MATCHING PARTON SHOWERS TO NLO COMPUTATION

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Abstract

In this review a new method is presented for attaching parton shower algorithms to NLO partonic jet cross sections in electron-positron annihilation. Our method is based on the Catani-Seymour dipole subtraction method and also uses an adaptation of the matching scheme of Catani, Krauss, Kuhn, and Webber.

1 Introduction

One often uses perturbation theory to produce predictions for the results of particle physics experiments in which the strong interaction is involved. In order to get useable predictions one has to calculate at least at next-to-leading order to avoid large uncertainties those come from the unphysical scale dependences.

Unfortunately, standard NLO programs have significant flaws. One flaw is that the final states consist just of a few partons, while in nature final states consist of many hadrons. A worse flaw is that the weights are often very large positive numbers or very large negative numbers.

There is another class of calculational tools, the shower Monte Carlo event generators, such as HERWIG [1] and PYTHIA [2]. These have the significant advantage that the objects in the final state consist of hadrons. Furthermore, the weights are never large numbers. Finally, the programs have a lot of important structure of QCD built into them. For at least some cases like this, the shower Monte Carlo programs can provide a good approximation for the cross sections. The chief disadvantage of typical shower Monte Carlo event generators is that they are based on leading order perturbation theory for the basic hard process and thus reproduce only the first term in the perturbative expansion when applied to an infrared safe observable.

It is possible to add the machinery of a shower Monte Carlo event generator to a next-to-leading order program in such a way that the complete program produces realistic final states made of hadrons, with weights are not unbounded in size. One example is the program of Frixione, Nason, and Webber [3], which so far has been applied to cases with massless incoming partons but not to cases with massless final state partons at the Born level of calculation. The other example is that of [4], which concerns three-jet observables in electron-positron annihilation and thus addresses massless final state partons but not massless initial state partons.

In this paper, we want, most of all, to have an algorithm that can be used by NLO practitioners in a reasonably straightforward manner. For this reason, we have based the algorithm on the dipole subtraction scheme of Catani and Seymour [5]. It is quite widely used for NLO calculations (for example in the programs NLOJET++ [6] and MCFM [7]).

We note that NLO calculations are generally limited to just one class of observables – for instance four-jet production but not at the same time three-jet production. We would like to overcome this limitation. For this reason we have adapted the k_T-jet matching scheme of Catani, Kuhn, Krauss, and Webber[8] to the present circumstances. We also seek be as independent as possible of the choice of any particular shower Monte Carlo event generator. That is, we do not think that practitioners of NLO calculations should need to do separate calculations for each present and future shower Monte Carlo.

The general idea of the algorithm that we present applies, we believe, to lepton-lepton collisions, lepton-hadron collisions, and hadron-hadron collisions.

In the next section we have a very brief review of the algorithm. The precise definition and the details of the algorithm can be found in Ref. [9].

2 Structure of the algorithm

The cross section computed with parton showers will consist of contributions from each available m,

$$\sigma^{\rm NLO+S} = \sum_{m=2}^{m_{\rm NLO}} \left[\sigma_m^{\rm B+S} + \sigma_m^{\rm R+S} + \sigma_m^{\rm V+S}\right] + \sum_{m=m_{\rm NLO}+1}^{m_{\rm max}} \sigma_m^{\rm B+S} \ . \tag{1}$$

For the contributions at NLO level, there are three terms, which correspond to Born, real emission, and virtual loop contributions with showers added ("$+S$"). For the remaining terms there is only a Born contribution. We will arrange that (for a suitably behaved observable)

$$\sigma_m^{\rm B+S} + \sigma_m^{\rm R+S} + \sigma_m^{\rm V+S} = \sigma_m^{\rm NLO} + \mathcal{O}(\alpha_{\rm s}^{B_m+2}) + \mathcal{O}(1\,{\rm GeV}/\sqrt{s}) \ , \tag{2}$$

that the NLO expansion of the partial shower cross sections gives the correct NLO partonc cross section ($\sigma_m^{\rm NLO}$) plus higher order and power corrections.

2.1 Born term with showers

Our discussion begins in this subsection with $\sigma_m^{\rm B+S}$. We define

$$\begin{aligned}\sigma_m^{B+S} = \frac{1}{m!}\sum_{\{f\}_m}\int d\Gamma(\{p\}_m)\,\theta(d_{\rm ini} < d_m(\{p,f\}_m))\,W_m(\{p,f\}_m) \\ \times\sum_{l=1}^{m}\sum_{k\neq l}\big\langle\mathcal{M}(\{p,f\}_m)\big|\int dY_l\ \boldsymbol{E}_{l,k}(Y_l)\,\big|\mathcal{M}(\{p,f\}_m)\big\rangle I(\{p,f\}_m;l,k,Y_l) \ .\end{aligned} \tag{3}$$

The first line contains integrals over Born level parton momenta ($d\Gamma(\{p\}_m)$)and a corresponding sum over parton flavors. The second line contains sums over choices l of the parton that splits and a spectator parton k along with an integral over the splitting variables[a] $Y_l = \{y_l, z_l, \phi_l, \hat f_{l,1}, \hat f_{l,2}\}$ and a matrix element of certain operators $\boldsymbol{E}_{l,k}$ acting on the Born amplitude $|\mathcal{M}(\{p,f\}_m)\rangle$. The operators $\boldsymbol{E}_{l,k}$ together with the other factors in the formula describe the formation of showers from the Born level partons.

The integration over the momenta $\{p\}_m$ is restricted by a factor $\theta(d_{\rm ini} < d_m(\{p,f\}_m))$. Here $d_m(\{p,f\}_m)$ is defined by applying the k_T jet finding algorithm to the m parton momenta. Given an n-parton final state, we apply the recursive "k_T" jet finding algorithm[10] to the parton momenta $\{p,f\}_n$, successively grouping the partons into jets. The algorthm gives a sequence of jet resolution parameters $d_J(\{p,f\}_n)$ at which two jets were joined, reducing J jets to $J-1$ jets. Typically one has $d_n < d_{n-1} < \cdots < d_3$.

There is also a factor $W_m(\{p,f\}_m)$, which is the product of factors associated with the splitting history that matches the found jet structure, following the method of Ref. [8].

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[a]The splitting variables are the y_l virtuality like variable, z_l momentum fraction variable, ϕ_l azimuthal angle and the flavors of the emitted partons. The integral over these variables is

$$\int_0^1 \frac{dy_l}{y_l}\int_0^1 dz_l \int_0^{2\pi}\frac{d\phi_l}{2\pi}\,\frac{1}{2}\sum_{\hat f_{l,1},\hat f_{l,2}}\delta^{f_l}_{\hat f_{l,1}+\hat f_{l,2}} \equiv \int dY_l \ .$$

The function $I(\{p,f\}_m;l,k,Y_l)$ is the interface function to secondary shower. Its important property is that the secondary shower provides only perturbative and power correction, thus we have

$$I(\{p,f\}_m;l,k,Y_l) = F_{m+1}(\{\hat{p}\}_{m+1}) + \mathcal{O}(\alpha_s) + \mathcal{O}(1\,\mathrm{GeV}/\sqrt{s}) \ , \tag{4}$$

where the function $F_{m+1}(\{\hat{p}\}_{m+1})$ is the measurement function of an infrared safe observable.

Now, we turn to the splitting function $\boldsymbol{E}_{l,k}$ in Eq. (3), which is an operator on the flavor and spin space of parton l in the vector $|\mathcal{M}\rangle$. This operator has the following form,

$$\begin{aligned}\boldsymbol{E}_{l,k} = \frac{\boldsymbol{T}_l\cdot\boldsymbol{T}_k}{-\boldsymbol{T}_l^2}\int_0^\infty dr\,\delta(r-R_l(\{p,f\}_m,y,z))\,\theta(\tilde{d}(\{p,f\}_m,l,y_l,z_l)<d_{\mathrm{ini}})\,\frac{\alpha_s(r)}{2\pi} \\ \times\,\boldsymbol{S}_l(p_l,f_l,Y_l)\exp\Big(-\int_r^\infty dr'\int_0^1\frac{dy'}{y'}\int_0^1 dz'\sum_{l'}\delta(r'-R_{l'}(\{p,f\}_m,y',z')) \\ \times\,\theta(\tilde{d}(\{p,f\}_m,l',y',z')<d_{\mathrm{ini}})\,\frac{\alpha_s(r')}{2\pi}\,\langle\boldsymbol{S}(y',z',f_{l'})\rangle\Big) \ .\end{aligned} \tag{5}$$

The parton splitting is organized according to an evolution parameter r, which is defined to be proportional to the transverse momentum square, $R_l(\{p,f\}_m,y,z) = s_l\,yz(1-z)$ and s_l is a virtuality scale appropriate to parton l. The simplest choice would be $s_l = s$. .

With the use of function[b] $\tilde{d}(\{p,f\}_m,l,y_l,z_l)$, we limit the splitting in $\boldsymbol{E}_l$ to be unresolvable at a scale d that is approximately $d_{\mathrm{ini}}\times 2p_l\cdot p_{k_l}/s_l$. With this cut the vetoing procedure is implemented in the first step of the shower ensuring the cancellation of the d_{ini} dependences al least at NLL level.

In each $\boldsymbol{E}_{l,k}$ operator, there is an operator on the parton color space, $\boldsymbol{T}_l\cdot\boldsymbol{T}_k/[-\boldsymbol{T}_l^2]$, that is for the soft color connections. The splitting function $\boldsymbol{S}_l$ acts in the spin space of the emitter and depends on the splitting parameters Y_l for parton l as well as on the momentum p_l. This functions are proportional to the dipole splitting functions.

The next factor, the Sudakov exponential, gives the probability that *none* of the partons has split at a higher evolution scale. The factor $\langle\boldsymbol{S}(y_{l'},z_{l'},f_{l'})\rangle$ in the Sudakov exponent is the average over angle and flavors of $\boldsymbol{S}$ for parton l'.

2.2 NLO corrections with shower

We turn to the discussion of the NLO corrections. Let us start with the real contribution. Define

$$\begin{aligned}\sigma_m^{R+S} = \frac{1}{(m+1)!}\sum_{\{\hat{f}\}_{m+1}}\int d\Gamma(\{\hat{p}\}_{m+1})\,\tilde{I}\big(\{\hat{p},\hat{f}\}_{m+1}\big) \\ \times\Big\{\big|\mathcal{M}(\{\hat{p},\hat{f}\}_{m+1})\big|^2\,\theta\big(d_{m+1}(\{\hat{p},\hat{f}\}_{m+1})<d_{\mathrm{ini}}<d_m(\{\hat{p},\hat{f}\}_{m+1})\big) \\ -\sum_{\substack{i,j\\ \text{pairs}}}\sum_{k\neq i,j}\mathcal{D}_{ij,k}\,\theta\big(\tilde{d}(\{p,f\}_m^{ij,k},\{l,y,z\}_{ij,k})<d_{\mathrm{ini}}<d_m(\{p,f\}_m^{ij,k})\big)\Big\} \ .\end{aligned} \tag{6}$$

The first term is the $m+1$ parton matrix element squared with the proper m-jet definition and the second term is the sum of the dipole contributions to eliminate the infrared singularities. The dipole function $\mathcal{D}_{ij,k}$ are based on the m-parton tree level color connected matrix elements and the correct definition can be taken from the paper by Catani and Seymour [5].

The definition of the virtual correction is

$$\begin{aligned}\sigma_m^{\mathrm{V+S}} = \frac{1}{m!}\sum_{\{f\}_m}\int d\Gamma(\{p\}_m)\,\theta\big(d_{\mathrm{ini}}<d_m(\{p,f\}_m)\big)\,\tilde{I}(\{p,f\}_m) \\ \times\Big\{V(\{p,f\}_m)-\frac{\alpha_s(\mu_R)}{2\pi}\,W_m^{(1)}(\{p,f\}_m)\,\big|\mathcal{M}(\{p,f\}_m)\big|^2\Big\} \ ,\end{aligned} \tag{7}$$

[b] It is an approximation of the jet resolution variable,

$$\tilde{d}(\{p,f\}_m,l,y_l,z_l) = \frac{s_l}{s}\,y_l\min\left\{\frac{1-z_l}{z_l},\frac{z_l}{1-z_l}\right\} \ .$$

where the function $V(\{p,f\}_m)$ represents sum of the 1-loop matrix element and integrated subtraction term given in the second term of Eq. (6) over the phase space of the unresolved particle. The function $W_m^{(1)}(\{p,f\}_m)$ is coefficient of the α_s term in the expansion of Sudakov reweighting factor $W_m(\{p,f\}_m)$.

2.3 Secondary shower

Consider, the function $I(\{p,f\}_m; l,k,Y_l)$ used for $\sigma_m^{\rm B+S}$. In this term the $\boldsymbol{E}$ operator describe the emission of the hardest splitting, thus all the further splittings are constrained.

All of the partons are allowed to split, and the one that does is parton l with aid of spectator k. The others did not split at an evolution variable above the value r. That is, parton l', with the aid of spectator k', did not split with $r' > r$. Further evolution of these partons should then be restricted to the range $r' < r$. This constraint for the transverse momentums is

$$|k_\perp'^2| < \frac{2p_{l'}\cdot p_{k'}}{s_{l'}}\,\frac{s_l}{2p_l\cdot p_k}\,|k_\perp^2|\ . \tag{8}$$

A restriction like this can be imposed in the chosen shower Monte Carlo program by using a veto algorithm, as described for instance in Ref. [8]. A sensible choice for the k' would be to let k' be one of the final state partons to which parton l' is color connected (at leading order in $1/N_c$). For the splitting of one of the daughters of parton l, one may simply impose $|k_\perp'^2| < |k_\perp^2|$.

Finally, in principle there should be a cut $d(p_i,p_j) < d_{\rm ini}$ imposed on further splittings. However, for most events passed to the Monte Carlo, $|k_\perp^2|$ will be much smaller than $d_{\rm ini}s$, so that this cut is not really needed.

3 Conclusion

We have proposed an algorithm for adding showers to next-to-leading order calculations for $e^+ + e^- \to N$ *jets*. This algorithm is based on the dipole subtraction scheme [5] that is widely used for next-to-leading order calculations. We also use the k_T-jet matching scheme of Ref. [8] in order to incorporate the possibility of calculating infrared safe N-jet cross sections for different values of N into the same computer program.

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