned} P_2 &= \gamma_2 m_2 (1, 0, 0, \dots, -v_2 \sin \theta_1, -v_2 \cos \theta_1), \\ P_2' &= (E_2', 0, 0, \dots, 0, 0). \end{aligned} \quad (15)$$

Momentum conservation leads to the additional relation  $\gamma_1 m_1 v_1 = \gamma_2 m_2 v_2$ . Substituting Eqs. (14) and (15) in

TABLE II: Efficiency for gravitational radiation generation in head-on collisions for different spacetime dimensions  $D$ , using the instantaneous collision method.

$D$	$\epsilon$ (%)
4	15.9
6	1.8
8	0.07
10	0.001
$\infty$	0

Eq. (13) and (12), we find

$$\frac{d^2 E}{d\omega d\Omega} = \frac{2G_D}{(2\pi)^{D-2}} \frac{D-3}{D-2} \frac{\gamma_1^2 m_1^2 v_1^2 (v_1 + v_2)^2 \sin^4 \theta_1 \omega^{D-4}}{(1 - v_1 \cos \theta_1)^2 (1 + v_2 \cos \theta_1)^2}. \quad (16)$$

For arbitrary (even) dimensions  $D > 4$  the spectrum is not flat. The integration of Eq. (16) from  $\omega = 0$  to a cutoff frequency  $\omega_c$  gives

$$\frac{dE}{d\Omega} = \frac{2G_D}{(2\pi)^{D-2}} \frac{1}{D-2} \frac{\gamma_1^2 m_1^2 v_1^2 (v_1 + v_2)^2 \sin^4 \theta_1 \omega_c^{D-3}}{(1 - v_1 \cos \theta_1)^2 (1 + v_2 \cos \theta_1)^2}. \quad (17)$$

If a BH forms in the collision, the effective timescale for the process is  $\tau \sim r_+$ . This suggests to approximate the cutoff frequency  $\omega_c$  by the inverse of the BH radius. For a collision between equal-mass particles ( $m_1 = m_2 = m$ ,  $v_1 = v_2 = v$ ) this yields a total energy

$$E = \frac{2^{3-D} \gamma^2 m^2}{M \Gamma^2[(D-1)/2]}, \quad (18)$$

where  $M \sim 2m\gamma$ . The efficiency  $E/(2m\gamma)$  is

$$\epsilon = \frac{2^{1-D}}{\Gamma^2[(D-1)/2]}. \quad (19)$$

Values of  $\epsilon$  are listed in Table II for different dimensions. The results are in agreement with the four-dimensional estimate of D'Eath and Payne [12]. However, the total energy decreases with the spacetime dimension, in disagreement with the estimate of Ref. [13].

The instantaneous collision approach seems to suggest that the trapped surface method overestimates the total energy emitted in the collision. The approach described in this section can be generalized to describe the collision of rotating bodies. Extension to rotating BHs would also be of interest for the computation of gravitational radiation from gamma-ray bursts [40].

### B. Perturbation around the black hole background

The collision of a BH with an ultra-relativistic particle was studied in detail by Cardoso and Lemos [41] in four dimensions, and by Berti *et al.* for  $D > 4$  [42]. In the perturbative approach, the BH-particle system is described

TABLE III: Efficiency for gravitational radiation generation in head-on collisions for different spacetime dimensions  $D$ , using the perturbed Schwarzschild-Tangherlini BH method.

$D$	$\epsilon$ (%)
4	13
6	10
8	7
10	8

by a perturbed Schwarzschild-Tangherlini BH, where the perturbation  $h_{\mu\nu}$  is induced by the infalling particle. Expanding Einstein's equations to first order in  $h_{\mu\nu}$ , the problem can be expressed as a second order differential equation for  $h_{\mu\nu}$ . Although the formalism is strictly valid only for a colliding particle with small energy  $E$  compared to the BH mass, the results can be extrapolated to  $E \sim M_{BH}$  [42]. The results of this analysis are given in Table III. They are in qualitative agreement with the predictions of the instantaneous collision approach.

The total energy radiated by a freely-falling particle in the four-dimensional Schwarzschild spacetime [43] is in very good agreement with numerical simulations of BH head-on collisions. An extensive comparison of numerical and perturbative results can be found in Refs. [22–25]. According to perturbation theory, the total energy radiated by a test particle of mass  $m$  falling radially from rest at infinity into a BH of mass  $M \gg m$  is  $E = 0.0104m^2/M$  [43]. If  $m$  is replaced by the reduced mass  $\mu$ , this result agrees with the numerical result  $E \simeq 0.0013$  (see Fig. 14 and Sec. IV of Ref. [25]). The slight discrepancy between perturbative and numerical results can be quantitatively explained by considering three fudge factors in the perturbative analysis [25]:  $F_{r_0}$ , which accounts for the finite initial infall distance in the numerical simulations;  $F_h$ , which accounts for tidal deformations heating up the BH horizon;  $F_{\text{abs}}$ , which accounts for the reabsorption of the gravitational waves by the BH. In our case the process starts at infinite separation, thus  $F_{r_0} = 1$ ,  $F_h \simeq 0.86$ , and  $F_{\text{abs}} \simeq 0.99$ .

Perturbation theory in the close-limit approximation and numerical simulations are also consistent for four-dimensional ultra-relativistic collisions. Numerical studies of boosted BHs with fixed initial separation show that the energy emission saturates at  $E \sim 0.01M$  for very large initial BH momenta (see, e.g., Fig. 2 of Ref. [27]).

Ref. [26] combines Newtonian dynamics and numerical simulations of boosted BHs to conclude that the maximum energy emission could actually be much lower than the above value,  $E \lesssim 0.0016M$ . Four-dimensional investigations also suggest that most of the radiation is emitted in the ringdown phase. This is confirmed by comparing perturbative calculations to post-Newtonian calculations [44]. The bremsstrahlung radiation of a particle at distance larger than  $4M$  (in Schwarzschild coordinates) contributes only  $\sim 3\%$  of the total energy emitted.

In conclusion, numerical results in four dimensions indicate that perturbation theory likely overestimates the emitted radiation. The generalization of these results to higher dimensions is nontrivial, and the values in Tables II and III could underestimate the efficiency for gravitational wave generation in high-energy collision of two equal mass particles.

#### IV. CONCLUSIONS

We reviewed and compared different approaches to computing the gravitational energy released during black hole formation in high-energy particle collisions. While in four dimensions there is good agreement between all these methods, results differ significantly in higher dimensions. The straightforward conclusion is that new techniques and refinements in previous calculations are needed to obtain a quantitative understanding of BH formation in higher-dimensional spacetimes. This is particularly important for high  $D$ , where different approaches do not yield consistent results, and for non head-on collisions, where the trapped surface method is expected to be less reliable.

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