

Conformally Sequestered SUSY Breaking in Vector-like Gauge Theories

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Abstract

We provide, in a framework of vector-like gauge theories, concrete models for conformal sequestering of dynamical supersymmetry (SUSY) breaking in the hidden sector. If the sequestering is sufficiently strong, anomaly mediation of the SUSY breaking may give dominant contributions to the mass spectrum of SUSY standard-model particles, leading to negative slepton masses squared. Thus, we also consider a model with gravitational gauge mediation to circumvent the tachyonic slepton problem in pure anomaly mediation models.

1 Introduction

It is widely believed that conformal field theory is dynamically realized in a large class of non-abelian gauge theories with a certain number of matter multiplets (see Ref.[1]). Conformal gauge theory is very attractive in the phenomenological point of view, since if it includes a SUSY-breaking sector, conformal sequestering [2, 3] of the SUSY breaking may occur, providing a solution to the flavor-changing neutral current (FCNC) problem in the supersymmetric standard model (SSM).¹ It is tempting to consider vector-like gauge theories for the SUSY breaking, since they are naturally incorporated into vector-like superconformal gauge theories, which are relatively well understood.

In this paper we extend vector-like gauge theories for the SUSY breaking [5] by adding massive hyperquarks to turn the full high-energy theory above the mass threshold into conformal gauge theory. We find, however, that this simple extension does not achieve the conformal sequestering due to the presence of an unwanted global $U(1)$ symmetry. To eliminate the unwanted global symmetry we introduce non-abelian gauge interactions acting on the additional massive hyperquarks. We find various examples realizing the sequestering.

We first discuss $SP(3N + 1) \times SP(N)$ ⁶ gauge theories where all gauge coupling constants at the infrared fixed point are small for $N > 1$ and perturbative calculations are applicable. We show by an explicit one-loop calculation that the theories have non-trivial fixed points and the sequestering of the SUSY-breaking effects indeed occurs. However, the sequestering is too mild to be applied to the phenomenology, since all the couplings are weak. Therefore, we dwell on strongly coupled conformal gauge theory such as an $SP(3) \times SP(1)$ ² theory in this paper.²

We also propose a Planck-suppressed gauge mediation which circumvents the tachyonic mass problem for sleptons in anomaly mediation.³ Owing to the gravitational nature of this gauge mediation, the size of the gauge-mediated SUSY breaking is at most comparable

¹See Ref.[4] for some other phenomenological applications of superconformal dynamics.

²We are unable to prove explicitly that such a theory has a non-trivial infrared fixed point and the required sequestering is obtained, since gauge couplings are all strong. We only state, in this paper, why we expect that is the case.

³This construction is essentially independent of the above model of conformal SUSY breaking and serves as a generic way to make anomaly mediation phenomenologically viable.

to the anomaly-mediation effects. For the lowest messenger scale, the total model provides a hybrid scheme [6] of anomaly [7] and gauge [8] mediations of SUSY breaking.

2 Conformal SUSY breaking

The IYIT model [5] for SUSY breaking is based on an $SP(N)$ gauge theory with $2(N+1)$ chiral superfields (hyperquarks), Q_α^i , in the fundamental ($2N$ -dimensional) representation.⁴ Here, $\alpha = 1, \dots, 2N$ and $i = 1, \dots, 2(N+1)$. We introduce $(N+1)(2N+1)$ gauge singlet chiral superfields, $S_{ij}(= -S_{ji})$, and impose the flavor $SU(2N+2)$ symmetry in the superpotential,

$$W = \lambda S_{ij} Q^i Q^j, \quad (1)$$

where S_{ij} are assumed to transform as an antisymmetric $(N+1)(2N+1)$ representation of the flavor $SU(2N+2)$ and we omit the color indices for simplicity. The reason why we impose the $SU(2N+2)$ symmetry becomes clear in the next section.

The effective low-energy superpotential is given by

$$W_{\text{eff}} = X(\text{Pf}V^{ij} - \Lambda^{2(N+1)}) + \lambda S_{ij} V^{ij}, \quad (2)$$

in terms of gauge invariant low-energy degrees of freedom $V^{ij} \sim Q^i Q^j$. Here, X is an additional chiral superfield and Λ denotes a dynamical scale of the $SP(N)$ gauge interaction. We see that the superfields S^{ij} have non-vanishing F terms in the vacuum and the SUSY is spontaneously broken. Notice here that the model possesses a $U(1)_R$ symmetry in addition to the flavor $SU(2N+2)$.

2.1 conformality

Now let us introduce $2n_F$ massive hyperquarks, Q^k , where $k = 1, \dots, 2n_F$. The mass term is written as

$$W_{\text{mass}} = \sum_i m Q^i Q^{i+n_F}. \quad (3)$$

Here, i runs from 1 to n_F . Above this mass scale, the high-energy theory is an $SP(N)$ gauge theory with $N_F = 2(N+1) + 2n_F$ hyperquarks. The SUSY $SP(N)$ gauge theory

⁴We adopt the notation where $SP(1) = SU(2)$.

with N_F hyperquarks is expected to be scale-invariant in the infrared for $3(N + 1) < N_F < 6(N + 1)$ [9].

We check, in the following, that the theory with the superpotential Eq.(1) can also be scale-invariant in the infrared. The NSVZ beta function [10] relates the running of the canonical gauge coupling constant to the anomalous dimension factors, γ_Q and $\gamma_{Q'}$, of the hyperquarks, Q and Q' , as

$$\mu \frac{d}{d\mu} \alpha_g = -\alpha_g^2 \left[\frac{3(N + 1) - (N + 1)(1 - \gamma_Q) - n_F(1 - \gamma_{Q'})}{2\pi - (N + 1)\alpha_g} \right], \quad (4)$$

where α_g is defined in terms of the gauge coupling constant g of $SP(N)$ as $\alpha_g = g^2/(4\pi)$ and μ denotes the renormalization scale. Here and hereafter in this section, we neglect the masses of the hyperquarks Q^k . The beta function of the Yukawa coupling constant in Eq.(1) is also given in terms of the anomalous dimension factors of the hyperquarks, γ_Q , and of the singlet chiral fields, γ_S , by

$$\mu \frac{d}{d\mu} \alpha_\lambda = \alpha_\lambda (\gamma_S + 2\gamma_Q), \quad (5)$$

where α_λ is defined in terms of the Yukawa coupling constant λ as $\alpha_\lambda = \lambda^2/(4\pi)$.

When the theory is scale-invariant with non-vanishing coupling constants, the beta functions in Eqs.(4) and (5) vanish. That is, we have, at the infrared fixed point,

$$3(N + 1) - (N + 1)(1 - \gamma_Q) - n_F(1 - \gamma_{Q'}) = 0, \quad (6)$$

$$\gamma_S + 2\gamma_Q = 0. \quad (7)$$

These conditions determine the anomalous dimensions at the fixed point. The anomalous dimensions at the fixed point are consistent with the unitarity of the theory for

$$-1 \leq \gamma_Q, \quad -1 \leq \gamma_{Q'}, \quad 0 \leq \gamma_S, \quad (8)$$

which comes from the restriction for unitary representation of the superconformal algebra [11].⁵ the above anomalous dimensions are consistent with the unitarity conditions for any gauge-singlet chiral multiplets such as QQ , $Q'Q'$, and S .

⁵Combining Eqs.(7) and (8), we also obtain $\gamma_Q \leq 0$ and $\gamma_S \leq 2$. The asymptotic freedom of $SP(N)$, namely, $n_F < 2(N + 1)$, results in $\gamma_{Q'} < -\gamma_Q/2 \leq 1/2$ from Eqs.(6) and (8).

Notice that the vanishing of the NSVZ beta function is consistent with the existence of the anomaly free $U(1)_R$ symmetry that enters in the superconformal algebra with the charges of the matter fields given by $R_i = (2 + \gamma_i)/3$; $i = Q, Q', S$ [11]. In this simple extension of the IYIT model, the anomalous dimensions cannot be determined uniquely from Eqs.(6) and (7), and hence, the charge assignment of the $U(1)_R$ is not determined only with this information.

Now, we show by a perturbative calculation that the fixed point is infrared stable. We first see that the gauge and Yukawa coupling constants at the infrared fixed point are small if N_F is just below $6(N + 1)$, as in the case of the Banks-Zaks fixed point [12]. In this case we can obtain the anomalous dimensions at the one-loop level as

$$\gamma_Q = \frac{2N + 1}{2\pi}\alpha_\lambda - \frac{2N + 1}{4\pi}\alpha_g, \quad (9)$$

$$\gamma_{Q'} = -\frac{2N + 1}{4\pi}\alpha_g, \quad (10)$$

$$\gamma_S = \frac{2N}{2\pi}\alpha_\lambda. \quad (11)$$

For $n_F = 2(N + 1) - \varepsilon$, we determine the coupling constants at the fixed point from Eqs.(6) and (7), as

$$\alpha_g^* = \frac{4\pi\varepsilon}{7N^2 + 9N + 2} \left(\frac{3N + 1}{2N + 1} \right) \left(1 + \mathcal{O}\left(\frac{\varepsilon}{N}\right) \right), \quad (12)$$

$$\alpha_\lambda^* = \frac{2\pi\varepsilon}{7N^2 + 9N + 2} \left(1 + \mathcal{O}\left(\frac{\varepsilon}{N}\right) \right), \quad (13)$$

and the one-loop approximation is justified a posteriori for small ε/N .⁶

We can explicitly examine the infrared stability of the fixed point by considering the renormalization group (RG) evolutions near the fixed point. The RG equations of the small deviations,

$$\Delta\alpha_g \equiv \alpha_g - \alpha_g^*, \quad \Delta\alpha_\lambda \equiv \alpha_\lambda - \alpha_\lambda^*, \quad (14)$$

are given by

$$\mu \frac{d}{d\mu} \Delta\alpha_g = \left. \frac{\partial\beta_g}{\partial\alpha_g} \right|_* \Delta\alpha_g + \left. \frac{\partial\beta_g}{\partial\alpha_\lambda} \right|_* \Delta\alpha_\lambda, \quad (15)$$

$$\mu \frac{d}{d\mu} \Delta\alpha_\lambda = \left. \frac{\partial\beta_\lambda}{\partial\alpha_g} \right|_* \Delta\alpha_g + \left. \frac{\partial\beta_\lambda}{\partial\alpha_\lambda} \right|_* \Delta\alpha_\lambda, \quad (16)$$

⁶A non-perturbative determination of the coupling constants through a -maximization [13] is given in the Appendix A.

where β_G and β_λ denote the beta functions of α_g and α_λ given by Eqs.(4) and (5), respectively, and the values with the subscript “*” are evaluated at the fixed point. By using Eqs.(9)-(13), we find that all the eigenvalues of the coefficient matrix $\{\partial\beta_k/\partial\alpha_l\}$ are positive at the fixed point in Eqs.(12) and (13), where $k, l = g, \lambda$. Therefore, the fixed point in Eqs.(12) and (13) is infrared stable at least against small deviations from the fixed point.⁷

2.2 non-sequestering

We are at the point to show that the sequestering of the SUSY breaking does not occur due to an unwanted global $U(1)$ symmetry in this simple extension. By following Luty and Sundrum [2], we consider the RG evolutions of the wave function renormalization factors near the fixed point,

$$\frac{d}{dt}\Delta\ln Z_i = -\gamma_i + \gamma_i^*, \quad \Delta\ln Z_i \equiv \ln Z_i + \gamma_i^* t, \quad t \equiv \ln(\mu/M_*), \quad (17)$$

where $i = Q, Q', S$ and γ_i^* are the anomalous dimensions at the fixed point given by Eqs.(9)-(11). Here, M_* denotes the scale where the theory enters the conformal regime below the reduced Planck scale $M_G \simeq 2.4 \times 10^{18}$ GeV. The deviations from the fixed point can be parameterized by $\Delta\alpha_g$ and $\Delta\alpha_\lambda$ which, in turn, can be expressed as⁸

$$\Delta\alpha_g = \frac{\alpha_g^2}{2\pi - (N+1)\alpha_g} \Big|_* ((N+1)\Delta\ln Z_Q + n_F\Delta\ln Z_{Q'}), \quad (18)$$

$$\Delta\alpha_\lambda = -\alpha_\lambda^*(2\Delta\ln Z_Q + \Delta\ln Z_S). \quad (19)$$

By using the above expressions, we rewrite the RG equation Eq.(17) as

$$\frac{d}{dt}\Delta\ln Z_i = -\left(\frac{\partial\gamma_i}{\partial\alpha_g}\right) \Big|_* \Delta\alpha_g - \left(\frac{\partial\gamma_i}{\partial\alpha_\lambda}\right) \Big|_* \Delta\alpha_\lambda \quad (20)$$

$$= -\left(\left(\frac{\partial\gamma_i}{\partial\alpha_g}\right) \frac{\alpha_g^2}{2\pi - (N+1)\alpha_g}\right) \Big|_* ((N+1)\Delta\ln Z_Q + n_F\Delta\ln Z_{Q'}) + \left(\left(\frac{\partial\gamma_i}{\partial\alpha_\lambda}\right) \alpha_\lambda\right) \Big|_* (2\Delta\ln Z_Q + \Delta\ln Z_S), \quad (21)$$

⁷Similar situations of the conformal fixed point with non-trivial Yukawa interactions are discussed in Refs. [14].

⁸Without loss of generality, we adopt the convention of the holomorphic gauge coupling in Ref.[2].

and we define the coefficient matrix L_{ij} by

$$\frac{d}{dt}\Delta \ln Z_i = \sum_{j=Q,Q',S} L_{ij}\Delta \ln Z_j. \quad (22)$$

When all the eigenvalues of L are positive, all $\Delta \ln Z_i$ go to zero as $t \rightarrow -\infty$ (the infrared limit) and hence the SUSY breaking is sequestered [2, 3]. Unfortunately, we find that the coefficient matrix L has a zero eigenvalue. Thus, one linear combination of $\Delta \ln Z_i$ is constant in the course of the RG evolution and it is not suppressed at the infrared fixed point. We call it as $\Delta \ln \bar{Z}$. Since the vanishing eigenvalue corresponds to the eigenvector $(\Delta \ln Z_Q, \Delta \ln Z_{Q'}, \Delta \ln Z_S) = (1, -(N+1)/n_F, -2)$, we find that the solution to the Eq.(22) in the infrared limit is

$$\Delta \ln Z_Q \propto (\Delta \ln \bar{Z})_0, \quad (23)$$

$$\Delta \ln Z_{Q'} \propto -\frac{N+1}{n_F}(\Delta \ln \bar{Z})_0, \quad (24)$$

$$\Delta \ln Z_S \propto -2(\Delta \ln \bar{Z})_0, \quad (25)$$

with an $\mathcal{O}(1)$ proportionality factor, where $(\Delta \ln \bar{Z})_0$ denotes the value at $t = 0$. In general, the initial value $(\Delta \ln \bar{Z})_0$ contains visible sector superfields q_i as weakly coupled spectators such as

$$(\Delta \ln \bar{Z})_0 \supset \frac{\kappa_{ab}}{M_G^2} q_a^\dagger q_b, \quad (26)$$

where κ_{ab} denote $\mathcal{O}(1)$ coefficients. Therefore, from Eq.(25), we find that the SUSY breaking effects to the visible sector are not sequestered.⁹

The reason of our failure can be traced to the existence of a global $U(1)$ symmetry [2, 3] under which the SUSY breaking superfield S_{ij} transforms non-trivially. In general, when an anomaly-free (non- R) $U(1)$ symmetry exists, the charge assignment ω_i determines the eigenvector of the coefficient matrix L_{ij} for a vanishing eigenvalue:

$$\sum_j L_{ij}\omega_j = 0. \quad (27)$$

In the present case, a linear combination $\Delta \ln \bar{Z}$ (of $\Delta \ln Z_i$) remains constant in the infrared limit and the SUSY breaking effects are not sequestered if the SUSY breaking

⁹In Eqs.(23)-(25), we assume that other eigenvalues of L are positive. Even if it is not the case, the conclusion is not changed.

	$SP(N)$	$SP(N')$	$SP(N')$	$SP(N')$	$SP(N')$	$SP(N')$	$SP(N')$
$Q \times 2(N+1)$	\square_{2N}	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Q'	\square_{2N}	$\square_{2N'}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Q'	\square_{2N}	$\mathbf{1}$	$\square_{2N'}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Q'	\square_{2N}	$\mathbf{1}$	$\mathbf{1}$	$\square_{2N'}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Q'	\square_{2N}	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square_{2N'}$	$\mathbf{1}$	$\mathbf{1}$
Q'	\square_{2N}	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square_{2N'}$	$\mathbf{1}$
Q'	\square_{2N}	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square_{2N'}$
S_{ij}	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

Table 1: The matter contents in our perturbative example, $SP(N) \times SP(N')^6$, ($N = 3N' + 1$). Here, the subscripts of the fundamental representations denote the dimensions of the representations. In terms of the $SP(N)$ gauge theory, the number of the fundamental representation is given by $N_F = 2(N + 1) + 6 \times 2N' = 6N - 2$, while the number of the fundamental representation of each $SP(N')$ gauge theory is given by $N'_F = 2N = 6N' + 2$.

superfields have non-vanishing charges. The eigenvector we have found above corresponds to the charge assignment $(\omega_Q, \omega_{Q'}, \omega_S) = (1, -(N + 1)/n_F, -2)$ of an anomaly-free $U(1)$ symmetry. Thus, in order to realize the sequestering, we should violate the global $U(1)$ symmetry under which the SUSY breaking superfields transform non-trivially, provided we do not take $(\Delta \ln \bar{Z})_0 = 0$ by fine tuning. In the next section, we introduce additional gauge symmetries, where the unwanted $U(1)$ symmetry is broken by anomaly due to the new gauge interactions.

3 Conformally sequestered extensions

We introduce gauge interactions acting on the massive hyperquarks, Q^k , where the unwanted global $U(1)$ symmetry is broken by anomaly due to the new gauge interactions. We deal, in this section, with $SP(N) \times SP(N')^6$, ($N = 3N' + 1$) gauge theory, where the former $SP(N)$ corresponds to the gauge group for the SUSY breaking and the latter $SP(N')^6$ gauge group is introduced to break the unwanted $U(1)$ symmetry. We list all the matter contents in Table 1. We take such a large gauge group to see explicitly by a perturbative calculation that the conformal sequestering occurs. Indeed all the couplings at the infrared fixed point are weak for $N' > 1$ in the present model.

3.1 conformality

Now, we check that the theory with the extended gauge symmetry can be scale-invariant in the infrared. In this model, the beta functions of the $SP(N)$ gauge coupling constant α_g , the $SP(N')$ gauge coupling constant $\alpha_{g'}$, and the Yukawa coupling constant α_λ in Eq.(1) are given by

$$\mu \frac{d}{d\mu} \alpha_g = -\alpha_g^2 \left[\frac{3(N+1) - (N+1)(1-\gamma_Q) - 6N'(1-\gamma_{Q'})}{2\pi - (N+1)\alpha_g} \right], \quad (28)$$

$$\mu \frac{d}{d\mu} \alpha_{g'} = -\alpha_{g'}^2 \left[\frac{3(N'+1) - N(1-\gamma_{Q'})}{2\pi - (N'+1)\alpha_{g'}} \right], \quad (29)$$

$$\mu \frac{d}{d\mu} \alpha_\lambda = \alpha_\lambda (\gamma_S + 2\gamma_Q), \quad (30)$$

where we have assumed that all the $SP(N')$ sectors are equivalent. Namely, we have imposed an exchange symmetry between any two $SP(N')$'s in the $SP(N')^6$ so that the $SP(N')^6$ has a common gauge coupling constant $\alpha_{g'}$. Then, by requiring all the beta functions to vanish, we determine the anomalous dimensions uniquely as

$$\gamma_Q = -\frac{2(N(N+1) - 9N'(N'+1))}{N(N+1)} = -\frac{4}{9N'^2 + 9N' + 2}, \quad (31)$$

$$\gamma_{Q'} = \frac{N - 3(N'+1)}{N} = -\frac{2}{3N'+1}, \quad (32)$$

$$\gamma_S = -2\gamma_Q. \quad (33)$$

Here, we have neglected the masses of the hyperquarks Q' .

We also determine the coupling constants at the infrared fixed point by a perturbative calculation. The anomalous dimensions at the one-loop level are given by

$$\gamma_Q = \frac{2N+1}{2\pi} \alpha_\lambda - \frac{2N+1}{4\pi} \alpha_g, \quad (34)$$

$$\gamma_{Q'} = -\frac{2N'+1}{4\pi} \alpha_{g'} - \frac{2N+1}{4\pi} \alpha_g, \quad (35)$$

$$\gamma_S = \frac{2N}{2\pi} \alpha_\lambda. \quad (36)$$

Then, Eqs.(31)-(36) determine the coupling constants at the infrared fixed point by

$$\frac{N+1}{2\pi} \alpha_g^* = \frac{8(9N'+4)}{3(2N'+1)(3N'+1)^2}, \quad (37)$$

	$\{a_g^*, a_{g'}^*, a_\lambda^*\}$	eigenvalues of M
$N' = 2$	$\{0.2, 0.07, 0.1\}$	$\{0.5, 0.1, 0.002\}$
$N' = 3$	$\{0.1, 0.1, 0.07\}$	$\{0.2, 0.05, 0.002\}$
$N' = 4$	$\{0.07, 0.1, 0.04\}$	$\{0.1, 0.03, 0.001\}$
$N' = 5$	$\{0.05, 0.09, 0.03\}$	$\{0.1, 0.02, 0.0006\}$

Table 2: Stability of the infrared fixed point. $a_{g,g',\lambda}$ denote the coupling constant, $a_g^* = \alpha_g^*(N+1)/(2\pi)$, $a_{g'}^* = \alpha_{g'}^*(N'+1)/(2\pi)$, and $a_\lambda^* = \alpha_\lambda^*N/\pi$. All the eigenvalues of the coefficient matrix M are positive for $N' > 1$.

$$\frac{N'+1}{2\pi}\alpha_{g'}^* = \frac{12(N'+1)(3N'^2 - 3N' - 2)}{(2N'+1)(3N'+2)(3N'+1)^2}, \quad (38)$$

$$\frac{N}{\pi}\alpha_\lambda^* = \frac{8}{(3N'+2)(3N'+1)}. \quad (39)$$

We see that all the coupling constants are small and the perturbative calculation is reliable for $N' > 1$.¹⁰

The above result enables us to explicitly analyze the infrared stability of the fixed point in the same way as done in the previous section. The RG equations of the small deviations $\Delta\alpha_k \equiv \alpha_k - \alpha_k^*$, ($k = g, g', \lambda$) are given by

$$\mu \frac{d}{d\mu} \Delta\alpha_k = \sum_{l=g,g',\lambda} M_{kl} \Delta\alpha_l, \quad (40)$$

where the coefficient matrix M is defined by

$$M_{kl} = \left. \frac{\partial\beta_k}{\partial\alpha_l} \right|_*. \quad (41)$$

Here, “*” indicates the values evaluated at the fixed point. If all the eigenvalues of the coefficient matrix are positive, the fixed point in Eqs.(37)-(39) is infrared stable. In Table 2, we show numerical results on the eigenvalues of the matrix M for the case of $N' > 1$. From the table, we see that eigenvalues are all positive for $N' > 1$. Therefore, we find that the $SP(3N'+1) \times SP(N')$ ⁶ gauge theory has the stable infrared fixed point in Eqs.(31)-(33).

¹⁰For $N' = 1$, although the anomalous dimensions in Eq.(33) satisfy the unitarity bound Eq.(8), the gauge coupling constants of $SP(N')$ in Eq.(37) turns out to be negative, which implies that the perturbative description is invalid.

3.2 sequestering

We now discuss the sequestering of the SUSY breaking. The RG equations of the wave function renormalization factors Z_i near the fixed point are given by

$$\frac{d}{dt}\Delta \ln Z_i = - \sum_{k=g,g',\lambda} \left(\frac{\partial \gamma_i}{\partial \alpha_k} \right) \Big|_* \Delta \alpha_k \quad (42)$$

$$= \sum_{i=Q,Q',S} L_{ij} \Delta \ln Z_j. \quad (43)$$

Here, the coefficient matrix L in the second line is given by using the following relations:

$$\Delta \alpha_g = \frac{\alpha_g^2}{2\pi - (N+1)\alpha_g} \Big|_* ((N+1)\Delta \ln Z_Q + 6N'\Delta \ln Z_{Q'}), \quad (44)$$

$$\Delta \alpha_{g'} = \frac{\alpha_{g'}^2}{2\pi - (N'+1)\alpha_{g'}} \Big|_* N\Delta \ln Z_{Q'}, \quad (45)$$

$$\Delta \alpha_\lambda = -\alpha_\lambda^* (2\Delta \ln Z_Q + \Delta \ln Z_S). \quad (46)$$

The sequestering of the SUSY breaking is realized when all the eigenvalues of L are positive.

Interestingly, as we show below, the coefficient matrix L has the same eigenvalues as the coefficient matrix M in Eq.(41). Therefore, the sequestering occurs automatically, if the infrared fixed point determined in Eqs.(31)-(33) is stable. To prove that, we rewrite the conditions for the vanishing beta functions as

$$\sum_{j=Q,Q',S} A_{kj} \gamma_j = b_k, \quad (47)$$

where the coefficient matrix A and the vector b can be read off from Eqs.(28)-(30) and $k = g, g', \lambda$. Then, we see the following relations:

$$M_{kl} = \sum_{j=Q,Q',S} A_{kj} \Gamma_{jl}, \quad (48)$$

$$L_{ij} = \sum_{k=g,g',\lambda} \Gamma_{ik} A_{kj}, \quad (49)$$

where we have defined

$$\Gamma_{ik} \equiv \left(\frac{\partial \gamma_i}{\partial \alpha_k} \right) \Big|_*. \quad (50)$$

Since the coefficient matrix A is invertible, the coefficient matrices M and L are similar to each other, so that they have the same eigenvalues. Therefore, the sequestering occurs automatically when the anomalous dimensions are uniquely determined by the conditions for the vanishing beta functions (i.e. A is invertible) and the fixed point is infrared stable (i.e. all the eigenvalues of M are positive). Notice that this is no accident: the conformal sequestering originates from nothing but the attractor structure of the infrared fixed point.

In our $SP(3N'+1) \times SP(N')^6$ model, we have shown that the fixed point is determined from the conditions of vanishing beta functions and the fixed point is infrared stable for $N' > 1$. Thus, we have found that the sequestering is realized in our model. Notice that the relation between the infrared stability and the sequestering holds independently of the perturbative calculation. Therefore, even if perturbative analysis is not applicable, we may argue that the sequestering occurs, if the fixed point is expected to be infrared stable.

It should be noted here that the unwanted global $U(1)$ symmetry discussed in the previous section is broken by anomalies of the $SP(N')^6$ gauge interactions and hence there is no conserved $U(1)$ current. This is the reason why the matrix M does not have a zero eigenvalue.

In addition to the above global $U(1)$ symmetry, there are many unbroken global $U(1)$'s acting on the gauge singlet superfields S_{ij} , which consist of the $U(1)$ subgroups of the flavor $SU(2N+2)$ of the hyperquarks Q^i . Thus, there are many linear combinations of the wave function renormalization factors which are not sequestered in the course of the RG evolutions to the infrared fixed point. For example, a linear combination $\Delta \ln Z_{S_{12}} - \Delta \ln Z_{S_{34}}$ is not sequestered, since this corresponds to the global $U(1) \subset SU(2N+2)$ symmetry. Fortunately, we can make such non-sequestered combinations vanishing by imposing the flavor $SU(2N+2)$ symmetry (or a sufficiently large discrete subgroup thereof) at high energies so that the conformal sequestering of the SUSY breaking is realized. Namely, by assuming that the Kähler potential inducing soft masses for squarks and sleptons is restricted by the flavor $SU(2N+2)$ symmetry as

$$\frac{\kappa_{ab}}{M_G^2} \sum_{ij} S_{ij}^\dagger S_{ij} q^{\dagger a} q^b, \quad (51)$$

we can set the linear combinations of $\Delta \ln Z$'s which are not sequestered to be zero.

Then, as we have discussed, the remaining combinations of $\Delta \ln Z$'s are sequestered and the squared masses of the sfermions from Eq.(51) are suppressed at the infrared fixed point. This is the reason why we have imposed the flavor $SU(2N + 2)$ symmetry in the SUSY-breaking sector.

Finally, in the rest of this section, we show that the sequestering is too mild in the present model to solve the FCNC problem. In view of the Table 2, the smallest eigenvalue β'_* of the coefficient matrix L (or equivalently M) is of the order of 10^{-3} for $N' \geq 2$. Thus, the linear combination of $\Delta \ln Z_i$ that corresponds to the smallest eigenvalue approaches to the fixed point very slowly, which, in turn, prevents $\Delta \ln Z_i$ from getting up to the fixed point immediately. That is, in the infrared regime ($t \ll 0$), we find

$$\Delta \ln Z_S(t) \sim e^{\beta'_* t} \Delta(\ln \bar{Z})_0, \quad (52)$$

$$(\Delta \ln \bar{Z})_0 = c_S(\Delta \ln Z_S)_0 + c_Q(\Delta \ln Z_Q)_0 + c_{Q'}(\Delta \ln Z_{Q'})_0, \quad (53)$$

where $\Delta \ln \bar{Z}$ corresponds to the eigenvector for the smallest eigenvalue, the subscript “0” indicates the value at $t = 0$, and c_i denote numerical coefficients. By explicit calculation, we find that the coefficients c_i are typically $\mathcal{O}(0.1 - 0.01)$ in our perturbative models. In order to solve the FCNC problem by sequestering, we should require $\Delta \ln Z_S \lesssim 10^{-7}$ at the SUSY-breaking scale [15].¹¹ Thus, without fine tuning among $\ln Z_i$, we should require $e^{\beta'_* t} \lesssim 10^{-7}$. However, since β'_* is of the order of 10^{-3} , it takes too long to achieve the sufficient sequestering. Therefore, we find that the sufficient sequestering cannot be expected in our perturbative models.

In the perturbative examples, we have seen that the size of the “sequestering speed” β'_* is not larger than the anomalous dimensions at the fixed point. Thus, in order to realize the sufficient sequestering (i.e. $\beta'_* = \mathcal{O}(1)$), we should require that the anomalous dimensions at the fixed point are of the order one.¹² This means that we must consider a strongly coupled conformal gauge theory.¹³ In the next section, we discuss such a strongly

¹¹Here, we assume that the flavor diagonal masses of the sfermions are of the order of 1 TeV, which are suppressed compared to the gravitino mass of the order of 100 TeV (see discussions in section 5).

¹²It is based on a naive expectation that the speed of the sequestering, $\beta'_* \sim (\partial\gamma/\partial\alpha)\alpha|_*$ or $(\partial\gamma/\partial\alpha)\alpha^2|_*$, is not so far from γ_* even in the strongly coupled case (see Eqs.(43)-(46)).

¹³Unfortunately, the strongly coupled case $N' = 1$ also seems inadequate since the anomalous dimensions are not sufficiently large.

	$SP(N)$	$SP(N')$	$SP(N')$
$Q \times 2(N + 1)$	\square_{2N}	$\mathbf{1}$	$\mathbf{1}$
Q'	\square_{2N}	$\square_{2N'}$	$\mathbf{1}$
Q'	\square_{2N}	$\mathbf{1}$	$\square_{2N'}$
S_{ij}	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

Table 3: The matter contents in strongly coupled model $SP(N) \times SP(N')^2$, ($N = 3, N' = 1$). Here, the subscripts of the fundamental representations denote the dimensions of the representations. In terms of the $SP(N)$ gauge theory, the number of the fundamental representation is given by $N_F = 2(N + 1) + 2 \times 2N' = 12$, while the number of the fundamental representation of each $SP(N')$ gauge theories is given by $N'_F = 2N = 6$.

coupled theory and present the reason why we consider the sequestering might be also realized there, although the perturbative calculation is not applicable.

4 Strongly coupled $SP(3) \times SP(1)^2$ model

In this section, we discuss $SP(3) \times SP(1)^2$ gauge theory as an example, where $SP(3)$ corresponds to the gauge group for the SUSY breaking and the $SP(1)^2$ gauge group acts on massive hyperquarks Q^{jk} . We list the matter contents in Table 3. We assume that such a gauge theory with the Yukawa interaction in Eq.(1) has a non-trivial fixed point. Then, the anomalous dimensions for Q, Q' , and S at the fixed point are determined as

$$\gamma_Q = -1, \quad \gamma_{Q'} = -1, \quad \gamma_S = 2, \quad (54)$$

which sit on a boundary of the unitarity bound Eq.(8).¹⁴

In Table 4, we list some other examples of the $SP(N) \times SP(N')^2$ gauge theories, which include the cases where the perturbative analysis is marginally applicable. In such examples with gauge symmetry structures similar to $SP(3) \times SP(1)^2$, we can explicitly check that the fixed points are infrared stable. Thus, based on these results (i.e. consistency with the unitarity and the presence of similar but calculable examples), we expect that the fixed point with Eq.(54) is infrared stable, although it is hard to check it by an explicit calculation.¹⁵

¹⁴The reason we take this example is only for simplicity. In the phenomenological point of view, we only require a large size of the anomalous dimensions which satisfy the unitarity bound Eq.(8).

¹⁵We know no calculable example that has an infrared unstable (non-trivial) fixed point in the present

	$\{a_g^*, a_{g'}^*, a_\lambda^*\}$	$\{\gamma_Q, \gamma_{Q'}, \gamma_S\}$	eigenvalues of M
$SP(3) \times SP(1)^2$	non-perturbative	$\{-1, -1, 2\}$	non-perturbative
$SP(5) \times SP(2)^2$	non-perturbative	$\{-0.8, -0.8, 1.6\}$	non-perturbative
$SP(7) \times SP(3)^2$	non-perturbative	$\{-0.7, -0.7, 1.4\}$	non-perturbative
$SP(13) \times SP(7)^2$	$\{0.5, 0.4, 0.3\}$	$\{-0.2, -0.8, 0.4\}$	$\{1.6, 0.7, 0.06\}$
$SP(20) \times SP(11)^2$	$\{0.4, 0.5, 0.2\}$	$\{-0.1, -0.8, 0.2\}$	$\{1.0, 0.7, 0.04\}$

Table 4: Fixed point in the $SP(N) \times SP(N')$ theory. $a_{g,g',\lambda}^*$ denote the coupling constant, $a_g = \alpha_g^*(N+1)/(2\pi)$, $a_{g'} = \alpha_{g'}^*(N'+1)/(2\pi)$, and $a_\lambda^* = \alpha_\lambda^* N/\pi$. For the calculable examples, all the eigenvalues of the coefficient matrix M are positive, and hence the fixed points are infrared stable.

Now, we discuss the sequestering of the SUSY breaking effects in the strongly coupled $SP(3) \times SP(1)^2$ model. As argued in the previous section, if the anomalous dimensions are determined uniquely from the conditions for the vanishing beta functions, then the sequestering is equivalent to the infrared stability of the fixed point. Hence, once we assume that the fixed point with Eq.(54) is infrared stable, the sequestering is guaranteed. By assuming that the ‘‘sequestering speed’’ β'_* is not so far from the values of γ_i (see Eqs.(43)-(46)), we expect $\beta'_* = \mathcal{O}(1)$ in our strongly coupled model. The flavor-changing soft masses are sufficiently suppressed by sequestering¹⁶ at the energy scale μ as high as

$$\left(\frac{\mu}{M_G}\right) \lesssim 10^{-\frac{7}{\beta'_*}}. \quad (55)$$

When the RG scale μ comes close to the physical mass scale m_{phys} of Q' , the theory ceases to be scale invariant and effectively becomes an asymptotically free $SP(3)$ gauge theory of strong coupling with 8 hyperquarks Q , and finally SUSY is broken dynamically at $\mu \lesssim m_{\text{phys}}$. Here, the physical mass is given by

$$m_{\text{phys}} = (mM_G^{-\gamma_{Q'}})^{\frac{1}{1-\gamma_{Q'}}} = \sqrt{mM_G}, \quad (56)$$

where the last equality results from the unitarity boundary value $\gamma_{Q'} = -1$ in the present model. Thus, the above condition Eq.(55) for the sufficient sequestering can be rewritten

class of $SP(N) \times SP(N')^n$, ($n = 1, 2, \dots$) gauge theories.

¹⁶In what follows, we assume that the theory is in the vicinity of the conformal fixed point at M_G , that is, we assume $M_* \simeq M_G$ (see below Eq.(17)).

in terms of the mass scale of Q' or the SUSY breaking scale Λ as

$$\left(\frac{\Lambda}{M_G}\right) \lesssim \left(\frac{m_{\text{phys}}}{M_G}\right) \lesssim 10^{-\frac{7}{\beta'_*}}. \quad (57)$$

As discussed in the next section, we are interested in the case where the gravitino mass is of the order of 100 TeV, which implies $\Lambda \sim 10^{11-12}$ GeV. Thus, we claim that the above conditions can be satisfied for $\beta'_* = \mathcal{O}(1)$, and hence, the FCNC can be suppressed in the present strongly coupled model.

5 Circumventing the tachyonic slepton problem

If the sequestering of SUSY breaking occurs sufficiently, the SUSY-breaking masses for the squarks and sleptons become negligibly small at low energies. In this situation we must invoke some mechanism to transmit sizable SUSY-breaking effects to the visible sector of the SSM. The most natural candidate is anomaly mediation [7]. This mechanism is not only theoretically interesting, but also phenomenologically attractive. This is because the gravitino mass is expected at $\mathcal{O}(100)$ TeV, which provides us with a solution to the gravitino problem [16, 17]. However, the anomaly mediation mechanism suffers from the tachyonic slepton mass problem [7].

In this section we consider a Planck-suppressed gauge mediation to remedy this phenomenological defect of the anomaly mediation.¹⁷ Let us introduce a messenger sector which consists of chiral superfields ψ , $\bar{\psi}$, ψ' , and $\bar{\psi}'$. Here, ψ , $\bar{\psi}$ and ψ' , $\bar{\psi}'$ transform as vector-like representations under the gauge group of the SSM, and we take them to fit in complete $SU(5)$ GUT representations, $\mathbf{5} + \mathbf{5}^*$, for simplicity.

Our additional superpotential terms are given by

$$\delta W = \frac{h}{M_G^2} S_{ij} Q^i Q^j \psi \bar{\psi} + m_m \psi \bar{\psi}' + m_m \psi' \bar{\psi}, \quad (58)$$

which let the SUSY breaking intact for a sufficiently large mass parameter m_m .¹⁸ Here, h denotes a coupling constant of order one and the combination $S_{ij} Q^i Q^j$ stands just for a SUSY-breaking superpotential term which has a non-vanishing F component (see Eq.(2)).

¹⁷In the Appendix B, we also provide a renormalizable setup for such a remedy.

¹⁸In fact, $m_m \gtrsim m_{3/2}$ is required, where $m_{3/2}$ denotes the gravitino mass.

The SUSY-breaking effects are transmitted to the sfermions and Higgs bosons by the SSM gauge interactions (see Ref.[18]).

In the SUSY-breaking dynamics, we expect $|\langle S \rangle| \lesssim \Lambda$ [19], which yields only Planck-suppressed R breaking effects in the gauge mediation. In this case, gauginos do not obtain sizable SUSY-breaking masses via the gauge mediation and the gaugino spectrum is virtually the same as in the purely anomaly-mediated one.¹⁹ On the other hand, the scalar field ϕ obtains the mass squared via the gauge mediation for $m_m < m_{\text{phys}}$ as

$$m_\phi^2 \simeq 2 \sum_{a=1,2,3} C_a^\phi \left(\frac{\alpha_a}{4\pi} \right)^2 \frac{|hF_S|^2}{m_m^2} \left(\frac{\Lambda}{M_G} \right)^4, \quad (59)$$

$$\simeq 18 \sum_{a=1,2,3} C_a^\phi \left(\frac{\alpha_a}{4\pi} \right)^2 \left(\frac{|h|m_{3/2}}{|\lambda|m_m} \right)^2 m_{3/2}^2, \quad (60)$$

where $C_a^\phi (a = 1, 2, 3)$ is the quadratic Casimir invariant for each gauge group relevant to the scalar ϕ .²⁰ In the above equation, we have used $\sqrt{F_S} \simeq \sqrt{\lambda}\Lambda$ and the gravitino mass $m_{3/2}$ given by

$$m_{3/2} \simeq \frac{|F_S|}{\sqrt{3}M_G} \simeq \frac{|\lambda|\Lambda^2}{\sqrt{3}M_G}. \quad (61)$$

As a result, we find that the gauge-mediated masses squared are comparable to the anomaly mediated ones for $m_m \sim m_{3/2}$.²¹ In particular, the positive contributions to the slepton masses squared in Eq.(60),

$$m_{\tilde{e}}^2_{\text{G.M.}} \simeq \frac{3}{5} 18 \left(\frac{|h|m_{3/2}}{|\lambda|m_m} \right)^2 \left(\frac{\alpha_1}{4\pi} \right)^2 m_{3/2}^2, \quad (62)$$

can overwhelm the negative contribution of the anomaly-mediated mass squared,

$$m_{\tilde{e}}^2_{\text{A.M.}} \simeq -\frac{6}{5} \frac{33}{5} \left(\frac{\alpha_1}{4\pi} \right)^2 m_{3/2}^2. \quad (63)$$

¹⁹Since the superpotential has the constant term which is required to obtain the flat universe, we may as well introduce an interaction term between $\psi\bar{\psi}$ and the constant term. Then, the R breaking effects in the gauge mediation possibly become sizable [18], which may result in the gaugino spectrum different from the one in the pure anomaly mediation. The expression of the gauge mediated mass squared in Eq.(60) may also be altered.

²⁰Here, we have neglected RG effects from the MSSM couplings.

²¹For $m_m \lesssim m_{3/2}$, even if the total SUSY-breaking were kept intact, messenger scalar particles would become tachyonic. Hence we restrict ourselves to $m_m \gtrsim m_{3/2}$. The choice $m_m \sim m_{3/2}$ realizes the lowest-scale model of gauge mediation (see Ref.[6, 20]) for $m_{3/2}$ of order 100 TeV. We note that $m_m \sim m_{3/2}$ is realized by a relation $m_m \sim m$ of the Lagrangian parameters in view of Eq.(56).

Therefore, we conclude that the tachyonic slepton problem is resolved in the total model of anomaly and gauge mediation hybrid by tuning scales of these two mediations with each other.

Note that the newly added superpotential term in Eq.(58) does not violate the global $SU(2N + 2)$ symmetry which is relevant for the conformal sequestering. Hence, we can apply the above mechanism to the conformally sequestered models. However, we should note that in the case of conformally sequestered models, the first term in Eq.(58) is also sequestered in the course of the RG evolution from M_* to m_{phys} .²² Thus, in order to realize the sizable gauge mediation effects as in Eq.(62), we need to compensate the sequestering effects by preparing the additional superpotential terms

$$\left(\frac{M_*}{m_{\text{phys}}}\right)^{\beta'_*} \frac{h}{M_G^2} S_{ij} Q^i Q^j \psi \bar{\psi} + m_m \psi \bar{\psi}' + m_m \psi' \bar{\psi} \quad (64)$$

at the scale M_* which effectively realize the Eq.(58) after the conformal sequestering. This implies that the higher-dimensional term stems from integrating out an intermediate matter of mass $(m_{\text{phys}}/M_*)^{\beta'_*} M_G$ with Planck-suppressed coupling to $S_{ij} Q^i Q^j$.²³ In the above analysis, we have simply used Eq.(58) as a resultant effective superpotential at the scale m_{phys} for the conformally sequestered models.

Finally, we comment on the cosmological aspects of this class of models. Since the relic density of the lightest messenger particle is too much to be consistent with the observation, we should require that they decay into the SSM particles at early stage of the universe. This is implemented by introducing small mixings between the messenger particles and the SSM particles. In addition, it should also be noted that there are Goldstone bosons in the SUSY breaking sector, which correspond to the spontaneous breaking of the global

²²The sequestering can be seen through a field redefinition $\tilde{S}_{ij} = (1 + \lambda^{-1} h \psi \bar{\psi} / M_G^2) S_{ij}$, which turns the effects of the superpotential coupling h into those of the Kähler couplings appearing as perturbations to the renormalization factors.

²³For example, we may consider a concrete model by introducing extra singlet supermultiplets X and \bar{X} with a superpotential

$$\frac{h_1}{M_G} X S Q Q + M_X X \bar{X} + h_2 \bar{X} \psi \bar{\psi}, \quad (65)$$

at the scale M_* . Here, $h_{1,2}$ denote coupling constants, M_X the mass parameter of X . Then, after integrating out X and \bar{X} , we can effectively obtain the first term in Eq.(58) at the scale m_{phys} for $M_X \sim M_G (m_{\text{phys}}/M_*)^{\beta'_*} ((h_1 h_2)/h)$.

$SU(2N+2)$ symmetry. However, those massless particles are decoupled from the thermal bath since they only couple with the SSM particles via the Planck-suppressed operator, and hence they do not affect the history of the universe.²⁴ Therefore, we find that the present hybrid scheme yields also a consistent scenario from the cosmological point of view.

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A Anomalous dimensions from a -maximization

Recently, Intriligator and Wecht proposed a powerful technique to compute the conformal R current in a certain class of conformal field theories in four dimensions and hence the anomalous dimensions thereof [13]. In this appendix we use this so-called a -maximization method to determine the anomalous dimensions of the fields in the conformally extended IYIT model beyond the Banks-Zaks approximation presented in section 3.

The a -maximization method simply states that the conformal R current appearing in the superconformal algebra maximizes a particular t'Hooft anomaly

$$a = \text{Tr}(3R^3 - R), \quad (66)$$

which is related to the conformal anomaly on a curved spacetime

$$\int_{S^4} \langle T^\mu_\mu \rangle. \quad (67)$$

In our model of the $SP(N)$ gauge theory, the candidate of the conformal R current contains one free parameter $x = \gamma_Q$, from which the corresponding R charges are determined by Eqs.(6) and (7) as

$$R_Q = \frac{2}{3}\left(1 + \frac{x}{2}\right), \quad R_{Q'} = \frac{2}{3}\left(1 + \frac{\gamma_{Q'}}{2}\right), \quad R_S = \frac{2}{3}\left(1 + \frac{-2x}{2}\right) \quad (68)$$

²⁴Here, we assume that the inflaton decays dominantly to the SSM particles.

with

$$\gamma_{Q'} = 1 - \frac{3(N+1) - (N+1)(1-x)}{2(N+1) - \varepsilon}, \quad (69)$$

where $\varepsilon = 2(N+1) - n_F$.

The claim is that among these one-parameter R currents, the conformal one maximizes the anomaly a , which is obtained as follows:

$$\begin{aligned} a = & 2N(2N+2) \left[3(R_Q - 1)^3 - (R_Q - 1) \right] \\ & + 2(2N+2-\varepsilon)2N \left[3(R_{Q'} - 1)^3 - (R_{Q'} - 1) \right] \\ & + (N+1)(2N+1) \left[3(R_S - 1)^3 - (R_S - 1) \right], \end{aligned} \quad (70)$$

where we note that the R charges appearing in a are those of fermions (i.e. $R_{\psi_Q} = R_Q - 1$) because only fermions contribute to the anomaly. By maximizing a with respect to x , we can determine $x_* = \gamma_Q|_*$. The unique local maximum is achieved by setting

$$\begin{aligned} x_* &= - \left(\varepsilon^2(2+3N) - 4\varepsilon(1+N)(2+3N) + (1+N)^2(8+13N) \right)^{-1} A; \\ A &\equiv 4 - 4\varepsilon + \varepsilon^2 + 22N - 16\varepsilon N + 3\varepsilon^2 N + 32N^2 - 12\varepsilon N^2 + 14N^3 + (\varepsilon - 2(1+N))B, \\ B &\equiv \sqrt{\varepsilon^2(1+2N)(1+6N) - 4\varepsilon(1+N)(1+2N)(1+6N) + (2+9N+7N^2)^2}. \end{aligned} \quad (71)$$

To compare this rather complicated expression with the perturbative results, we expand Eq.(71) in terms of ε . Remarkably, the first order approximation is given by

$$x_1 = - \frac{N}{2+9N+7N^2} \varepsilon, \quad (72)$$

which completely agrees with our Banks-Zaks-like calculation. Furthermore we can systematically study higher order corrections.²⁵ It is quite intriguing that the a -maximization determines all-order loop effects only from the one-loop result Eq.(70).

We can also study a of the gauged version of extended IYIT model in section 3, which leads to conformal sequestering. Since the gauging enforces yet another constraint on the anomaly free R charge, we obtain a unique R charge assignment without using the a -maximization procedure. It is important to realize, following the general argument of

²⁵For instance, the two loop contribution should be

$$x_2 = - \frac{3N(1+N(7+11N))}{(1+N)^2(2+7N)^3} \varepsilon^2.$$

the monotonically decreasing a , that a_{gauged} is less than a_{ungauged} . This is obvious when x_*^{gauged} is sufficiently close to x_*^{ungauged} , since x_*^{ungauged} yields the local maximum of $a(x)$. For example, we can show by a direct computation that a of the gauged extended IYIT model presented in section 3 is always less than that of the ungauged version presented in section 2 for a fixed gauge group. This result is consistent with the fact that our conformal fixed point is a stable one. In particular, it is worthwhile to notice that this is even true for $N = 1$ case, which cannot be treated in the one-loop approximation.

Finally, it would be an interesting but challenging problem to obtain the speed of the conformal sequestering from the interpolating a -function. In Ref.[21], the off-shell a -function is proposed as solving a -maximization condition with a Lagrange multiplier ξ that enforces the constraint on the R charge:

$$a(R(\xi), \xi) = \text{Tr} \left(3R^3 - R \right) + \sum \xi(\text{constraint}) , \quad (73)$$

where $R(\xi)$ is obtained by maximizing a with respect to R for fixed ξ , and the constraint is either ABJ anomaly free condition or the requirement that the superpotential be marginal. As was observed in [21], the first derivative of $a(\xi)$ is related to the β function of the coupling constant.²⁶ Furthermore, the second derivative (Hessian) of $a(\xi)$ at a fixed point ξ_* is proportional to the slope of the β function

$$\left. \frac{\partial^2 a(\xi)}{\partial \xi_i \partial \xi_j} \right|_* \propto \left. \frac{\partial \beta_i(\xi)}{\partial \xi_j} \right|_* . \quad (74)$$

Consequently there is a chance to read the conformal sequestering matrix without performing the explicit loop calculation even for a strongly coupled theory. Unfortunately, we do not know the proportionality factor (related to the denominator of the NSVZ beta function evaluated at the fixed point) and the transformation matrix $\{\partial \xi_i / \partial g_j\}$ non-perturbatively, so we cannot determine the conformal sequestering matrix. Since the conformal sequestering matrix is a physical renormalization invariant quantity while $a(\xi)$ is not, we need an off-shell scheme-independent a -function for our purposes.

²⁶Since the number of the Lagrange multipliers ξ agrees with that of marginal deformations, it is conjectured that ξ can be regarded as a coupling constant in a certain scheme.

B Another example of the hybrid scheme

In section 5, we have considered the hybrid model of the anomaly and gauge mediated SUSY breaking, which solves the tachyonic slepton problem in a pure anomaly mediation model. In this appendix, we propose another example of the hybrid model which can be constructed with renormalizable interactions between the SUSY breaking sector and the messenger sector. That is, in addition to the anomaly mediated SUSY breaking, we consider the gauge mediated SUSY breaking discussed in Ref. [18], where messenger sector consists of N_m flavors of chiral superfields ψ_i and $\bar{\psi}_j$ ($i, j = 1, \dots, N_m$) and N_m flavors of chiral superfields ψ'_i and $\bar{\psi}'_j$ ($i, j = 1, \dots, N_m$). Here, ψ_i, ψ'_i and $\bar{\psi}_j, \bar{\psi}'_j$ transform as $\mathbf{5}$ and $\mathbf{5}^*$ of $SU(5)$ GUT, respectively. Then, with the superpotential,

$$hS_{ij}\psi_i\bar{\psi}_j + m_m\psi_i\bar{\psi}'_i + m_m\psi'_i\bar{\psi}_i, \quad (75)$$

the SUSY-breaking effects are transmitted to the sfermions and Higgs bosons by the gauge interactions. Here, h denotes the coupling constant, m_m the mass parameter, and we assume that $h = h_0$ of order one at $M_* \lesssim M_G$. As discussed in section 3, we impose the global $SU(2N+2)$ symmetry to the SUSY breaking sector, and in order for the interaction in Eq.(75) to respect this symmetry, we assume that $N_m = 2(N+1)$ and $\psi, \bar{\psi}$ and $\psi', \bar{\psi}'$ transform as $\overline{\mathbf{2N+2}}$ and $\mathbf{2N+2}$ representations, respectively, of the $SU(2N+2)$.²⁷

In this case, the scalar field ϕ obtains the mass squared via the gauge mediation, and at the messenger scale, it is given by,

$$m_\phi^2 \simeq 2 \sum_{a=1,2,3} C_a^\phi \left(\frac{\alpha_a}{4\pi}\right)^2 \frac{|hF_S|^2}{m_m^2}, \quad (76)$$

$$\simeq 6 \sum_{a=1,2,3} C_a^\phi \left(\frac{\alpha_a}{4\pi}\right)^2 \left(\frac{m_{\text{phys}}}{M_*}\right)^{\gamma_S} \left(\frac{|h_0|M_G}{m_m}\right)^2 m_{3/2}^2. \quad (77)$$

Here, we have used RG evolution of hF_S ,

$$hF_S \Big|_{m_m} = hF_S \Big|_{m_{\text{phys}}} = \left(\frac{m_{\text{phys}}}{M_*}\right)^{\frac{\gamma_S}{2}} h_0 F_S \Big|_{m_{\text{phys}}} \simeq \left(\frac{m_{\text{phys}}}{M_*}\right)^{\frac{\gamma_S}{2}} h_0 m_{3/2} M_G, \quad (78)$$

²⁷Here, we assume $\langle S \rangle = 0$, that is, R symmetry is not broken. In this case, gauginos do not obtain the SUSY-breaking masses via the gauge mediation and the gaugino spectrum is the same as in the pure anomaly mediation.

where subscripts (m_m, m_{phys}) denote the RG scale. Furthermore, in the course of RG running, the scalar mass squared is suppressed by a factor of η which denotes the sequestering effects between the scales m_m and m_{phys} ,

$$\eta = \begin{cases} (m_{\text{phys}}/m_m)^{\beta'_*} & \text{for } m_m > m_{\text{phys}}, \\ 1 & \text{for } m_m \leq m_{\text{phys}}. \end{cases} \quad (79)$$

For example, for the $SP(3) \times SP(1)^2$ model, the positive contribution to the slepton masses squared in Eq.(77) is given by

$$m_{\tilde{e}}^2_{\text{G.M.}} \simeq \frac{3}{5} 6 \left(\frac{m_{\text{phys}}}{m_m} \right)^2 \left(\frac{M_G}{M_*} \right)^2 \left(\frac{\alpha_1}{4\pi} \right)^2 m_{3/2}^2 \eta, \quad (80)$$

where we are using $\gamma_S = 2$ and are assuming $h_0 = 1$.²⁸ Thus, when m_m satisfies

$$m_m \lesssim \begin{cases} m_{\text{phys}} (M_G/M_*)^{2/(2+\beta'_*)} & \text{for } m_m > m_{\text{phys}}, \\ m_{\text{phys}} (M_G/M_*) & \text{for } m_m \leq m_{\text{phys}}, \end{cases} \quad (81)$$

this contribution overcomes the negative contribution from the anomaly mediated mass squared in Eq.(63).²⁹ Therefore, we find that this hybrid model also provides a solution to the tachyonic slepton problem with an appropriate choice of the mass scale of messengers.³⁰

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²⁸Here, we have neglected RG effects from the MSSM couplings.

²⁹In this model, the gauge coupling constants in the SSM remain perturbative up to Grand Unification scale $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV for $m_m \gtrsim 10^{13}$ GeV.

³⁰Since the messenger particles are very heavy $m_m \gtrsim 10^{13}$ GeV, they are not produced thermally for the reheating temperature of the universe around $T_R \simeq 10^{10}$ GeV, which is very advantageous for the thermal leptogenesis [22].

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