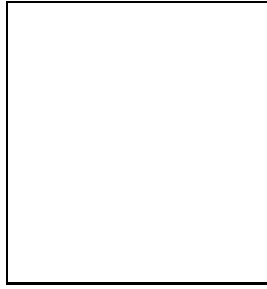


Instanton traces in lattice gluon correlation functions

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Strong coupling constant computed in Landau gauge and MOM renormalization scheme from lattice two and three gluon Green Functions exhibits an unexpected behavior in the deep IR, showing a maximum value around 1GeV. We analyse this coupling below this maximum within a semiclassical approach, where gluon degrees of freedom at very low energies are described in terms of the classical solutions of the lagrangian, namely instantons. We provide some new results concerning the relationship between instantons and the low energy dynamics of QCD, by analysing gluon two- and three-point Green functions separately and with the help of a cooling procedure to eliminate short range correlations.

The running of the strong coupling constant is one of the first predictions that one can extract from perturbation theory in QCD and its behavior at high energies (where perturbation theory makes sense) has been measured in several experiments¹. At lower energies, non perturbative effects appear, and some other alternative to perturbation theory has to be used. In this note we will report on the analysis at the light of semiclassical approximations of numerical results about the low-energy QCD coupling obtained by the use of lattice calculations. The IR behavior of the coupling constant, being a non physically measurable quantity, is nevertheless an interesting object in order to get a better understanding of the non perturbative nature of the theory, as one can compare the results from lattice simulations with some phenomenological model, gaining insight on the phenomena happening at low energies on QCD. In this sense, this paper will recall the lattice calculation of the coupling constant from two- and three-point gluon Green functions, and will present some evidences concerning the influence of semiclassical solutions of the theory (in particular, instantons) over low energy dynamics.

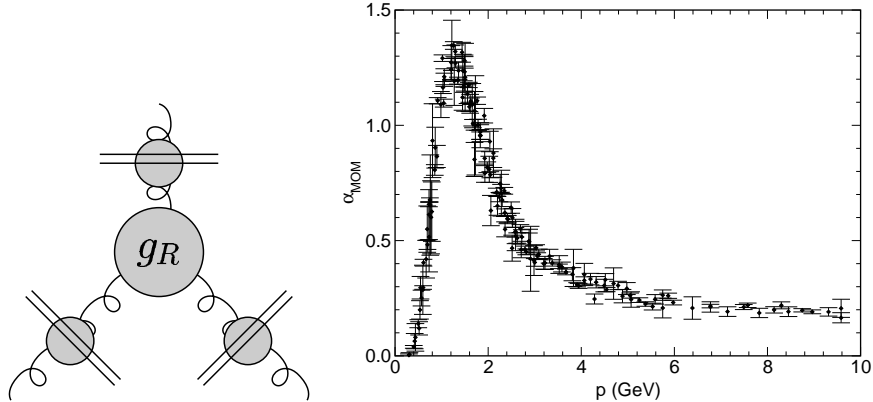


Figure 1: (a) Schematic representation of the renormalization scheme used to define $\alpha_s(p)$, the renormalized vertex is decomposed as the renormalized coupling and the three renormalized propagators and (b) running of the coupling obtained for a wide ensemble of lattice parameters.

1 QCD coupling constant

We will present here results concerning the strong coupling constant computed in MOM renormalization scheme, where renormalized quantities are defined when evaluated at the renormalization scale to take their tree level form, changing bare by renormalized quantities. When applied to the coupling, this defines its renormalized value as the amputated three gluon vertex (See figure 1(a)). In terms of the scalar parts of two and three gluon Green functions², $G^{(2)}(p^2)$ and $G^{(3)}(p^2)$, this can be stated as:

$$\alpha_{\text{MOM}}(p^2) = \frac{p^6 (G^{(3)}(p^2))^2}{4\pi (G^{(2)}(p^2))^3}. \quad (1)$$

Of course, $G^{(3)}$ depends on the three entering momenta (p_1, p_2, p_3) . In this work we will be limited to the simplest case, with $p_1^2 = p_2^2 = p_3^2$, known as symmetric MOM scheme.

The running of the coupling constant computed in the lattice in Landau gauge and MOM renormalization scheme has been analyzed in a series of papers², concluding the presence of power corrections $\sim 1/p^2$ to the purely perturbative running. These corrections appear in a natural way when one performs an Operator Product Expansion (OPE) of the Green functions involved. The OPE relates these power corrections to the existence of a gluon condensate, $\langle A^2 \rangle$, whose value was fixed in the range $\sim 2 - 3 \text{ GeV}^2$. This value can be understood if gluons propagate in a background field provided by instantons⁸.

2 Instantons and low energy dynamics

With this ingredients, the running of the coupling constant for momenta $\gtrsim 2.5 \text{ GeV}$ can be precisely described. Nevertheless, the perturbative description, even with OPE corrections, becomes meaningless for smaller momenta, and in particular beyond the lump (See figure 1(b)). Understanding this unexpected behavior of the renormalized coupling is a key point³ for a full description of low energy QCD dynamics, fundamental for problems like confinement, etc.

Yang-Mills equations have classical solutions like instantons⁴, related to the existence of zero modes in Dirac equation, and explain, for example, the difference between η and η' masses through the De Witten Veneziano formula⁵. They could also give the key to understand the confinement⁶. In Landau gauge, gluon fields for an instanton centered at the origin can be

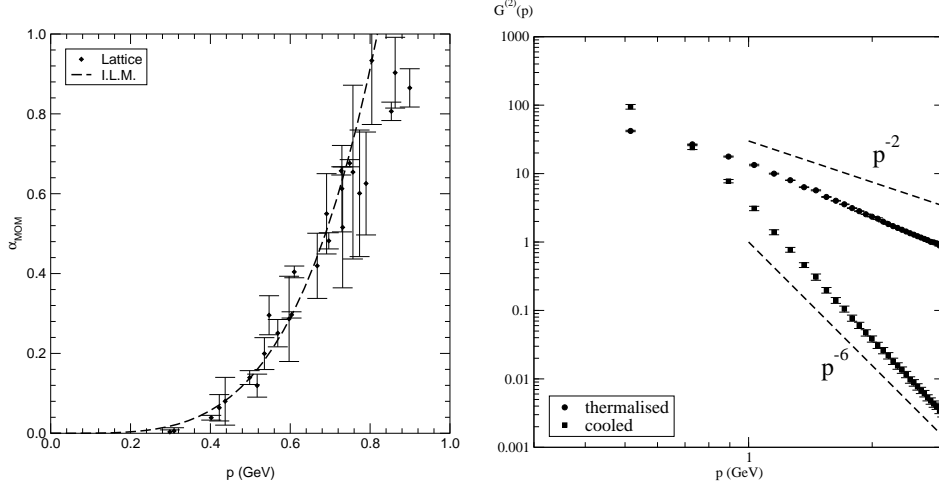


Figure 2: (a) and (b) gluon propagator for the original (thermalised) configurations and for the cooled one; clearly the effect of cooling makes the propagator to fall at high energies like that of an ILM more than that of a massless free field.

expressed as:

$$A_{\mu}^a(x) = \frac{1}{g} \frac{2\bar{\eta}_{\mu\nu}^a x_{\nu} \rho^2}{x^2(x^2 + \rho^2)}, \quad (2)$$

with $\bar{\eta}_{\mu\nu}^a$ t'Hooft's tensor and ρ a parameter that gives instanton size. Instanton liquid models⁵ (ILM) in general treat with ensembles of these structures placed randomly in space with a density n . In principle, in an ILM, there will be correlations between instantons modifying the form (2). The effect of having a different x -dependence is crucial when studying gluon Green functions^a, and has already been treated extensively⁷, but there are some results that do not depend on this spatial “profile” of instantons.

If we assume that long distance (low energy) correlations are dominated by instantons, gluon Green Functions should be described by the equivalent Green functions in the field (2). In this independent pseudo-particle approach (called sum-ansatz in the literature), the combination of Green functions (1) does depend only on the instanton density, n , and not on their spatial profile or size. The prediction of an ILM for the renormalized coupling is then:

$$\alpha_s^{Ins}(p^2) = \frac{1}{18\pi n} p^4. \quad (3)$$

This formula describes nicely the results obtained from the lattice (combining very different lattice spacings and volumes) in the deep IR, below 1 GeV (see plot 2(a)). From a fit to the lattice results⁷ one can extract a value of instanton density of around $5 - 7 \text{ fm}^{-4}$. The same kind of analysis can be made separately for the Green functions⁷ that, in this case, do depend on the profile of instantons. The conclusions are nevertheless the same: for low energies (say below 1 GeV) the running of gluon Green functions can be understood with a simple semiclassical model. This could be related to the confinement, as for very small energies, quantum effects over correlation functions disappears and only classical effects survive.

3 Green functions after cooling

As a further crosscheck, we make use of an extended method to study instanton physics called cooling⁹ consisting in a local iterative procedure that progressively eliminates quantum fluctua-

^aAlso the effect of having instantons with different sizes is important for those cases.

tions, leading to a smooth field distribution, where one can locate instanton lumps. This method has nevertheless a number of known biases as instanton annihilation, modifications of instanton size, etc. that makes hard a quantitative study. Instead of it, we can use cooled configurations to check the results obtained for the thermalised configurations, for example, via the calculation of gluon green functions after cooling.

Although gluon propagator in an ILM depend on the profile of instantons, its high energy limit does not, as it is fixed by boundary conditions to:

$$G^{(2)}(p^2) \rightarrow \frac{1}{g^2} \frac{32n}{p^6}, \quad (4)$$

while, for instance, a boson propagator behaves like $1/p^2$ (excepting logarithms). In lattice simulations, when one applies cooling methods, gluon propagator passes from a behavior $1/p^2$ in the thermalised configuration to $1/p^6$, as predicted by instanton models. This is a clear indication that after eliminating short range correlations all is given by the classical solutions. The same can be stated for three gluon vertex⁷, and therefore for the combination of Green functions that we use to define the coupling^b that grows like p^4 in all the range of energies for cooled configurations.

4 Conclusions

In conclusion, results coming both from the thermalised and cooled configurations confirm the proposed image of the dominance in the large distance regime of QCD of instantons. Altogether with the OPE description of gluon propagator and coupling, this leads to an encouraging image of the running of the coupling where only the lump (where both classical and quantum effects are important) lacks of explanation.

Acknowledgments

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^bOf course in cooled configurations we can not properly speak of a ‘‘coupling constant’’ as it has no physical meaning.