

Kaon Electromagnetic Form Factor in the Light-Front Formalism¹

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Abstract

Numerical calculations are performed and compared to the experimental data for the electromagnetic form factor of the kaon, which is extracted from both components of the electromagnetic current, J^+ and J^- , with a pseudo-scalar coupling of the quarks to the kaon. In the case of J^+ there is no pair term contribution in the Drell-Yan frame ($q^+ = 0$). However, for J^- , the pair term contribution is different from zero and necessary in order to preserve the rotational symmetry of the current. The free parameters are the quark masses and the regulator mass.

1. Introduction

The convenience of using light-front variables in QCD descriptions of hadron properties and interactions, has been established long time ago. In particular, we have a clarifying article by N.N. Bogolyubov and coworkers published in 1983 [1]. In the subsections 3.5.3 and 3.5.4 of this article, the QCD description of the simplest composite systems (the mesons) and the corresponding form factors at high momentum transfer are discussed. The formalism is developed from a gauge invariant two-point function of the Bethe-Salpeter amplitude. The pion electromagnetic form factors are shown as example using the light-front formalism. Models for wave-functions in the light front are originally developed in [2]. In more recent years, the use of light-front formalism has become a common procedure in QCD description of hadrons [3]. Here, we can mention a few works that we are concerned, as [4, 5, 6, 7, 8, 9, 10], dedicated to study pseudoscalar properties of mesons, structure wave functions and quark-antiquark correlations. From such references, one can trace a more complete and detailed bibliography.

In the present communication, we report results for the kaon electromagnetic form factor that are extracted from both components of the electromagnetic current, $J^+ = J^0 + J^3$ and $J^- = J^0 - J^3$, with a pseudo-scalar coupling of the quarks. In the case of J^+ there is no pair term contribution in the Drell-Yan frame ($q^+ = 0$). However, for the J^- component of the electromagnetic current, the pair term contribution is different from zero and necessary in order to preserve the rotational symmetry of the current. We note that, when considering the case of vector particles, even the J^+ electromagnetic current has contribution from pair terms to have a full covariant theory [11, 12] (For a more recent

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application of this ideas in the vector anomaly problem, see ref.[?]). In order to satisfy the angular condition for spin one particles, it is necessary to consider pair terms in the electromagnetic current J^+ [11]. Besides the valence contribution to the J^- current, the pair term is necessary for both, pseudoscalar and vector particles to keep the rotational symmetry properties of the current in the light-front formalism.

2. Electromagnetic Current Model

In order to extract the electromagnetic form factor for the kaon, the components J^+ and J^- of the electromagnetic current are used. The $J^{(\mu=\pm)}$ components of the electromagnetic current for the kaon have contribution, from the quark (q) and the antiquark (\bar{q}), are given by

$$\begin{aligned} J_q^\mu(q^2) &= i e_q g^2 N_c \int \frac{d^4 k}{(2\pi)^4} \times \\ &\times \text{Tr}[S(k, m_{\bar{q}}) \gamma^5 S((k - P'), m_q) \gamma^\mu S((k - P), m_q) \gamma^5] \Lambda(k, P') \Lambda(k, P), \\ J_{\bar{q}}^\mu(q^2) &= q \leftrightarrow \bar{q} \text{ in } J_q^\mu(q^2), \end{aligned} \quad (1)$$

where the number of colors is $N_c = 3$, g is the coupling constant and e_q ($e_{\bar{q}}$) is the quark (anti-quark) charge. We use the Breit frame, where the momentum transfer is $q^2 = -(\vec{q}_\perp)^2$, $P^0 = P'^0$ and $\vec{P}'_\perp = -\vec{P}_\perp = \vec{q}_\perp/2$. The function $\Lambda(k, p) = N/[(p - k)^2 - m_R^2 + i\epsilon]$ is used in order to regulate the divergent integral, where m_R is the regulator mass and m_q and $m_{\bar{q}}$ are, respectively, the quark and anti-quark masses. The function $S(p)$ is the fermion propagator:

$$S(p, m) = \frac{1}{\not{p} - m + i\epsilon}. \quad (2)$$

The light-front coordinates are defined as $k^+ = k^0 + k^3$, $k^- = k^0 - k^3$, $k_\perp = (k^1, k^2)$. In the following, for the calculation of the pair terms, we consider the model given in [11] for a composite boson bound state and in the study of the Ward-Takahashi identity in the light-front formalism [14]. The contribution of the pair term for J^+ and J^- components of the electromagnetic current comes from the matrix elements proportional to k^- in both cases (antiquark and quark on-shell).

3. Electromagnetic Form Factor

The most general expression for the form factor of the spin zero particles is given by:

$$\langle P | J^\mu | P' \rangle = (P' + P)^\mu F(q^2) + (P' - P)^\mu G(q^2) \quad (3)$$

In this elastic process, the form factor depends only on q^2 , and $G(q^2) = 0$ in all q^2 . Here, off-shell effects are not explored. However, the off-shell effects are important and relevant in many topics for particles and nuclear physics.

In order to extract the form factor for the kaon, $F_{K^+}(q^2)$, we used both J^+ and J^- components of the electromagnetic current. One can verify that only the on-shell pole

$\bar{k}^- = (k_\perp^2 + m_{\bar{q}}^2)/k^+$ contribute to the k^- integration in the interval $0 < k^+ < P^+$:

$$F_{\bar{q}}^+(q^2) = -e_{\bar{q}} \frac{N^2 g^2 N_c}{P^+} \int \frac{d^2 k_\perp dk^+}{4\pi^3} \frac{\mathcal{N}_{\bar{q}}^+ \theta(k^+) \theta(P^+ - k^+)}{k^+ (P^+ - k^+)^2 (P'^+ - k^+)^2 (P^- - \bar{k}^- - \frac{f_{2,q} - i\epsilon}{P^+ - k^+})} \times \frac{1}{(P'^- - \bar{k}^- - \frac{f_{3,q} - i\epsilon}{P'^+ - k^+})(P^- - \bar{k}^- - \frac{f_{4,q} - i\epsilon}{P^+ - k^+})(P'^- - \bar{k}^- - \frac{f_{5,q} - i\epsilon}{P'^+ - k^+})}, \quad (4)$$

$$F_q^+(q^2) = [q \leftrightarrow \bar{q} \text{ in } F_{\bar{q}}^+(q^2)], \quad (5)$$

where $f_{2,q} \equiv (P - k)_\perp^2 + m_q^2$, $f_{3,q} \equiv (P' - k)_\perp^2 + m_q^2$, $f_4 \equiv (P - k)_\perp^2 + m_R^2$ and $f_5 \equiv (P' - k)_\perp^2 + m_R^2$. In the numerator, $\mathcal{N}_{\bar{q}}^+$ is given by

$$\mathcal{N}_{\bar{q}}^+ = \frac{-1}{4} \text{Tr}[(\not{k} + m_{\bar{q}}) \gamma^5 (\not{k} - \not{P}' + m_q) \gamma^+ (\not{k} - \not{P} + m_q) \gamma^5] \Big|_{k_- = \bar{k}^-}. \quad (6)$$

The kaon light-front wave function of the model can be extracted from (4) and (5) as

$$\Phi_Q^i(x, k_\perp) = \frac{1}{(1-x)^2} \frac{N}{(m_{K^+}^2 - M_0^2)(m_{K^+}^2 - M^2(m_Q, m_R))}, \quad (7)$$

where $x = k^+/P^+$ is the momentum fraction, $Q = \bar{q}, q$ and

$$M^2(m_Q, m_R) = \frac{k_\perp^2 + m_Q^2}{x} + \frac{(P - k)_\perp^2 + m_R^2}{1-x} - P_\perp^2. \quad (8)$$

The squared free quark mass is given by $M_0^2 = M^2(m_{\bar{q}}, m_q)$. For the final wave-functions, $\Phi_{\bar{q}}^f$ and Φ_q^f , we just need to exchange $P \leftrightarrow P'$ in (7) and (8).

The expression obtained for the electromagnetic form factor in terms of the initial ($\Phi_{\bar{q}}^i$) and final ($\Phi_{\bar{q}}^f$) wave functions is

$$F_{\bar{q}}^+(q^2) = -e_{\bar{q}} \frac{N^2 g^2 N_c}{4\pi^3 P^+} \int \frac{d^2 k_\perp dx}{x} \mathcal{N}_{\bar{q}}^+ \theta(x) \theta(1-x) \Phi_{\bar{q}}^{*f}(x, k_\perp) \Phi_{\bar{q}}^i(x, k_\perp), \quad (9)$$

$$F_q^+(q^2) = [q \leftrightarrow \bar{q} \text{ in } F_{\bar{q}}^+(q^2)]. \quad (10)$$

The final expression for the electromagnetic form factor obtained with J^+ is the sum of two contributions from the quark and the antiquark currents:

$$F_{K^+}^+(q^2) = F_q^+(q^2) + F_{\bar{q}}^+(q^2), \quad (11)$$

where the normalization is given by $F_{K^+}^+(0) = 1$. The calculation of the kaon electromagnetic form factor in the light-front with J^+ , without pair term, gives the same result as the covariant one (see Fig.1).

The contribution to the electromagnetic form factor obtained with J^- after the integration in k^- from the interval $0 < k^+ < P^+$ is given by

$$F_{\bar{q}}^{-(I)}(q^2) = -e_{\bar{q}} \frac{N^2 g^2 N_c}{4\pi^3 P^+} \int \frac{d^2 k_\perp dx}{x} \theta(x) \theta(1-x) \mathcal{N}_{\bar{q}}^{-(I)} \Phi_{\bar{q}}^{*f}(x, k_\perp) \Phi_{\bar{q}}^i(x, k_\perp), \quad (12)$$

$$F_q^{-(I)}(q^2) = [q \leftrightarrow \bar{q} \text{ in } F_{\bar{q}}^{-(I)}(q^2)], \quad (13)$$

where

$$\mathcal{N}_{\bar{q}}^{-(I)} = \frac{k_{\perp}^2 + m_{\bar{q}}^2}{xP^+} \left[(m_q - m_{\bar{q}})^2 - \frac{q^2}{4} \right] + P^+ \left[2m_{\bar{q}}(m_q - m_{\bar{q}}) + xP^{+2} \right] \quad (14)$$

When using J^- to extract the electromagnetic form factor, besides the contribution of the interval (I), the pair term contributes to the electromagnetic form factor in the interval (II) ($P^+ < k^+ < P'^+$). The pair term contribution for the form factor, as shown in [15, 10], is given by $F^{-(II)}(q^2)$.

$$F^{-(II)}(q^2) = \frac{N^2 g^2 N_c}{P^+} \left[e_q \Delta_q^{-(II)}(q^2) + e_{\bar{q}} \Delta_{\bar{q}}^{-(II)}(q^2) \right] \quad (15)$$

where $\Delta_{\bar{q}}^{-(II)}$ and $\Delta_q^{-(II)}$ given below. These terms correspond to the pair contribution in the J^- component of the electromagnetic current, which are obtained after the integration in k^- and the limit $P'^+ \rightarrow P^+$ are performed. Then one gets the following equations for the pair terms:

$$\Delta_{\bar{q}}^{-(II)} = \frac{-1}{4\pi^3} \int \frac{d^2 k_{\perp}}{(2\pi)^4} \mathcal{N}_{\bar{q}}^{-(II)} \sum_{i=2}^5 \frac{\ln(f_i)}{\prod_{j=2, i \neq j}^5 (f_j - f_i)}, \quad (16)$$

where $f_2 \equiv f_{2,q}$, $f_3 \equiv f_{3,q}$ and

$$\mathcal{N}_{\bar{q}}^{-(II)} = \frac{-1}{P^+} \left[P^{+2} + \frac{q^2}{4} - (m_q - m_{\bar{q}})^2 \right]. \quad (17)$$

We obtain the corresponding quark current contribution as in the above Eqs. (16) and (17) just by replacing $q \leftrightarrow \bar{q}$.

In the limit $P'^+ \rightarrow P^+$, the pair term contribution (zero mode) is non zero and responsible for the covariance of the J^- component of the electromagnetic current. The sum of the contributions from the intervals (I) and (II) for J^- in the light-front gives the same result as in the covariant calculation [11, 14].

The final expression for the electromagnetic form factor for the kaon, extracted from J^- is

$$F_{K^+}^-(q^2) = [F_q^{-(I)}(q^2) + F_{\bar{q}}^{-(I)}(q^2) + F^{-(II)}(q^2)], \quad (18)$$

which is normalized by the charge conservation, $F_{K^+}^-(0) = 1$.

4. Results and Conclusion

Next, we present results obtained considering the light-front formalism, as well as the covariant formalism. The parameters of the model are the constituent quark masses $m_q = m_u = 0.220$ GeV, $m_{\bar{q}} = m_{\bar{s}} = 0.419$ GeV, and the regulator mass $m_R = 0.946$ GeV, which are adjusted to fit the electromagnetic radius of the kaon. With these parameters, the calculated electromagnetic radius of the kaon is $\langle r_{k^+}^2 \rangle = 0.354$ fm², very close to the experimental radius $\langle r_{k^+}^2 \rangle = 0.340$ fm² [16].

The electromagnetic form factor is presented in Fig.1. Due to the fact that J^+ does not have light-front pair term contributions, the electromagnetic form factor results equal

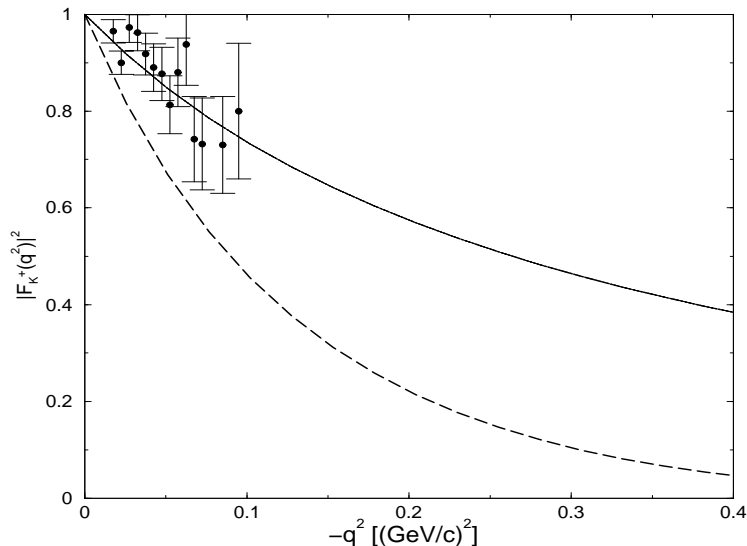


Figure 1: Kaon (K^+) electromagnetic form factor calculated within the covariant and the light-front formalisms, using J^+ and J^- components of the electromagnetic current. The dashed line give the results from J^- without the light-front pair term. Adding the pair term to it, we obtain the result given by the solid line, which coincides with the light-front and covariant calculations with J^+ . Experimental data are from [16].

to the one obtained in a covariant calculation. In the case of J^- , the light-front calculation gives results quite different from the covariant results, as shown in figure 1. After the inclusion of the pair terms, with J^- , we observe a complete agreement between the light-front and covariant results. In conclusion, the J^+ and J^- components of the electromagnetic current of the kaon are obtained in the light-front and in the covariant formalisms, in a constituent quark model. In the case of J^- , we note that the pair terms are essential to obtain a complete agreement between the covariant and the light-front results for the kaon electromagnetic form factor.

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