

Observation potential for χ_b at the Tevatron and LHC

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We confirm the results of previous works that the internal motion of quarks inside charmonium mesons increases the cross section of the process $e^+e^- \rightarrow J/\psi\eta_c$. We also show, that this effect increases the widths of the scalar meson decay into two vector ones and state that the decays $\chi_{b0,2} \rightarrow 2J/\psi$ can be used to detect these scalar mesons at Tevatron and LHC colliders.

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I. INTRODUCTION

Recently Belle collaboration has studied the production of pseudoscalar and vector charmonium mesons in electron-positron annihilation at $\sqrt{s} = 10.6$ GeV [1]. The lower bound of the cross section measured at Belle

$$\sigma(e^+e^- \rightarrow \psi\eta_c) > 33 \text{ fb}$$

is about an order of magnitude higher, than the theoretical predictions [2]. Some efforts were made to explain this discrepancy. For example, in [3] it was assumed, that some of the Belle's $\psi\eta_c$ signal could actually be $\psi\psi$ events and the value of the cross-section of two photon process $e^+e^- \rightarrow \psi\psi$ was presented. However in later works [4, 5] it was shown that QCD corrections decrease this cross section and the subsequent experimental [6] analysis excluded this possibility completely. Another possible exclamations could be the contributions of glueball [7] and color octet states to this process

Lately the discrepancy between theoretical predictions and experimental value for the cross section of the process $e^+e^- \rightarrow \psi\eta_c$ has found the surprisingly simple explanation. In the works [8, 9] it was shown, that taking into consideration intrinsic motion of quarks inside mesons one can significantly decrease the virtuality of intermediate particles and hence raise the value of the cross section. The method of light cone expansion, used in these works, is actually an expansion in a small parameter $r \sim M/\sqrt{s}$, where M is the meson mass. It should be mentioned however that this method can lead to large uncertainties, caused by poor knowledge of quark distribution functions. As it was shown in [8] one can significantly change the result by changing the expressions of these functions. Another method, used prior to these works was the expansion in internal velocity v of quarks in a meson. In most of the works considering the process $e^+e^- \rightarrow \psi\eta_c$ only the leading term of this expansion was used and the decrease of virtuality of internal particles was neglected. The purpose of our paper is to consider of this effect in the framework of amplitude expansion over the internal velocity v and comparison of our results with light cone approximations.

Besides the reaction $e^+e^- \rightarrow \psi\eta_c$ there are also other processes in which the intermediate particles propagate with large virtuality when one neglects the internal motion of quarks inside mesons. Among these processes we would like to mention the decays of scalar and tensor mesons into two vector ones, i.e. $\chi_{0,2} \rightarrow VV$. In this paper we will consider some of these decays, namely $\chi_{c0,2} \rightarrow \rho\rho, \phi\phi$ and $\chi_{b0,2} \rightarrow \psi\psi$. The main reasons for this interest are:

- The only known experimental value for such processes [10]

$$\text{Br}(\chi_{c0} \rightarrow \phi\phi) = (1.0 \pm 0.6) \cdot 10^{-3} \quad (1)$$

is much greater than the theoretical predictions made in the assumption that the intrinsic motion of quarks can be neglected (in what follows this assumption will be called "δ-approximation") [11, 12] and one could expect that taking into account internal motion would increase them.

- As it was mentioned in [11], the decays of $\chi_{b0,2}$ can be used to study these mesons in high energy experiments (for example in the reaction $p\bar{p} \rightarrow \chi_{bJ}X \rightarrow \psi\psi X$). The branching ratio of the decay $\chi_{b0} \rightarrow \psi\psi$ obtained in this work

$$\text{Br}(\chi_{b0} \rightarrow \psi\psi) = 3 \cdot 10^{-5}$$

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is however too small to use this mode in studying the properties of χ_{b0} meson. When estimating this branching fraction the authors have neglected the internal motion of c quarks in the hard part of the amplitude. As it will be shown bellow, taking it into account one can increase the value of this branching fraction by an order of magnitude. In this conditions the observation of χ_b mesons in $\psi\psi$ mode is a feasible task.

II. $e^+e^- \rightarrow \psi\eta_c$

The diagrams contributing to the process $e^+e^- \rightarrow \psi\eta_c$ at the leading order in the strong coupling constant α_s are shown on fig.1 and the corresponding amplitude can be written in the form [2]:

$$\mathcal{M} = -\frac{e_c e^2}{s} \bar{v}(k_2) \gamma^\mu u(k_1) \langle \psi + \eta_c | J_\mu(0) | 0 \rangle.$$

If one neglects the internal motion of quarks in mesons, the matrix element of the electromagnetic current is equal to

$$\langle \psi(p, e) + \eta_c(p') | J^\mu(0) | 0 \rangle = i \frac{1024\pi\alpha_s}{3s^2} \Psi_{\eta_c}(0) \Psi_\psi(0) \epsilon^{\mu\nu\lambda\sigma} p_\nu p'_\lambda e_\sigma, \quad (2)$$

where $\Psi_{\psi, \eta_c}(0)$ are the values of ψ and η_c wave functions in the origin. In this approximation the value of the cross section of this process is $\sigma = 1.9$ fb, about an order of magnitude smaller, than the experimental result [1]. As it was mentioned above, a possible reason for this discrepancy is that internal motion of quarks was neglected in the hard scattering part of this process. This effect was studied in the framework of light cone expansion [8, 9], i.e. an expansion in the parameter $r \sim M/\sqrt{s} \sim 1/3$. The calculations show, that one increases greatly the value of the cross section and allows to achieve the same values. It should be mentioned, that the method used in these works can lead to uncertainties caused by the poor knowledge of the distribution functions. One can obtain almost arbitrary result by changing these functions [8].

Another method used in studying the process $e^+e^- \rightarrow \psi\eta_c$ is the expansion of the amplitude in the relative velocity v of quarks in a meson. The potential models [13] give the value $v^2 = 0.23$, so the expansion parameter $v \sim 0.5$, that is certainly larger, than the expansion parameter of the light cone expansion formalism. In our article we intend to consider the decrease of virtuality of intermediate quarks and gluons in the framework of the following simple model. The contribution of the diagram fig.1a to the amplitude of the e^+e^- annihilation into two $c\bar{c}$ pairs and the hadronization of these pairs into final mesons ψ and η_c is proportional to

$$\begin{aligned} \mathcal{M} &\sim l^\mu \frac{\bar{u}(p'/2 + k') \gamma^\nu v(p/2 - k)}{((p + p')/2 + k' - k)^2 ((p + p')/2 + k')^2 - m_c^2} \times \\ &\times \bar{u}\left(\frac{p}{2} + k\right) \gamma_\mu \left(\hat{k} - \hat{p}' - \frac{\hat{p}}{2} + m_c\right) \gamma_\nu v\left(\frac{p'}{2} - k'\right), \end{aligned} \quad (3)$$

where k and k' are the relative momenta of quarks in ψ and η_c mesons, u and v are quark and antiquark bispinors and l^μ is the leptonic current. If the motion of quark-antiquark pair in the meson is considered as non-relativistic, one can expand the expression (3) over the relative momenta k and k' . The expression (2) for example is derived from the first term of this expansion. Taking the higher terms of this expansion we however encounter the difficulty caused by the strong dependence of propagators in eq. (3) on k and k' (actually, at large values of this variables the series will not converge at all). To avoid this difficulty we will use the exact expressions for the denominators in eq.(3) and neglect internal motion of quarks in the numerator of this expression. In the c.m.s. of the $\psi\eta_c$ pair the following formulae are valid:

$$\begin{aligned} p &= \gamma M \{1, \mathbf{0}_\perp, \beta\}, \\ p' &= \gamma M \{1, \mathbf{0}_\perp, -\beta\}, \\ k &= \{\gamma q_\parallel \beta, \mathbf{q}_\perp, \gamma q_\parallel\}, \\ k' &= \{-\gamma q'_\parallel \beta, \mathbf{q}'_\perp, \gamma q'_\parallel\}, \end{aligned}$$

where M is the mass of the charmonium meson (for simplicity we neglect the difference between vector and pseudoscalar meson masses), β is the velocity of one of the charmonia (directed along the Oz axes), \mathbf{q} and \mathbf{q}' are the space parts of the relative momenta k and k' in the rest frames of the corresponding mesons, q_\parallel and q'_\parallel are z-projections of these vectors and \mathbf{q}_\perp and \mathbf{q}'_\perp are their two-dimensional components perpendicular to this axes. From these equations

the following expressions for the scalar products can be obtained:

$$\begin{aligned}
pk' &= -2\beta\gamma^2 M q_{\parallel}', \\
p'k &= 2\beta\gamma^2 M q_{\parallel}, \\
kk' &= -\gamma^2 q_{\parallel} q_{\parallel}' (1 + \beta^2) - \mathbf{q}_{\perp} \mathbf{q}_{\perp}', \\
k^2 &= -q_{\parallel}^2 - \mathbf{q}_{\perp}^2, \\
k'^2 &= -q_{\parallel}'^2 - \mathbf{q}_{\perp}'^2.
\end{aligned}$$

It is clear that the scalar products (pk') and $(p'k)$ are enhanced by the factor $\gamma = \sqrt{s}/2M$, in (kk') there are both enhanced and unenhanced terms and scalar products k^2 and k'^2 do not contain this factor at all. In the light cone expansion formalism the perpendicular components of vectors \mathbf{q} and \mathbf{q}' (i.e. terms that are not enhanced by γ) are neglected. This approximation is valid for reactions with high c.m. energy, where $\gamma \gg 1$, but can lead to sufficient errors in other cases. That is why we will carry out our calculations using two methods — both neglecting contributions of this terms and leaving them nonzero and afterwards we will compare the results.

The internal motion of quarks inside ψ meson will be described by setting its quark and antiquark momenta to $P/2+k$ and $P/2-k$ respectively. Similarly, the momenta of quark and antiquark hadronizing into η_c will be $P' \pm k'/2$. After this substitution the gluon propagator will have the form

$$\left(\frac{p+p'}{2} + k' - k \right)^2 = \frac{s}{4} \left(1 + \frac{4}{s}(p'k - pk') + \frac{4}{s}(k - k')^2 \right) = \frac{s}{4}d, \quad (4)$$

and quark propagators on fig.1 will be equal to

$$\left(\frac{p}{2} + p' - k \right)^2 - m_c^2 = \frac{s}{2} \left(1 - \frac{4}{s}p'k + \frac{2}{s} \left(k^2 + \frac{M^2}{4} - m_c^2 \right) \right) = \frac{s}{2}s_1, \quad (5)$$

$$\left(\frac{p'}{2} + p + k' \right)^2 - m_c^2 = \frac{s}{2} \left(1 + \frac{4}{s}pk' + \frac{2}{s} \left(k'^2 + \frac{M^2}{4} - m_c^2 \right) \right) = \frac{s}{2}s_2. \quad (6)$$

From equations (4), (5), (6) it is clear, that in δ -approximation (i.e. neglecting internal motion and assuming the quark mass $m_c = M/2$) the dimensionless propagators d and $s_{1,2}$ will be equal to 1. Keeping this in mind it is easy to understand that to obtain the expression for any diagram of fig.1 one should divide the expression (2), where this motion was neglected, by this dimensionless propagators and integrate it with the proper chosen wave functions. It is useful to notice, that the diagrams 1c can be obtained by charge conjugation of the diagram 1a, so it is not necessary to calculate it separately (the same is also valid for diagrams 1b and 1d) Thus, the amplitude of the process $e^+e^- \rightarrow \psi\eta_c$ can be written in the form

$$\mathcal{M} = \mathcal{M}_0 \frac{1}{\int d\rho} \int d\rho \frac{1}{2d} \left(\frac{1}{s_1} + \frac{1}{s_2} \right) = \mathcal{M}_0 K, \quad (7)$$

where the integration is hold over the wave functions of ψ and η_c mesons in momentum representation:

$$d\rho = d^3q d^3q' \phi_{\psi}(\mathbf{q}) \phi_{\eta_c}(\mathbf{q}').$$

In equation (7) \mathcal{M}_0 is the amplitude of this process with no internal motion and the factor K describes the decrease of momenta of virtual quarks and gluon. The variation of wave function width results in change of this factor and therefore the variation of annihilation cross section. It is clear, that if we fix the value of the amplitude and tend the width of the wave function to zero (i.e. neglect the internal motion of quarks), than the factor K will tend to 1 and the equation (7) will reduce to (2). It should be mentioned also, the values $\Psi_{J/\Psi}(0)$, $\Psi_{\eta_c}(0)$ also depend on the choice of the wave functions — it is obvious, that with increase of the width of the distribution in \mathbf{q} the values of the wave function in the origin will grow. Thus, the variation of wave function width will lead to double effect: the variation of the momenta of internal particles and the change of the values of wave functions in the origin. Both these effects are taken into account in the equation (7).

III. WAVE FUNCTIONS

In our model we assign the momentum $P/2+k$ to c quark in ψ . It is evident, that this quark is not on mass shell: its virtuality depends on the relative momentum k and equals to $M^2/4+k^2$. The formal use of this formula

can lead to unphysical situations, when the virtuality of c quark is zero or negative. It is obvious, that such cases would occur only if the motion of $c\bar{c}$ pair in the meson is relativistic, whereas in the model used here is valid only for non-relativistic movement, when the virtuality of this quark does not differ significantly from $M/2$. The calculations described below will show, that even in the broad wave-function case ($v^2 = 0.4$) the maximum of the amplitude is accumulated at $-k^2 \sim 1 \text{ GeV}^2$, that corresponds to virtuality $\sim 1.2 \text{ GeV}$.

Another unphysical singularity caused by the fact that the virtuality of quarks and antiquarks can be zero or negative is that the propagators (4), (5),(6) can vanish at some values of quarks' momenta. As a result one can obtain almost arbitrary values for the cross section of the process under consideration by using different distribution functions.

In the previous works the potential quark model was often used to derive the form of the wave function. Such a function, obtained from Schrödinger equation solution allows, although with a small probability, any values of the relative momentum of quarks and we will get an infinitive result when substituting it in the equation (7). This problem is obviously caused by the singularity of propagators in the region $q \sim M/2$, where the motion of the quarks is relativistic and the usage of non-relativistic wave functions is not valid. To avoid this difficulty, when deriving the expression for the wave function we will use another method, proposed in [14].

In what follows we will restrict ourself to the valence quark approximation, i.e. we will suppose that ψ and η_c mesons are built only from $c\bar{c}$ pair. Let the momentum of the quark in the meson rest frame be $q = (q_0, \mathbf{q})$ and the meson's wave function in the momentum representation is $\phi(\mathbf{q})$. Then, in the frame of infinitive momentum the c quark distribution function will be

$$f(x) = \int d^3q |\phi(\mathbf{q})|^2 \delta\left(x - \frac{q_0 + q_z}{M}\right),$$

where M is the meson mass and x is the c -quark momentum fraction. For the mesons with hidden flavour in valence quark approximation we have $q_0 = M/2$. It is useful to introduce the following integration variables:

$$d^3q = \frac{\pi M^2}{2} v dv dq_{\parallel},$$

where $v = 2|\mathbf{q}|/M$. After integration over q_z

$$\begin{aligned} f(x) &= \frac{\pi M^3}{4} \int_{\Phi(x)}^1 dv^2 |\phi(v^2)|^2, \\ \Phi(x) &= (1 - 2x)^2. \end{aligned}$$

Differentiating this relation with respect to x we get

$$f'(x) = -\frac{\pi M^3}{4} \Phi'(x) |\phi(\Phi(x))|^2.$$

From the last formula one can obtain the relation between meson wave function and the structure function in the infinitive momentum frame:

$$|\phi(v^2)|^2 = -\frac{4}{\pi M^3} \frac{df(\Phi^{-1}(v^2))}{dv^2}. \quad (8)$$

Let us use the Regge parametrization of the structure function [15, 16, 17]:

$$f(x) \sim x^{-\alpha} (1-x)^{\gamma-\alpha}.$$

Since we restrict ourself to two-body state $c\bar{c}$ and neglect the contribution of the sea quarks the parameter γ should be set equal to zero. Substituting the expression for the wave function into (8) we get

$$\phi(q) = \left(\frac{-\alpha}{\pi M^3 B(1-\alpha, 1-\alpha)} \right)^{1/2} \left(\frac{1-v^2}{4} \right)^{(-\alpha-1)/2}. \quad (9)$$

Let us determine the value of the parameter α . We can do this by fixing either the parameters of the ψ regge trajectory, or mean square of the velocity of c -quark in meson. In what follows we will use the latter method. The calculations will be held using four different values of mean quark velocity : $v^2 = 0.2, 0.3, 1/3$ and 0.4 . These values correspond to $\alpha = -6.0, -3.5, -3.0$ and -2.25 . It is interesting to notice, that for $v^2 = 1/3$, $\alpha = -3.0$ [16] the wave

function integrated over transverse momentum differs from light cone function used in [9] no more than by 5%. We have chosen the value $v^2 = 1/3$ because the calculation of $\psi \rightarrow e^+e^-$ and $\eta_c \rightarrow \gamma\gamma$ decay widths with this function leads to the values $\Gamma(\psi \rightarrow e^+e^-) = 5.45$ keV, $\Gamma(\eta_c \rightarrow \gamma\gamma) = 7.3$ keV, that are in agreement with the experiment [18, 19].

The results of our calculations for listed values of v^2 are presented in table I. In the first column there are values for $|R(0)|^2 = 4\pi|\Psi(0)|^2$, the second one contains the values of the cross section obtained from equation (2) neglecting internal quark motion in the hard part of the amplitude, the last two ones contain the values of the factor K^2 from formula (7) neglecting and leaving the transverse components of internal momenta k and k' . Finally, in last column we give the values of the total cross section.

The results of our calculations, presented in table I show, that the calculation held with exact dependence of propagators on internal momenta gives the values that are larger, than the values obtained neglecting this motion. For example, for the case $v^2 = 0.4$ the cross section is multiplied by 4. So our assumption that the dependence of propagators on the internal momenta is strong proved to be correct and the expansion of this propagators in relative velocity v leads to the series with rather poor convergence. Although, the effect described above increases the value of the cross section, this enhancement is not large enough to explain the experimental data obtained at Belle. As it was mentioned above, the calculations in the framework of the light cone expansion method [9] give the values that are in agreement with experimental data and, if there is no other reasons that can increase the cross section obtained in the framework of NRQCD by an order of magnitude we can assume that the light cone expansion parameter $M/\sqrt{s} \sim 1/3$ is more suitable for describing the double quarkonia states in the electron- positron annihilation at $\sqrt{s} = 10.6$ GeV, then the relative velocity $v \sim 0.5$.

Another interesting feature that can be seen from table I is that taking into account the terms suppressed by s leads to significant variation of the results. For the narrow wave function with $v^2 = 0.2$ this correction is about 20%, while for the wide one with $v^2 = 0.4$ the cross section is almost doubled. Thus, the transverse components of internal momenta have to be taken into account. In the light cone expansion formalism the terms suppressed by s are neglected and it results in a large error. However, taking this terms into account will only increase the one, and a new result does not contradict the experiment.

IV. $\chi_J \rightarrow VV$

In addition to the $e^+e^- \rightarrow \psi\eta_c$ process considered in the previous sections there are also other reactions in which intermediate particles have large virtuality when δ -approximation is used. Among these processes we would like to note the decays $\chi_{0,2} \rightarrow VV$. In the δ -approximation these decays were already studied in literature [11, 12]

Two diagrams contributing to such decays at the leading order on the strong coupling constant α_s are shown in fig.2, the others can be obtained by interchanging final vector mesons. It can be easily seen, that the virtuality of gluons on these diagrams in the δ -approximation equals $M_\chi^2/4$ and one could expect, that similarly to the results of previous sections it will decrease if the internal motion of quarks in the mesons is taken into account.

Analytical formulae for the width of the decay $\chi_0 \rightarrow VV$ were presented in the work [20] and we will use this results in our paper. The nonzero helicity amplitudes $\mathcal{A}_{\lambda_1, \lambda_2}^{(0)}$ of the decay of scalar meson χ_0 into vector mesons V_1 and V_2 with the helicities λ_1 and λ_2 are given by the expressions

$$\begin{aligned} \mathcal{A}_{1,1}^{(0)} &= \mathcal{A}_{-1,-1}^{(0)} = i \frac{2^{13}}{9\sqrt{3}} \pi^3 \alpha_s^2 \epsilon^2 \frac{|R'(0)|}{M^4} f_T^2 I_{1,1}^{(0)}(\epsilon), \\ \mathcal{A}_{0,0}^{(0)} &= -i \frac{2^{12}}{9\sqrt{3}} \pi^3 \alpha_s^2 \frac{|R'(0)|}{M} f_L^2 I_{0,0}^{(0)}(\epsilon), \end{aligned}$$

where

$$\epsilon = m/M,$$

m and M are masses of vector and scalar mesons respectively, $R(r)$ is the radial part of the scalar meson's wave function, $f_{L,T}$ are longitudinal and transverse leptonic constants of the vector meson and coefficients $I_{\lambda_1, \lambda_2}^{(0)}$ are equal to

$$\begin{aligned} I_{1,1}^{(0)} &= -\frac{1}{32} \int_0^1 dx dy \phi_T(x) \phi_T(y) \frac{1}{xy + (x-y)^2 \epsilon^2} \frac{1}{(1-x)(1-y) + (x-y)^2 \epsilon^2} \times \\ &\times \frac{1}{2xy - x - y + 2(x-y)^2 \epsilon^2} \left[1 + \frac{1}{2} \frac{(x-y)^2 (1-4\epsilon^2)}{2xy - x - y + 2(x-y)^2 \epsilon^2} \right], \end{aligned}$$

$$\begin{aligned}
I_{0,0}^{(0)} &= -\frac{1}{32} \int_0^1 dx dy \phi_T(x) \phi_T(y) \frac{1}{xy + (x-y)^2 \epsilon^2} \frac{1}{(1-x)(1-y) + (x-y)^2 \epsilon^2} \times \\
&\times \frac{1}{2xy - x - y + 2(x-y)^2 \epsilon^2} \left\{ 1 - \frac{1}{2} \frac{(x-y)^2 (1-4\epsilon^2)}{2xy - x - y + 2(x-y)^2 \epsilon^2} - \right. \\
&\left. - 2\epsilon^2 \left[1 + \frac{1}{2} \frac{(x-y)^2 (1-4\epsilon^2)}{2xy - x - y + 2(x-y)^2 \epsilon^2} \right] \right\}.
\end{aligned}$$

In the above equations x and y are the momentum fractions of the final mesons, carried by quarks and $\phi_{L,T}(x)$ are longitudinal and transverse distribution functions of these quarks in mesons. Later we will show, that the contribution of longitudinally polarized mesons to this process is small and one can safely neglect it, as it was done in [11].

In [20] the similar formulae for nonzero helicity amplitudes of tensor meson decay are also presented:

$$\mathcal{A}_{\lambda_1 \lambda_2; \mu}^{(2)} = \tilde{\mathcal{A}}_{\lambda_1 \lambda_2} e^{i\mu\varphi} d_{m, \lambda_1 - \lambda_1}^{(2)}(\theta),$$

where μ is the meson spin projection on fixed axe, θ and φ are polar and azimuthal angles of one of the final mesons in χ_2 rest frame and reduced amplitudes $\tilde{\mathcal{A}}_{\lambda_1 \lambda_2}$ are given by the expressions

$$\begin{aligned}
\tilde{\mathcal{A}}_{1,1} &= \tilde{\mathcal{A}}_{-1,-1} = -i \frac{2^{13} \sqrt{2}}{9\sqrt{3}} \pi^3 \alpha_s^2 \epsilon^2 \frac{|R'(0)|}{M^4} f_T^2 I_{1,1}^{(2)}, \\
\tilde{\mathcal{A}}_{1,0} &= \tilde{\mathcal{A}}_{0,1} = \tilde{\mathcal{A}}_{-1,0} = \tilde{\mathcal{A}}_{0,-1} = -i \frac{2^{12} \sqrt{2}}{9} \pi^3 \alpha_s^2 \epsilon \frac{|R'(0)|}{M^4} f_T f_L I_{1,0}^{(2)}, \\
\tilde{\mathcal{A}}_{1,-1} &= \tilde{\mathcal{A}}_{-1,1} = -i \frac{2^{12}}{9} \pi^3 \alpha_s^2 \frac{|R'(0)|}{M^4} f_T^2 I_{1,-1}^{(2)}, \\
\tilde{\mathcal{A}}_{0,0} &= -i \frac{2^{11} \sqrt{2}}{9\sqrt{3}} \pi^3 \alpha_s^2 \frac{|R'(0)|}{M^4} f_L^2 I_{0,0}^{(2)},
\end{aligned}$$

$$\begin{aligned}
I_{1,1}^{(2)} &= I_{1,1}^{(0)}, \\
I_{1,0}^{(2)} &= -\frac{1}{32} \int_0^1 \int_0^1 dx dy \phi_T(x) \phi_L(y) \frac{1}{xy + (x-y)^2 \epsilon^2} \frac{1}{(1-x)(1-y) + (x-y)^2 \epsilon^2} \times \\
&\times \frac{1}{2xy - x - y + 2(x-y)^2 \epsilon^2} \left[1 + \frac{1}{2} \frac{(x-y)^2 (1-4\epsilon^2)}{2xy - x - y + 2(x-y)^2 \epsilon^2} \right], \\
I_{1,-1}^{(2)} &= -\frac{1}{32} \int_0^1 \int_0^1 dx dy \phi_T(x) \phi_L(y) \frac{1}{xy + (x-y)^2 \epsilon^2} \frac{1}{(1-x)(1-y) + (x-y)^2 \epsilon^2} \times \\
&\times \frac{1}{2xy - x - y + 2(x-y)^2 \epsilon^2}, \\
I_{0,0}^{(2)} &= -\frac{1}{32} \int_0^1 \int_0^1 dx dy \phi_T(x) \phi_L(y) \frac{1}{xy + (x-y)^2 \epsilon^2} \frac{1}{(1-x)(1-y) + (x-y)^2 \epsilon^2} \times \\
&\times \frac{1}{2xy - x - y + 2(x-y)^2 \epsilon^2} \left\{ 1 + \frac{(x-y)^2 (1-4\epsilon^2)}{2xy - x - y + 2(x-y)^2 \epsilon^2} + \right. \\
&\left. + 4\epsilon^2 \left[1 + \frac{1}{2} \frac{(x-y)^2 (1-4\epsilon^2)}{2xy - x - y + 2(x-y)^2 \epsilon^2} \right] \right\}.
\end{aligned}$$

In the literature plenty of different expressions for distribution functions $\phi_{L,T}(x)$ can be found (review is given in [21]) and, as it was shown in [8], the result depends strongly on the their choice. That is why in our calculations we will use several variants:

- Chernyak, Zhitnitsky [20] (in the tables this set will be labeled as "CZ") :

$$\phi_L(x) = \phi_{CZ}(x), \quad \phi_L(x) = \phi_2(x),$$

- Asymptotic [20] (" ϕ_1 ") :

$$\phi_L(x) = \phi_T(x) = \phi_1(x), \tag{10}$$

- Symmetric [16] (" ϕ_3 ")

$$\phi_L(x) = \phi_T(x) = \phi_3(x),$$

- δ -approximation (" δ ")

$$\phi_L(x) = \phi_T(x) = \delta\left(x - \frac{1}{2}\right).$$

All these distributions are normalized by the condition

$$\int_0^1 \phi(x) dx = 1$$

and the effective width of the distribution ϕ is defined as

$$\langle \delta x \rangle_\phi = \int_0^1 dx \phi^2(x) \left(x - \frac{1}{2}\right)^2.$$

In the above equations

$$\begin{aligned} \phi_{CZ}(x) &= 13.2x(1-x) - 36x^2(1-x)^2, & \langle \delta x \rangle_{\phi_{CZ}} &\approx 6.7 \cdot 10^{-2}, \\ \phi_1(x) &= 6x(1-x), & \langle \delta x \rangle_{\phi_1} &\approx 5 \cdot 10^{-2}, \\ \phi_2(x) &= 30x^2(1-x)^2, & \langle \delta x \rangle_{\phi_2} &\approx 3.6 \cdot 10^{-2}, \\ \phi_3(x) &= 140x^3(1-x)^3, & \langle \delta x \rangle_{\phi_3} &\approx 2.8 \cdot 10^{-2}. \end{aligned}$$

By analogy with the reaction $e^+e^- \rightarrow \psi\eta_c$ one can expect, the branching fraction of the decay $\chi_{b0,2} \rightarrow \psi\psi$ will increase with the increase of the width of the distribution.

In our work we consider the decays $\chi_{c0,2} \rightarrow \rho\rho$, $\chi_{c0,2} \rightarrow \phi\phi$ and $\chi_{b0,2} \rightarrow \psi\psi$. Numerical values for parameters are listed in table II. The values of branching fractions and decay widths obtained with the help of these distributions are presented in table III. From this table one can easily see that the branching fractions do strongly depend on the distribution functions and increase with increase of their "widths". As it was mentioned above, the experimental value of the $\chi_{c0} \rightarrow \phi\phi$ branching fraction is much greater than the theoretical predictions made in the framework of δ -approximation. Now internal motion of quarks in mesons raises these predictions. For example, with the distribution (10) we get

$$\text{Br}(\chi_{c0} \rightarrow \phi\phi) = 0.9 \cdot 10^{-3},$$

while the experimental result is $(1.0 \pm 0.6) \cdot 10^{-3}$. The same can be said about the decays $\chi_{b0,2} \rightarrow \psi\psi$. The calculations held in δ -approximation give $\text{Br}(\chi_{b0} \rightarrow \psi\psi) = 1.4 \cdot 10^{-5}$, and now this value can be raised to $2 \cdot 10^{-4}$.

In table IV we present the ratio of helicity amplitudes for production of longitudinally and transversely polarized vector mesons. From this table it is clear that this ratio is less than 15% for all the considered decays and the production of longitudinally polarized mesons can be safely neglected, as it was done in [11].

V. $p\bar{p} \rightarrow \psi\psi X$

As it was mentioned in [11], the decay $\chi_{b0,2} \rightarrow \psi\psi$ can be utilized in experiment to study the properties of χ_{bJ} mesons in high energy reactions (for example, in the inclusive process $p\bar{p} \rightarrow ggX \rightarrow \chi_{b0,2}X \rightarrow \psi\psi X$). It should be mentioned, that in addition to resonance production of $\psi\psi$ pair via χ_{bJ} decays there is also a non-resonance production of $\psi\psi$ pair in continuum and it is important to study the possibility of separating of the resonance signal from this background.

The cross section of the double ψ production in the proton-antiproton annihilation can be related to the cross section of its partonic subprocess $\hat{\sigma}(ab \rightarrow \psi\psi)$ and the structure functions of the initial hadrons according to the relation

$$\sigma(p\bar{p} \rightarrow abX \rightarrow \psi\psi X) = \sum_{a,b} \int d\xi_a d\xi_b f_a^{(p)}(\xi_a) f_b^{(\bar{p})}(\xi_b) \hat{\sigma}(ab \rightarrow \psi\psi), \quad (11)$$

where ξ_a and ξ_b are the momentum fractions of partons a and b , $f_a^{(p)}(\xi_a)$ and $f_b^{(\bar{p})}(\xi_b)$ are the distribution functions of partons a and b in proton and antiproton. It is convenient to use new integration variables $\hat{s} \approx s\xi_a\xi_b$ — the square of the invariant mass of the partonic pair and $\xi = \xi_a - \xi_b$. If we are interested in the central production of $\psi\psi$ pair in the resonance region, we have $\xi_a = \xi_b = M_\chi/\sqrt{s} \ll 1$. In this case the gluonic pair (i.e. $a = b = g$) gives the main contribution to the reaction give and the equation (11) can be written in the form

$$\frac{d\sigma(p\bar{p} \rightarrow \psi\psi X)}{d\hat{s}} = \frac{1}{16} \frac{\hat{\sigma}_{gg}}{\hat{s}} L, \quad (12)$$

where the first factor corresponds to averaging over gluon colors and helicities, $\hat{\sigma}_{gg} = \hat{\sigma}(gg \rightarrow \psi\psi)$, and the dimensionless factor

$$L(\hat{s}) = 2 \int_0^{1-\hat{s}/s} \frac{\xi_a \xi_b}{\xi_a + \xi_b} f_g^{(p)}(\xi_a) f_g^{(\bar{p})}(\xi_b) d\xi \quad (13)$$

describes the partonic luminosity.

The partonic cross section $\hat{\sigma}_{gg}$ equals the sum of cross sections of resonance reaction $p\bar{p} \rightarrow ggX \rightarrow \chi_{bJ}X \rightarrow \psi\psi X$ and the non-resonance background:

$$\hat{\sigma}_{gg} = \hat{\sigma}_r + \hat{\sigma}_{nr}$$

Resonance part of the cross section can be obtained from the values presented in table III with the help of Breit-Wigner formula:

$$\hat{\sigma}_r = \sum_{J=0,2} \frac{\pi}{M_{gg}^2} (2J+1) \frac{\Gamma(\chi_{bJ} \rightarrow gg) \Gamma(\chi_{bJ} \rightarrow \psi\psi)}{(M_{gg} - M_{\chi_{bJ}})^2 + \Gamma_{\chi_{bJ}}^2/4}, \quad (14)$$

where $M_{gg} = \sqrt{\hat{s}}$ — invariant mass of the colliding gluons and total widths of χ_{bJ} -mesons can be obtained from the values of the radial part $R(r)$ of their wave functions in the origin [22]:

$$\begin{aligned} \Gamma_{\chi_{b0}} &= \Gamma(\chi_{b0} \rightarrow gg) = 96 \frac{\alpha_s^2}{M_{\chi_{b0}}^4} |R'(0)|^2 \approx 0.544 \text{ MeV}, \\ \Gamma_{\chi_{b2}} &= \Gamma(\chi_{b2} \rightarrow gg) = \frac{128}{5} \frac{\alpha_s^2}{M_{\chi_{b2}}^4} |R'(0)|^2 \approx 0.14 \text{ MeV}. \end{aligned}$$

The implementation error Δ could however be much smaller than this widths and it can be useful to take it into account. This can be done by means of simple substitution

$$\hat{\sigma}_r^\Delta = \sum_{J=0,2} \frac{\pi}{M_{gg}^2} (2J+1) \frac{\Delta}{\Gamma_{\chi_{bJ}}} \frac{\Gamma(\chi_{bJ} \rightarrow gg) \Gamma(\chi_{bJ} \rightarrow \psi\psi)}{(M_{gg} - M_{\chi_{bJ}})^2 + \Delta^2/4}. \quad (15)$$

It can be easily seen that this substitution does not change the value of the total integrated cross section (i.e. $\int \hat{\sigma}_r dM_{gg} = \int \hat{\sigma}_r^\Delta dM_{gg}$).

Non-resonance part of the cross section (i.e. cross section of the double ψ production in continuum) was obtained in [23, 24]. According to this works

$$\frac{d\hat{\sigma}_{nr}}{d\hat{t}} = \frac{3}{4\pi} \left(\frac{\pi\alpha_s}{3} \right)^4 \left(\frac{|\psi(0)|^2}{M_\psi} \right)^2 \frac{\hat{s}^4 \sum_{n=0}^4 y^n C_n}{(\hat{t} - M_\psi^2)^4 (\hat{u} - M_\psi^2)^4}, \quad (16)$$

where \hat{s} , \hat{t} and \hat{u} are the Mandelstam variables of the partonic subprocess, M_ψ and $\psi(0)$ are mass and wave function of ψ , $y = (\hat{t} + \hat{u})/2\hat{s}$, $z = (\hat{t} - \hat{u})/2\hat{s}$, and the coefficients C_i equals

$$\begin{aligned} C_0 &= \frac{2}{3} (335 - 13568 z^2 + 98144 z^4 + 35584 z^6 - 1186048 z^8 + 258048 z^{10} + 3981312 z^{12}), \\ C_1 &= \frac{8}{3} (137 - 14288 z^2 + 168064 z^4 - 403840 z^6 - 596224 z^8 + 1935360 z^{10}), \\ C_2 &= \frac{8}{3} (-1 - 11472 z^2 + 239008 z^4 - 1010944 z^6 + 1134336 z^8), \\ C_3 &= \frac{64}{3} (5 + 828 z^2 + 688 z^4 - 10176 z^6), \\ C_4 &= 256 (1 + 8 z^2 + 16 z^4). \end{aligned}$$

On fig.3 the dependence of partonic cross section from the invariant mass of colliding gluons in the resonance mass region and the value for the implementation error $\Delta = 50$ MeV is shown. It is clearly seen from this figure that the separation of χ_b signal from the background for most of distributions should not make any troubles. On the other hand for the δ -approximation the signal/background ratio

$$\mathcal{R}^\Delta = \frac{\hat{\sigma}_r^\Delta}{\hat{\sigma}_{nr}} \Big|_{M_{gg}=M_\chi}$$

is not high enough and it may be useful to study the possibilities of increasing this ratio. It can be made by applying certain cuts on the kinematics of final ψ mesons. As it can be seen in fig.4, where the angular distribution of the background cross section over the cosine of scattering angle $x = \cos\theta$ is shown, final ψ mesons fly mainly along the beam axes, while the distribution of ψ produced in χ_b decays is isotropic. That's why applying the condition $|x| < x_{max}$ we can increase the ratio \mathcal{R}^Δ . On fig. 5 the dependence of this ratio from the cut angle is shown.

Let us now return to the total cross section of the $\chi_{b0,2}$ -meson production. According to formula (12) this cross section is equal to

$$\sum_{J=0,2} \sigma(p\bar{p} \rightarrow \chi_{bJ} + X) = \frac{1}{16} \int \frac{d\hat{s}}{\hat{s}} L(\hat{s}) \hat{\sigma}_r(\hat{s}), \quad (17)$$

where $\hat{\sigma}_r(\hat{s})$ is the resonance cross section (14). Because the resonances are narrow, we can neglect the dependence of the partonic luminosity $L(\hat{s})$ on \hat{s} , so the integral over \hat{s} in (17) will reduce to a simple expression:

$$\sum_{J=0,2} \sigma(p\bar{p} \rightarrow \chi_{bJ} + X) = \frac{\pi^2}{4M_\chi^3} L \sum_{J=0,2} (2J+1) \Gamma(\chi_{bJ} \rightarrow gg),$$

where the value of the partonic luminosity L is defined according to formula (13). For the gluon distribution functions in the initial (anti)protons we have used the parameterization presented in [25] at the value of virtuality $Q^2 = M_\chi^2$. The values of χ_{b0} and χ_{b2} production cross sections obtained using this formula are equal to

$$\sigma(p\bar{p} \rightarrow \chi_{b0} + X) = 250 \text{ nb}, \quad \sigma(p\bar{p} \rightarrow \chi_{b2} + X) = 320 \text{ nb}$$

at the Tevatron energies ($\sqrt{s} = 2$ TeV) and

$$\sigma(p\bar{p} \rightarrow \chi_{b0} + X) = 1.5 \mu\text{b}, \quad \sigma(p\bar{p} \rightarrow \chi_{b2} + X) = 2 \mu\text{b}$$

at LHC energies ($\sqrt{s} = 14$ TeV). In the $\psi\psi$ mode these values correspond to

$$\sigma(p\bar{p} \rightarrow \chi_{b0} + X) \text{Br}(\chi_{b0} \rightarrow \psi\psi) = 53 \text{ pb}, \quad \sigma(p\bar{p} \rightarrow \chi_{b2} + X) \text{Br}(\chi_{b2} \rightarrow \psi\psi) = 170 \text{ pb}$$

at $\sqrt{s} = 2$ TeV and

$$\sigma(p\bar{p} \rightarrow \chi_{b0} + X)\text{Br}(\chi_{b0} \rightarrow \psi\psi) = 330 \text{ pb}, \quad \sigma(p\bar{p} \rightarrow \chi_{b2} + X)\text{Br}(\chi_{b2} \rightarrow \psi\psi) = 1 \text{ nb}$$

at $\sqrt{s} = 14$ TeV.

The possibility of quarkonia observation in their $\psi\psi$ decays was also proposed in the works [26, 27]. The value of the η_b production cross section at Tevatron presented there

$$\sigma(p\bar{p} \rightarrow \eta_b + X) = 2.5 \mu\text{b}$$

is about an order of magnitude higher, then our results. This difference can be explained by the difference in the total widths of η_b and χ_b mesons. Inspired by these works the CDF collaboration has measured the $\psi\psi$ mass distribution and has observed a small cluster in the η_b region [28]¹. Since this cluster is small only upper 95% CL upper limit on $\sigma_{\eta_b}(|y| < 0.4)\text{Br}(\eta_b \rightarrow \psi\psi)[\text{Br}(\psi \rightarrow \mu^+\mu^-)]^2$ was found in this experiment. As it can be seen from the $\psi\psi$ mass spectrum, there are also some events in the χ_b region, so one can hope that with the increase of luminosity it will become possible to observe χ_b mesons in the reaction studied in this paper.

It should be mentioned, that our estimates were obtained in the collinear gluon approximation, i.e. using the integrated over the transverse momentum gluon structure functions. As a result of this approximation we have underestimated the values of the χ_b meson production cross section and it is comparable with the cross section of direct Υ production [29]. So, our estimates should be considered as lower bounds and the situation could be more favorable for the χ_b observation in $\psi\psi$ mode.

VI. CONCLUSION

The discrepancy of experimental results and theoretical predictions for the cross section of $\psi\eta_c$ production in e^+e^- annihilation have found a natural explanation by taking into account intrinsic motion of quarks in ψ and η_c mesons in the amplitude of heavy quarks production. At this point our results coincide with the results of the works [8, 9].

We have also studied another example of the reaction $\chi_{0,2} \rightarrow VV$, and specifically the decays of $\chi_{b0}(0^{++})$ and $\chi_{b2}(2^{++})$ into a pair of ψ mesons. It is shown, that taking into account the relative motion of quarks in the amplitude of the decay of χ_b meson to c -quarks increase the branching fractions of these decays by an order of magnitude. The agreement with the experiment was achieved for the only experimentally known decay $\chi_{c0} \rightarrow \phi\phi$.

Later we have considered the possibility of the observation of $\chi_b \rightarrow \psi\psi$ decay in the experiment. The values of the cross sections of χ_b production in $p\bar{p}$ annihilation and its subsequent decay into $\psi\psi$ pair presented in our paper show, that it is possible to observe this reaction at Tevatron and LHC energies.

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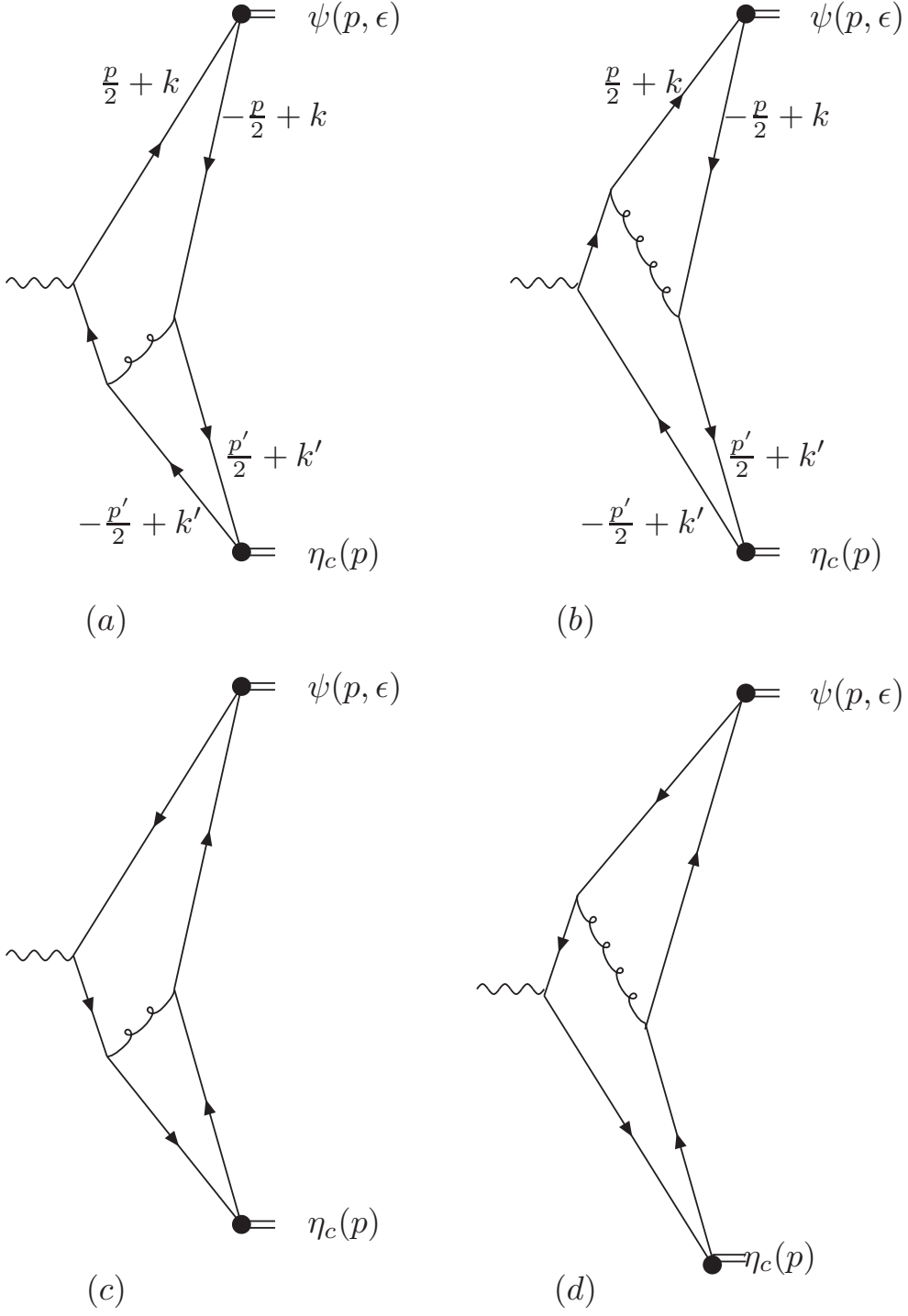
FIG. 1: $e^+e^- \rightarrow \psi\eta_c$

TABLE I: Cross sections of the $e^+e^- \rightarrow \psi\eta_c$ process using different parametrizations of meson wave functions

v^2	$ R(0) ^2(\text{GeV})$	$\sigma_0(\text{fb})$	σ/σ_0	$(\sigma/\sigma_0)_T$	$\sigma(\text{fb})$
0.2	0.31	0.78	1.9	2.3	1.7
0.3	0.50	1.9	2.4	3.5	6.5
1/3	0.55	2.3	2.6	4.0	9.2
0.4	0.65	3.2	2.9	5.2	16.6

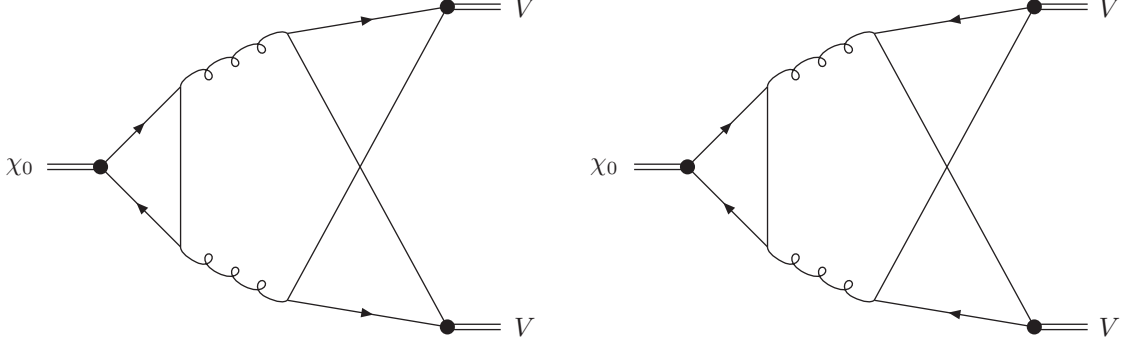
FIG. 2: $\chi_J \rightarrow VV$

TABLE II: Parameters

	M ,	m , GeV	α_s	f_T , GeV	f_L , GeV	$ R'(0) ^2$, GeV^5
$\chi_{c0} \rightarrow \rho\rho$	3.415	0.776	0.3	0.2	0.2	0.11
$\chi_{c0} \rightarrow \phi\phi$	3.415	1.02	0.3	0.2	0.2	0.11
$\chi_{b0} \rightarrow \psi\psi$	9.8599	3.09692	0.2	0.4	0.4	1.34
$\chi_{c2} \rightarrow \rho\rho$	3.55626	0.776	0.3	0.2	0.2	0.11
$\chi_{c2} \rightarrow \phi\phi$	3.55626	1.02	0.3	0.2	0.2	0.11
$\chi_{b2} \rightarrow \psi\psi$	9.9126	3.09692	0.2	0.4	0.4	1.34

TABLE III: $\Gamma(\chi \rightarrow VV)$, keV / $\text{Br}(\chi \rightarrow VV)$, 10^{-4}

	CZ	ϕ_1	ϕ_3	δ
$\chi_{c0} \rightarrow \rho\rho$	24.9 / 35.6	8.98 / 12.9	2.75 / 3.93	1.36 / 1.94
$\chi_{c0} \rightarrow \phi\phi$	16.3 / 23.3	6.31 / 9.02	2.10 / 3.01	1.09 / 1.57
$\chi_{b0} \rightarrow \psi\psi$	0.117 / 2.15	0.0462 / 0.849	0.0157 / 0.288	0.00827 / 0.152
$\chi_{c2} \rightarrow \rho\rho$	6.94 / 43.8	10.6 / 66.6	3.39 / 21.4	1.89 / 12.
$\chi_{c2} \rightarrow \phi\phi$	7.14 / 45.1	9.79 / 61.8	3.38 / 21.3	1.96 / 12.4
$\chi_{b2} \rightarrow \psi\psi$	0.0757 / 5.32	0.100 / 7.05	0.0355 / 2.5	0.0208 / 1.47

TABLE IV: $2|\mathcal{A}_{1,1}^0/\mathcal{A}_{0,0}^{(0)}|^2, \%$

	CZ	ϕ_1	ϕ_3	δ
$\chi_{c0} \rightarrow \rho\rho$	0.306	1.99	2.18	2.65
$\chi_{c0} \rightarrow \phi\phi$	1.23	7.41	7.95	9.43
$\chi_{b0} \rightarrow \psi\psi$	1.61	9.59	10.3	12.1

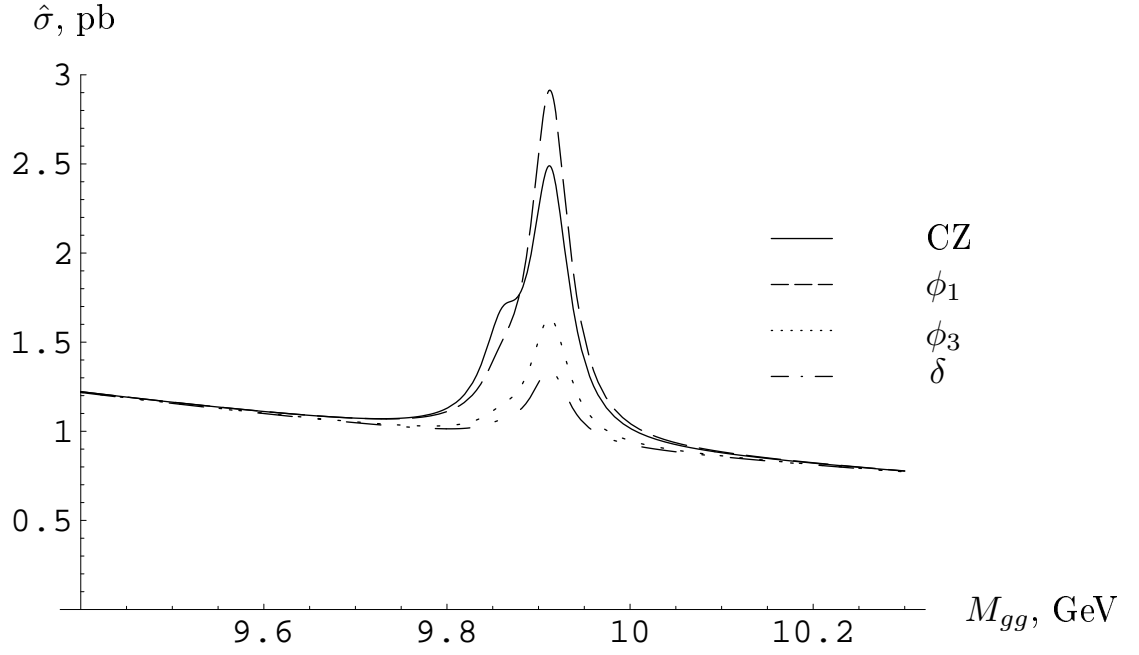


FIG. 3: Distribution of the partonic cross section versus gluon-gluon mass

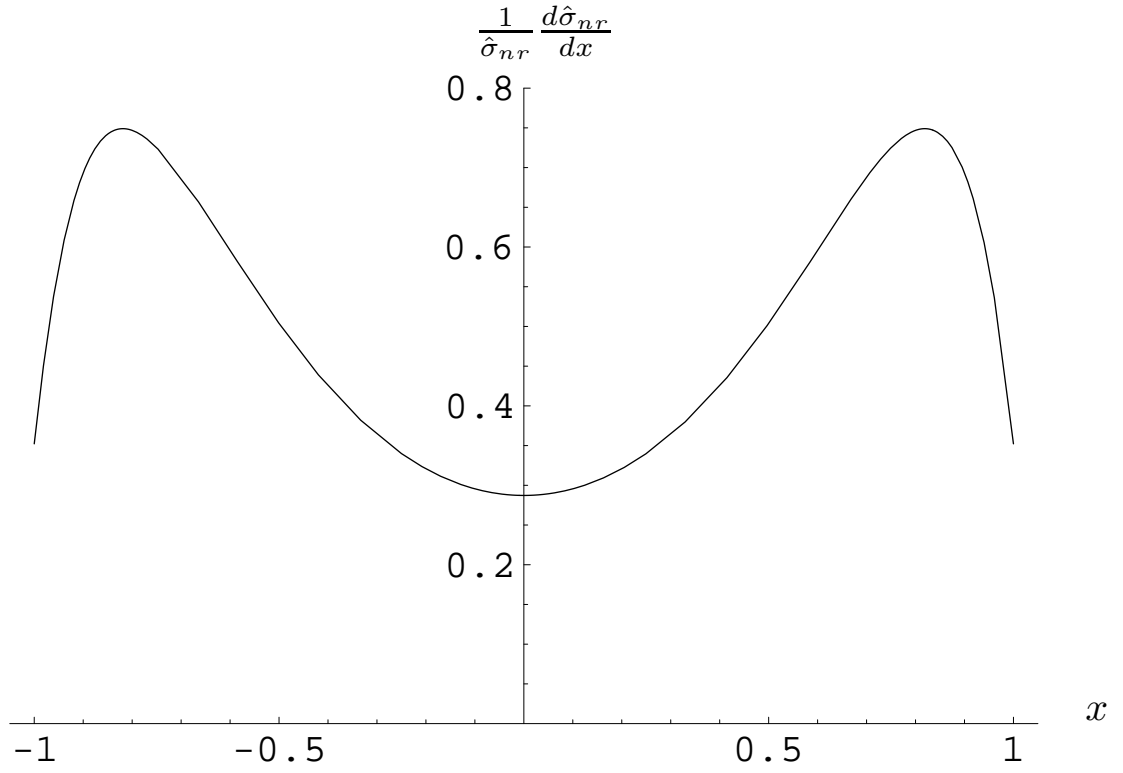


FIG. 4: Angular distribution for the background process

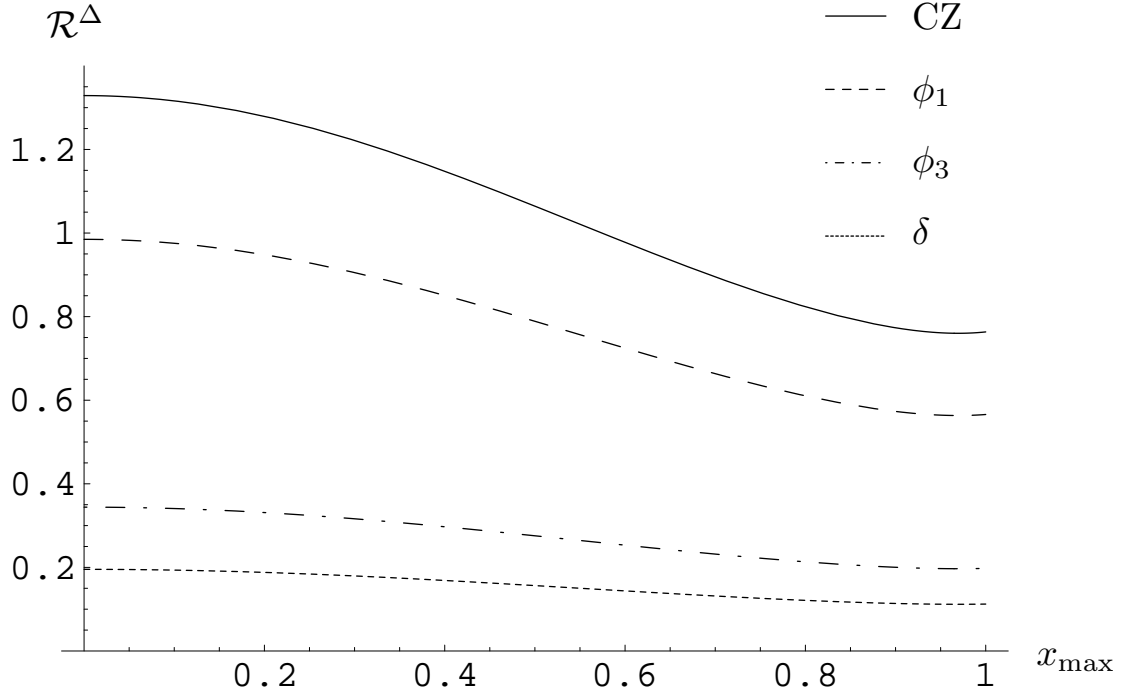


FIG. 5: Signal/background ratio for different distribution functions