AN ANALYTICAL DESCRIPTION OF SPIN EFFECTS IN HADRON-HADRON SCATTERING VIA PMD-SQS OPTIMUM PRINCIPLE

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Abstract

In this paper an analytical description of spin-effects in hadron-hadron scattering is presented by using PMD-SQS-optimum principle in which the differential cross sections in the forward and backward c.m. angles are considered fixed from the experimental data. The experimental tests of the optimal predictions, obtained by using the available phase shifts, are discussed.

1 Introduction

Recently, in Ref. [1], by using reproducing kernel Hilbert space (RKHS) methods [2-4], we described the quantum scattering of the spinless particles by a *principle of minimum distance in the space of quantum states* (PMD-SQS). Some preliminary experimental tests of the PMD-SQS, even in the crude form [1] when the complications due to the particle spins are neglected, showed that the actual experimental data for the differential cross sections of all principal hadron-hadron [nucleon-nucleon, antiproton-proton, meson-nucleon] scatterings at all energies higher than 2 GeV, can be well systematized by PMD-SQS predictions (see the papers [1]). Moreover, connections between the PMD-SQS and the *maximum entropy principle* for the statistics of the scattering quantum channels was also recently established by introducing quantum scattering entropies: S_θ and S_J [5]-[7]. Then, it was shown that the experimental pion-nucleon as well as pion-nucleus scattering entropies are well described by optimal entropies and that the experimental data are consistent with the principle of minimum distance in the space of quantum states (PMD-SQS) }[1]. However, the PMD-SQS in the crude form [1] cannot describe the polarization J-spin effects.

In this paper an analytical description of spin-effects in hadron-hadron scattering is presented by using PMD-SQS-optimum principle in which the differential cross sections in the forward (x=+1) and backward (x=-1) directions are considered fixed from the experimental data. An experimental test of the optimal prediction on the logarithmic slope b is performed for the pion-nucleon scattering at the forward c.m. angles.

2 Optimal state description of pion-nucleon scattering

First we present the basic definitions on the the pion-nucleon scattering:

$$\pi + N \to \pi + N \tag{1}$$

Therefore, let $f^{++}(x)$and $f^{+-}(x)$, be the scattering helicity amplitudes of the meson-nucleon scattering process (see ref.[14]) written in terms of the partial helicities f_{J-} sand f_{J+}as follows

$$\begin{aligned} f_{++}(x) &= \sum_{J=\frac{1}{2}}^{J_{\max}} \left(J+\tfrac{1}{2}\right)(f_{J-}+f_{J+})\, d^J_{\frac{1}{2}\frac{1}{2}}(x) \\ f_{+-}(x) &= \sum_{J=\frac{1}{2}}^{J_{\max}} \left(J+\tfrac{1}{2}\right)(f_{J-}-f_{J+})\, d^J_{-\frac{1}{2}\frac{1}{2}}(x) \end{aligned} \tag{2}$$

where the rotation functions are defined as

$$
\begin{aligned}
d^J_{\frac{1}{2}\frac{1}{2}}(x) &= \tfrac{1}{l+1}\cdot\left[\tfrac{1+x}{2}\right]^{\frac{1}{2}}\left[\underset{l+1}{\overset{\bullet}{P}}(x) - \underset{l}{\overset{\bullet}{P}}(x)\right] \\
d^J_{-\frac{1}{2}\frac{1}{2}}(x) &= \tfrac{1}{l+1}\cdot\left[\tfrac{1-x}{2}\right]^{\frac{1}{2}}\left[\underset{l+1}{\overset{\bullet}{P}}(x) + \underset{l}{\overset{\bullet}{P}}(x)\right]
\end{aligned} \tag{3}
$$

where $P_l(x)$are Legendre polynomials, $\underset{l}{\overset{o}{P}}(x) = \frac{d}{dx}P_l(x)$, x being the c.m. scattering angle. The normalization of the helicity amplitudes $f^{++}(x)$ and $f^{+-}(x)$, is chosen such that the c.m. differential cross section is given by

$$\frac{d\sigma}{d\Omega}(x) = |f_{++}(x)|^2 + |f_{+-}(x)|^2 \tag{4}$$

Then, the elastic integrated cross section is given by

$$\frac{\sigma_{el}}{2\pi} = \sum (j+\frac{1}{2})\left[|f_{j+}|^2 + |f_{j-}|^2\right] \tag{5}$$

Now, let us consider the following optimization problem:

$$\min\left\{\sum (j+\frac{1}{2})\left[|f_{j+}|^2 + |f_{j-}|^2\right]\right\} \tag{6}$$

when $\frac{d\sigma}{d\Omega}(+1)$ and $\frac{d\sigma}{d\Omega}(-1)$ are fixed.

We proved that the solution of this optimization problem is given by the following results :

$$f_o^{++}(x) = f^{++}(+1)\frac{K_{\frac{1}{2}\frac{1}{2}}(x,y)}{K_{\frac{1}{2}\frac{1}{2}}(+1,+1)} \tag{7}$$

$$f_o^{+-}(x) = f^{+-}(-1)\frac{K_{\frac{1}{2}-\frac{1}{2}}(x,y)}{K_{\frac{1}{2}-\frac{1}{2}}(-1,-1)} \tag{8}$$

where the functions K(x,y) are the reproducing kernels [2] expressed in terms of the rotation function by

$$K_{\frac{1}{2}\frac{1}{2}}(x,y) = \sum_{1/2}^{J_o}(j+\frac{1}{2})d^j_{\frac{1}{2}\frac{1}{2}}(x)d^j_{\frac{1}{2}\frac{1}{2}}(y) \tag{9}$$

$$K_{\frac{1}{2}-\frac{1}{2}}(x,y) = \sum_{1/2}^{J_o}(j+\frac{1}{2})d^j_{\frac{1}{2}-\frac{1}{2}}(x)d^j_{\frac{1}{2}-\frac{1}{2}}(y) \tag{10}$$

while the optimal angular momentum is given by

$$J_0 = \sqrt{\frac{4\pi}{\sigma_{el}}\left[\frac{d\sigma}{d\Omega}(+1) + \frac{d\sigma}{d\Omega}(-1)\right] + \frac{1}{4}} - 1 \tag{11}$$

Now, let us consider the logarithmic slope b of the forward diffraction peak defined by

$$b = \frac{d}{dt}\left[\ln \frac{d\sigma}{dt}(s,t)\right]_{t=0} \tag{12}$$

Then, using the definition of the rotation functions, from (70-(6) we obtain the following optimal slope b_o:

$$b_o = \frac{\lambda^2}{4}\left[\frac{4\pi}{\sigma_{el}}\left(\frac{d\sigma}{d\Omega}(+1) + \frac{d\sigma}{d\Omega}(-1)\right) - 1\right] \tag{13}$$

Optimal predictions on the differential cross section $\frac{d\sigma_o}{d\Omega}(x)$ and also for the spin-polarization parameters (P_o, R_o, A_o), are as follows.

$$\frac{d\sigma_o}{d\Omega}(x) = \frac{d\sigma}{d\Omega}(+1)\left[\frac{K_{\frac{1}{2}\frac{1}{2}}(x,+1)}{K_{\frac{1}{2}\frac{1}{2}}(+1,+1)}\right]^2 + \frac{d\sigma}{d\Omega}(-1)\left[\frac{K_{-\frac{1}{2}\frac{1}{2}}(x,-1)}{K_{-\frac{1}{2}\frac{1}{2}}(-1,-1)}\right]^2 \tag{14}$$

$$P_o\frac{d\sigma_o}{d\Omega}(x) = 2\sqrt{\frac{d\sigma}{d\Omega}(+1)}\sqrt{\frac{d\sigma}{d\Omega}(-1)}\left[\frac{K_{\frac{1}{2}\frac{1}{2}}(x,+1)}{K_{\frac{1}{2}\frac{1}{2}}(+1,+1)}\right]\left[\frac{K_{-\frac{1}{2}\frac{1}{2}}(x,-1)}{K_{-\frac{1}{2}\frac{1}{2}}(-1,-1)}\right]\sin\phi(x) \tag{15}$$

$$R_o\frac{d\sigma_o}{d\Omega}(x) = 2\sqrt{\frac{d\sigma}{d\Omega}(+1)}\sqrt{\frac{d\sigma}{d\Omega}(-1)}\left[\frac{K_{\frac{1}{2}\frac{1}{2}}(x,+1)}{K_{\frac{1}{2}\frac{1}{2}}(+1,+1)}\right]\left[\frac{K_{-\frac{1}{2}\frac{1}{2}}(x,-1)}{K_{-\frac{1}{2}\frac{1}{2}}(-1,-1)}\right]\cos\phi(x) \tag{16}$$

$$A_o\frac{d\sigma_o}{d\Omega}(x) = \frac{d\sigma}{d\Omega}(+1)\left[\frac{K_{\frac{1}{2}\frac{1}{2}}(x,+1)}{K_{\frac{1}{2}\frac{1}{2}}(+1,+1)}\right]^2 - \frac{d\sigma}{d\Omega}(-1)\left[\frac{K_{-\frac{1}{2}\frac{1}{2}}(x,-1)}{K_{-\frac{1}{2}\frac{1}{2}}(-1,-1)}\right]^2 \tag{17}$$

where

$$\cos\phi(x) = \frac{Re\{[f^{++}(+1)]^* f^{+-}(-1)\}}{|f^{++}(+1)||f^{+-}(-1)|}, \quad \sin\phi(x) = \frac{Im\{[f^{++}(+1)]^* f^{+-}(-1)\}}{|f^{++}(+1)||f^{+-}(-1)|} \tag{18}$$

Finally, we note that in ref. [15] we proved the following optimal inequality

$$b_o = \frac{\lambda^2}{4}\left[\frac{4\pi}{\sigma_{el}}\left(\frac{d\sigma}{d\Omega}(+1) + \frac{d\sigma}{d\Omega}(-1)\right) - 1\right] \leq b_{\text{exp}} \tag{19}$$

which includes in a more general and exact form the unitarity bounds derived by Martin [8] and Martin-MacDowell [9] (see also ref.[10]) and Ion [1],[11].

3 Experimental test of the PMD-SQS-optimal predictions

For an experimental test of the optimal result (13) the numerical values of the slopes b_o and b_{exp} are calculated directly by reconstruction of the helicity amplitudes from the experimental phase shifts (EPS) solutions of Holer et al. [14]. These results are given in the Tables 1-2 and are displayed in Fig 1-2.

4 Conclusions

The main results and conclusions obtained in this paper can be summarized as follows:

(i) The optimal state dominance in hadron-hadron scattering at small transfer momenta for $p_{LAB} \geq 2$ GeV/c is a fact well evidenced experimentally by the results presented in Figs. 1-2.

(ii) In the low energy region, the optimal slope (13) is in good agreement with the experimental data at discrete values of energy between the resonances positions or/and in the region corresponding to the diffractive resonances see Figs. 1-2.

Finally, we hope that our results as well as those from ref. [15] are new steps towards an analytic description of the quantum scattering in terms of an optimum principle, namely, the *principle of minimum distance in space of quantum state* (PMD-SQS) [1].

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Table 1: PMD-SQS optimal predictions for $\pi^+ P \to \pi^+ P$, calculated using the experimental phase shifts solutions from ref. [14]

P_{LAB} (GeV/c)	$(1/p_{cm})^2$ (GeV^{-2})	$\frac{d\sigma}{d\Omega}(+1)$ (mb/sr)	$\frac{d\sigma}{d\Omega}(-1)$ (mb/sr)	σ_{el} (mb)	J_0	b_o (GeV^{-2})	b_{o+1} (GeV^{-2})	b_{exp} (GeV^{-3})
0.020	3305.477	0.185	0.185	2.329	0.500	826.369	0.000	0.000
0.040	831.948	0.117	0.273	2.426	0.507	212.175	-81.659	-255.100
0.060	373.735	0.042	0.473	2.955	0.562	111.242	-76.667	-655.969
0.079	218.466	0.003	0.834	4.255	0.650	80.360	-54.150	-6650.37
0.097	147.078	0.027	1.254	6.057	0.705	60.956	-34.742	-382.911
0.112	111.845	0.133	1.831	8.638	0.763	51.949	-22.531	14.998
0.130	84.501	0.425	2.911	13.489	0.832	44.518	-12.769	46.680
0.153	62.493	1.294	4.971	24.018	0.878	35.590	-5.047	45.665
0.172	50.486	2.621	7.570	37.559	0.913	30.414	-1.554	36.764
0.185	44.276	4.091	9.466	49.167	0.927	27.285	0.505	31.826
0.200	38.530	6.621	12.743	68.315	0.952	24.677	2.099	28.020
0.218	33.100	11.275	18.196	101.316	0.976	21.973	3.297	24.066
0.247	26.651	22.300	28.265	169.036	1.002	18.383	4.383	19.386
0.267	23.333	30.173	32.196	203.762	1.024	16.604	5.021	16.972
0.280	21.531	32.650	31.802	207.611	1.037	15.616	5.255	15.690
0.290	20.299	33.087	30.510	202.992	1.046	14.905	5.320	14.776
0.295	19.727	33.442	29.397	199.639	1.051	14.575	5.450	14.316
0.301	19.076	33.287	27.945	193.442	1.056	14.201	5.543	13.785
0.305	18.662	32.962	26.816	188.205	1.059	13.956	5.603	13.460
0.310	18.166	32.487	25.413	180.947	1.067	13.720	5.705	13.111
0.320	17.238	31.198	22.436	165.767	1.077	13.213	5.883	12.391
0.331	16.308	28.798	19.733	150.463	1.074	12.448	5.729	11.601
0.351	14.822	25.715	14.507	122.777	1.090	11.550	6.047	10.492
0.378	13.156	20.937	10.134	92.640	1.113	10.573	6.052	9.381
0.408	11.654	16.983	7.542	71.111	1.141	9.713	5.830	8.490

Table 1 – continued from previous page

P_{LAB} (GeV/c)	$(1/p_{cm})^2$ (GeV^{-2})	$\frac{d\sigma}{d\Omega}(+1)$ (mb/sr)	$\frac{d\sigma}{d\Omega}(-1)$ (mb/sr)	σ_{el} (mb)	J_0	b_o (GeV^{-2})	b_{o+1} (GeV^{-2})	b_{exp} (GeV^{-3})
0.427	10.850	15.042	5.519	60.807	1.121	8.813	5.720	7.679
0.456	9.797	12.345	3.528	47.050	1.119	7.934	5.626	6.381
0.490	8.774	10.059	2.446	37.329	1.112	7.041	5.234	5.382
0.532	7.748	8.407	1.372	28.181	1.147	6.510	5.325	4.729
0.573	6.936	6.766	0.700	21.827	1.133	5.720	5.020	4.864
0.614	6.266	5.409	0.233	17.710	1.062	4.706	4.447	4.805
0.658	5.668	4.386	0.089	14.446	1.035	4.099	3.989	5.937
0.675	5.463	3.896	0.082	13.190	1.010	3.810	3.704	5.662
0.705	5.134	3.358	0.057	11.125	1.027	3.667	3.585	7.180
0.725	4.934	2.959	0.037	10.166	0.988	3.335	3.279	7.121
0.750	4.704	2.523	0.021	8.767	0.974	3.112	3.076	7.092
0.777	4.476	2.728	0.081	11.235	0.842	2.397	2.295	4.894
0.800	4.298	1.981	0.003	7.050	0.946	2.726	2.720	2.383
0.822	4.139	1.851	0.002	6.467	0.962	2.691	2.687	5.419
0.851	3.946	1.868	0.016	6.383	0.989	2.671	2.640	3.371
0.875	3.798	2.035	0.060	6.818	1.028	2.719	2.613	4.051
0.895	3.683	2.183	0.159	7.683	1.020	2.606	2.367	3.029
0.923	3.532	2.458	0.315	8.999	1.030	2.536	2.147	2.115
0.954	3.378	2.812	0.505	10.365	1.067	2.551	2.034	2.057
0.975	3.281	3.012	0.618	11.061	1.091	2.562	1.986	2.634
1.000	3.171	3.507	0.754	11.758	1.192	2.818	2.179	1.543
1.030	3.049	3.579	0.986	11.731	1.267	2.965	2.160	2.684
1.055	2.954	3.687	1.034	11.903	1.288	2.942	2.136	3.021
1.080	2.864	3.816	1.093	12.011	1.321	2.961	2.142	2.310
1.113	2.753	4.249	1.326	12.298	1.439	3.233	2.300	4.100
1.154	2.625	4.902	1.594	12.951	1.560	3.481	2.466	5.165
1.174	2.567	5.248	1.714	13.390	1.605	3.552	2.520	6.220
1.210	2.469	6.112	2.065	14.198	1.736	3.850	2.722	6.513
1.235	2.405	6.754	2.239	14.778	1.810	3.996	2.851	6.239
1.280	2.297	8.371	2.494	15.826	1.979	4.379	3.242	6.929
1.324	2.200	10.306	2.774	16.680	2.179	4.869	3.720	7.720
1.360	2.126	11.929	2.728	17.133	2.317	5.182	4.119	8.469
1.400	2.049	13.468	2.608	17.546	2.430	5.387	4.430	8.160

Table 1 – continued from previous page

P_{LAB} (GeV/c)	$(1/p_{cm})^2$ (GeV^{-2})	$\frac{d\sigma}{d\Omega}(+1)$ (mb/sr)	$\frac{d\sigma}{d\Omega}(-1)$ (mb/sr)	σ_{el} (mb)	J_0	b_o (GeV^{-2})	b_{o+1} (GeV^{-2})	b_{exp} (GeV^{-3})
1.430	1.995	14.381	2.487	17.605	2.506	5.508	4.622	8.869
1.473	1.923	15.282	2.189	17.401	2.587	5.584	4.824	8.308
1.505	1.872	15.777	1.912	17.265	2.623	5.557	4.906	7.235
1.550	1.804	15.023	1.591	16.169	2.628	5.373	4.816	7.097
1.590	1.748	14.457	1.236	15.287	2.626	5.201	4.757	6.626
1.640	1.683	13.875	0.879	14.218	2.646	5.065	4.738	6.513
1.680	1.634	13.428	0.602	13.232	2.684	5.033	4.799	6.425
1.720	1.587	13.034	0.450	12.729	2.683	4.885	4.709	6.383
1.760	1.543	12.617	0.324	12.065	2.705	4.814	4.684	6.210
1.800	1.502	12.279	0.241	11.602	2.716	4.715	4.617	5.858
1.840	1.462	11.843	0.211	11.032	2.739	4.653	4.566	5.896
1.880	1.425	11.883	0.135	10.757	2.780	4.644	4.587	5.958
1.920	1.389	11.822	0.098	10.320	2.843	4.693	4.651	5.883
1.980	1.338	11.836	0.054	9.958	2.906	4.686	4.663	5.865
2.03	1.299	11.976	0.043	9.603	2.997	4.783	4.765	6.044
2.07	1.269	12.061	0.037	9.465	3.039	4.779	4.763	5.881
2.15	1.213	12.500	0.054	9.179	3.176	4.910	4.887	5.919
2.20	1.181	12.973	0.086	9.075	3.282	5.043	5.007	5.997
2.28	1.132	13.706	0.154	8.894	3.453	5.258	5.197	6.228
2.34	1.098	14.547	0.187	8.829	3.607	5.482	5.409	6.387
2.46	1.036	15.840	0.273	8.702	3.849	5.765	5.663	6.485
2.56	0.989	16.725	0.274	8.585	4.013	5.903	5.804	6.458
2.75	0.910	18.072	0.239	8.328	4.280	6.059	5.977	6.479
3.00	0.824	19.115	0.167	7.803	4.595	6.190	6.135	6.848
3.40	0.715	20.284	0.093	7.098	5.027	6.269	6.240	6.942
3.65	0.660	21.180	0.053	6.886	5.245	6.231	6.215	6.708
4.00	0.596	23.088	0.062	6.590	5.663	6.432	6.415	7.018
5.00	0.467	26.653	0.042	5.751	6.654	6.690	6.680	6.989
6.00	0.383	30.776	0.031	5.259	7.594	6.957	6.950	7.302
10.00	0.223	44.885	0.097	4.414	10.328	7.087	7.072	7.369

Table 2: PMD-SQS optimal predictions for $\pi^+ P \to \pi^+ P$, calculated using the experimental phase shifts solutions from ref. [14]

P_{LAB} (GeV/c)	$(1/p_{cm})^2$ (GeV^{-2})	$\frac{d\sigma}{d\Omega}(+1)$ (mb/sr)	$\frac{d\sigma}{d\Omega}(-1)$ (mb/sr)	σ_{el} (mb)	J_0	b_o (GeV^{-2})	b_{o+1} (GeV^{-2})	b_{exp} (GeV^{-3})
0.020	3305.477	0.146	0.146	1.840	0.500	826.369	0.000	0.000
0.04o	831.948	0.135	0.144	1.755	0.498	206.915	-7.183	-17.101
0.060	373.735	0.147	0.124	1.739	0.486	89.461	5.960	3.735
0.079	218.466	0.214	0.099	2.024	0.481	51.553	17.938	22.395
0.097	147.078	0.240	0.072	2.076	0.463	32.714	16.711	17.401
0.112	111.845	0.277	0.038	2.095	0.461	24.770	18.437	18.375
0.130	84.501	0.360	0.007	2.247	0.519	22.326	21.460	25.665
0.153	62.493	0.510	0.011	2.853	0.596	20.242	19.456	24.444
0.172	50.486	0.766	0.070	3.982	0.699	20.666	17.890	24.759
0.185	44.276	0.984	0.168	5.009	0.772	20.934	16.255	24.908
0.200	38.530	1.371	0.390	6.860	0.864	21.439	14.564	25.689
0.218	33.100	1.985	0.867	10.142	0.945	20.963	12.078	25.069
0.247	26.651	3.357	1.867	17.184	1.017	18.788	9.693	20.694
0.267	23.333	4.112	2.667	21.502	1.052	17.276	8.184	17.930
0.280	21.531	4.144	2.977	22.281	1.066	16.236	7.198	16.414
0.290	20.299	4.054	3.079	22.208	1.070	15.409	6.568	15.127
0.295	19.727	3.956	3.128	21.966	1.074	15.055	6.231	14.511
0.301	19.076	3.842	3.140	21.569	1.078	14.632	5.906	13.870
0.305	18.662	3.716	3.100	21.084	1.077	14.288	5.668	13.312
0.310	18.166	3.526	3.029	20.248	1.078	13.932	5.396	12.668
0.320	17.238	3.251	2.900	19.093	1.073	13.136	4.911	11.270
0.331	16.308	2.900	2.696	17.726	1.054	12.096	4.305	9.950
0.351	14.822	2.314	2.301	15.157	1.019	10.474	3.403	7.325
0.378	13.156	1.651	1.806	12.043	0.964	8.574	2.377	3.563
0.408	11.654	1.210	1.483	10.323	0.879	6.640	1.380	-0.334
0.427	10.850	1.079	1.050	10.050	0.706	4.507	0.946	-3.973
0.456	9.797	1.185	1.067	9.564	0.791	4.798	1.364	-0.107
0.490	8.774	1.637	1.071	10.688	0.853	4.790	2.028	2.468
0.532	7.748	1.866	1.246	10.636	0.982	5.185	2.334	2.193
0.573	6.936	2.495	1.379	11.918	1.082	5.349	2.829	4.136
0.614	6.266	3.341	1.320	13.260	1.160	5.354	3.394	6.863

Table 2 – continued from previous page

P_{LAB} (GeV/c)	$(1/p_{cm})^2$ (GeV^{-2})	$\frac{d\sigma}{d\Omega}(+1)$ (mb/sr)	$\frac{d\sigma}{d\Omega}(-1)$ (mb/sr)	σ_{el} (mb)	J_0	b_o (GeV^{-2})	b_{o+1} (GeV^{-2})	b_{exp} (GeV^{-3})
0.658	5.668	5.251	1.516	15.402	1.402	6.406	4.654	11.622
0.675	5.463	5.869	1.585	16.407	1.441	6.431	4.774	10.935
0.705	5.134	7.139	1.566	17.806	1.529	6.602	5.183	10.529
0.725	4.934	7.238	1.388	17.790	1.519	6.283	5.073	10.413
0.750	4.704	6.794	1.041	16.931	1.463	5.663	4.754	8.870
0.777	4.476	5.950	0.719	14.958	1.419	5.151	4.475	9.318
0.800	4.298	5.567	0.424	13.119	1.447	5.092	4.655	11.247
0.822	4.139	5.332	0.296	12.151	1.464	4.989	4.671	12.973
0.851	3.946	5.873	0.189	12.277	1.541	5.135	4.944	14.188
0.875	3.798	7.117	0.151	13.584	1.641	5.435	5.303	14.491
0.895	3.683	8.524	0.131	15.314	1.712	5.619	5.520	13.478
0.923	3.532	11.592	0.135	18.379	1.875	6.197	6.116	13.609
0.954	3.378	15.096	0.189	21.664	2.019	6.643	6.550	13.139
0.975	3.281	17.059	0.237	23.422	2.087	6.790	6.686	13.470
1.000	3.171	19.475	0.253	25.402	2.164	6.945	6.846	11.733
1.030	3.049	18.243	0.350	24.370	2.137	6.546	6.408	11.839
1.055	2.954	16.727	0.203	22.918	2.088	6.116	6.034	10.378
1.080	2.864	14.490	0.177	20.993	2.005	5.569	5.494	8.474
1.113	2.753	12.188	0.105	18.286	1.949	5.126	5.076	7.380
1.154	2.625	10.354	0.063	15.651	1.935	4.833	4.800	6.605
1.174	2.567	9.985	0.085	14.774	1.969	4.856	4.810	7.245
1.210	2.469	8.142	0.072	11.935	1.983	4.721	4.674	8.381
1.235	2.405	9.990	0.131	13.495	2.110	5.064	4.991	7.676
1.280	2.297	10.063	0.188	12.973	2.190	5.127	5.022	7.619
1.324	2.200	10.496	0.258	12.483	2.328	5.403	5.261	7.343
1.360	2.126	10.588	0.280	11.947	2.418	5.544	5.388	8.192
1.400	2.049	10.944	0.304	11.481	2.544	5.795	5.625	8.576
1.430	1.995	11.214	0.369	11.298	2.624	5.928	5.724	9.010
1.473	1.923	11.226	0.309	10.762	2.704	5.993	5.820	8.693
1.505	1.872	11.443	0.308	10.893	2.716	5.876	5.709	7.948
1.550	1.804	11.496	0.298	10.650	2.764	5.826	5.667	7.940
1.590	1.748	10.504	0.443	10.029	2.737	5.558	5.315	8.458
1.640	1.683	11.858	0.206	10.190	2.889	5.838	5.731	8.038

Table 2 – continued from previous page

P_{LAB} (GeV/c)	$(1/p_{cm})^2$ (GeV^{-2})	$\frac{d\sigma}{d\Omega}(+1)$ (mb/sr)	$\frac{d\sigma}{d\Omega}(-1)$ (mb/sr)	σ_{el} (mb)	J_0	b_o (GeV^{-2})	b_{o+1} (GeV^{-2})	b_{exp} (GeV^{-3})
1.680	1.634	12.158	0.177	9.899	2.988	5.986	5.894	8.250
1.720	1.587	12.398	0.141	9.833	3.034	5.962	5.890	8.025
1.760	1.543	12.827	0.127	9.688	3.129	6.097	6.033	8.204
1.800	1.502	13.270	0.102	9.649	3.203	6.163	6.113	8.103
1.840	1.462	13.716	0.020	9.393	3.316	6.351	6.341	8.462
1.880	1.425	14.604	0.069	9.731	3.382	6.393	6.361	8.368
1.920	1.389	15.142	0.049	9.738	3.456	6.459	6.437	8.194
1.980	1.338	16.207	0.027	9.768	3.597	6.654	6.642	8.299
2.030	1.299	16.747	0.014	9.641	3.701	6.770	6.764	8.233
2.070	1.269	17.193	0.006	9.587	3.774	6.836	6.834	8.181
2.150	1.213	17.646	0.000	9.328	3.901	6.907	6.907	7.983
2.200	1.181	18.039	0.003	9.112	4.013	7.050	7.048	8.226
2.280	1.132	18.320	0.005	8.908	4.109	7.032	7.030	8.119
2.340	1.098	18.611	0.011	8.732	4.201	7.081	7.077	8.103
2.460	1.036	18.712	0.021	8.401	4.317	6.996	6.988	7.853
2.560	0.989	18.970	0.031	8.233	4.409	6.921	6.910	7.782
2.750	0.910	19.759	0.033	7.824	4.660	7.006	6.994	7.648
3.000	0.824	21.451	0.025	7.593	4.983	7.115	7.106	7.919
3.400	0.715	23.638	0.014	7.095	5.492	7.309	7.305	8.036
3.650	0.660	24.337	0.008	6.800	5.726	7.262	7.259	8.097
4.000	0.596	25.774	0.014	6.544	6.055	7.234	7.230	7.829
5.000	0.467	30.771	0.006	5.818	7.169	7.641	7.639	8.281
6.000	0.383	34.735	0.028	5.329	8.068	7.759	7.753	8.507
10.000	0.223	52.147	0.019	4.601	10.947	7.891	7.888	8.418

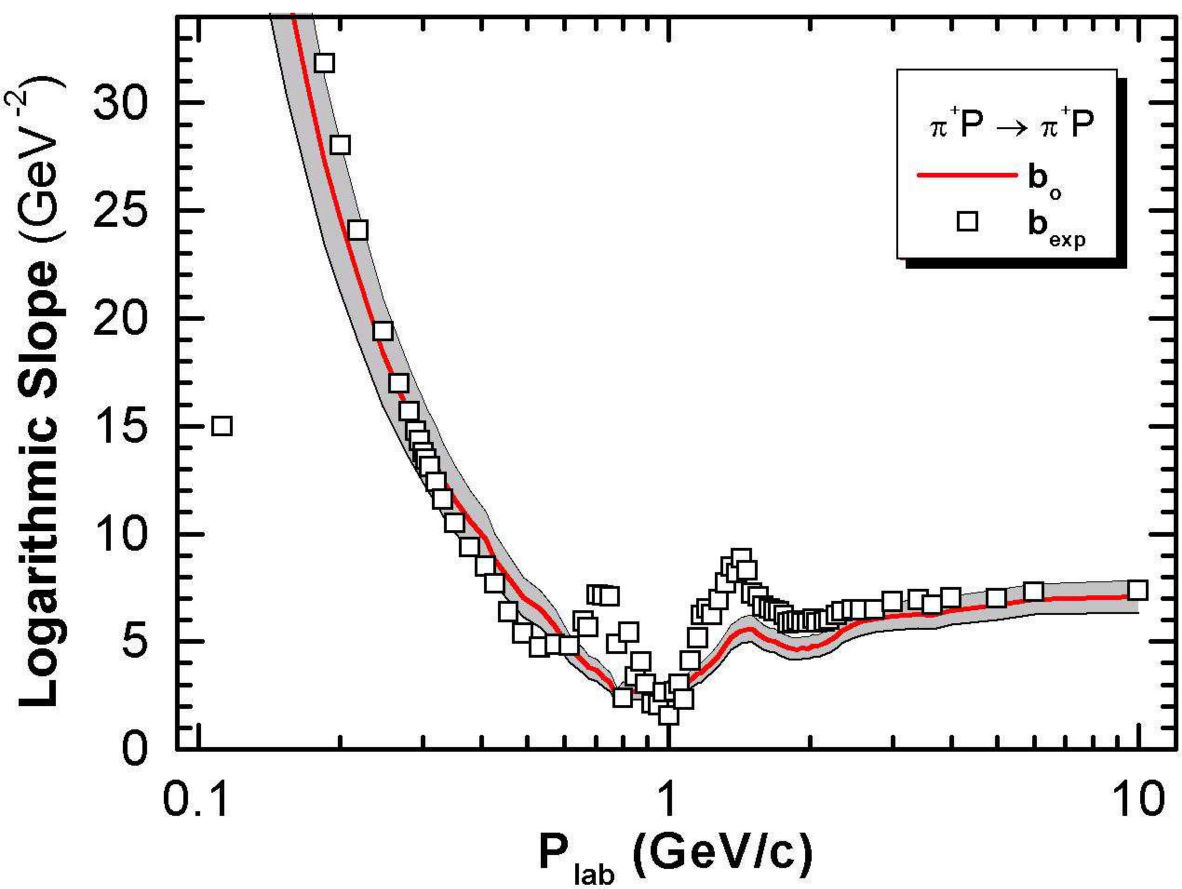

Figure 1: The experimental logarithmic slopes (b$_{exp}$) of the diffraction peak, for the forward $\pi^+ P \to \pi^+ P$ scattering, are compared with the PMD-SQS-optimal predictions b$_o$ (13) (see the text and Table 1).

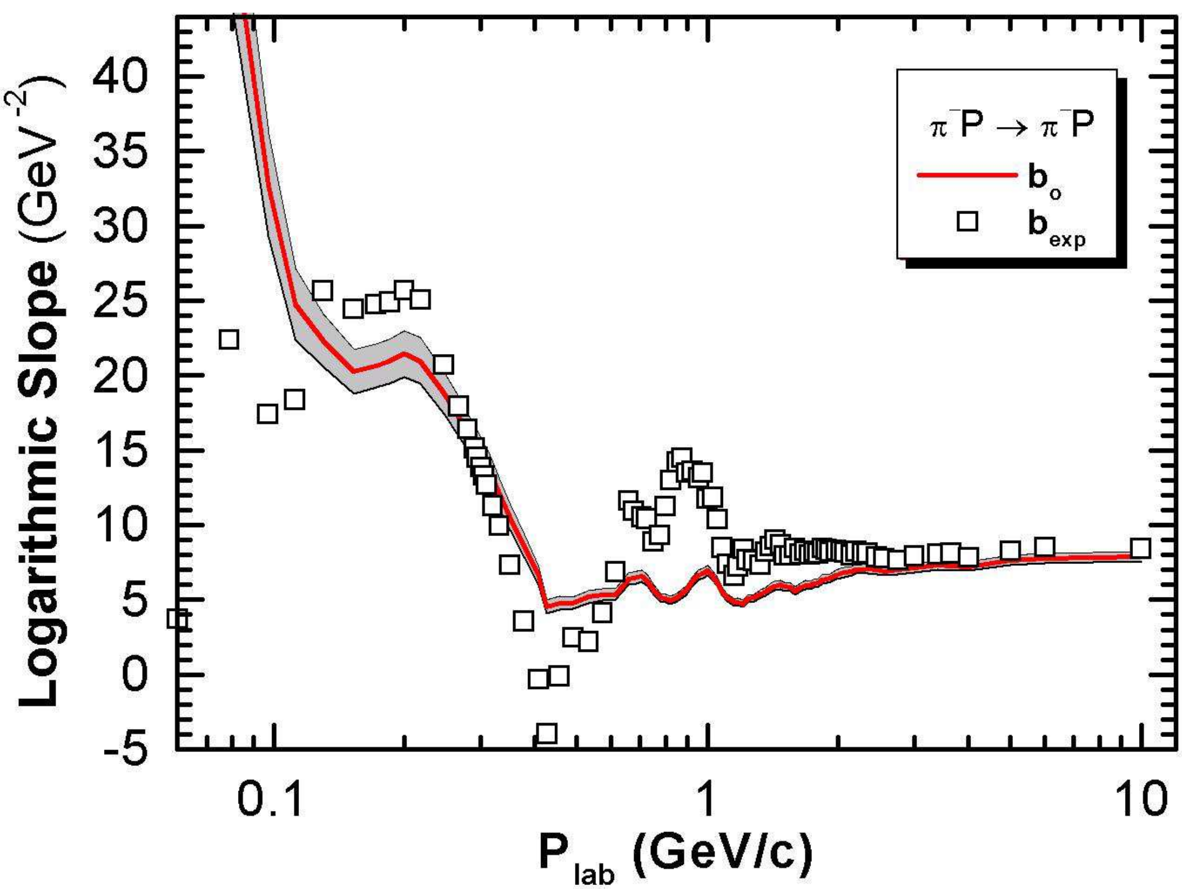

Figure 2: The experimental logarithmic slopes (b_{exp}) of the diffraction peak, for the forward $\pi^+ P \to \pi^+ P$ scattering, are compared with the PMD-SQS-optimal predictions b_o (13) (see the text and Table 2).