

A ‘Known’ CP Asymmetry in τ Decays

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Dedicated to our late colleague and friend P. Kabir

Abstract

We point out that dynamics known from the observed **CP** violation in $K_L \rightarrow \pi^\mp l^\pm \nu$ coupled with **CPT** invariance generate a **CP** asymmetry of $3.3 \cdot 10^{-3}$ in $\tau^\pm \rightarrow \nu K_S \pi^\pm \rightarrow \nu[\pi^+\pi^-]_K \pi^\pm$. The equality of the τ^+ and τ^- lifetimes required by **CPT** symmetry is restored in an intriguing way as the combined effect of long-lived $K \rightarrow \pi^+\pi^-$ as well as contributions from $K_L - K_S$ interference. While little new can be learnt from this **CP** asymmetry, the latter has to be accounted for, since **CP** asymmetries in $\tau \rightarrow \nu K \pi$ channels are prime candidates for revealing the intervention of New Physics. This ‘known’ **CP** asymmetry provides a very useful calibration point in such searches. It also provides a test of **CPT** symmetry (as well as the $\Delta Q = \Delta S$ selection rule).

1 Introduction

With the CKM prediction of large **CP** asymmetries in B decays [1, 2] like $B_d \rightarrow \psi K_S$ confirmed [3], the main task of B factories of all stripes is to look for ‘New Physics’, i.e. dynamics beyond the Standard Model (SM). This goal is being pursued by studying B decays of ever greater rarity. We want to stress that τ decays likewise deserve extensive efforts for three reasons at least:

- No **CP** violation has been observed yet in leptodynamics. Finding it there would represent a qualitative step forward.
- There are intriguing scenarios, where baryogenesis is driven by leptogenesis as primary effect [4]. **CP** violation is then required in leptodynamics. This is the main justification for undertaking Herculean efforts to find **CP** violation in neutrino oscillations. Searching for **CP** asymmetries in τ decays provides another of the few meaningful avenues towards that goal.

- Like for B mesons, studies of τ decays very likely provide different and presumably complementary perspectives onto the anticipated New Physics connected with the electroweak phase transition.

An optimal environment for studying τ decays is provided by $e^+e^- \rightarrow \tau^+\tau^-$. It offers a high rate relative to all other final states in a ‘clean’ and well-understood environment that allows searching for SM forbidden modes like $\tau^\pm \rightarrow l^\pm\gamma$, $\mu^\pm l^+l^-$ with $l = e, \mu$. Maybe even more importantly is another unique opportunity such τ factories offer, whether they are of the τ -charm or B factory of Giga-Z variety: they enable searches for novel **CP** asymmetries. Since the τ pair is produced with its spins aligned, one can use the decay of one τ to ‘tag’ the spin of the other τ and thus probe for spin dependent **CP** asymmetries *without* needing polarized beams.

In this short note we want to point out that contrary to a wide spread perception known dynamics generate **CP** asymmetries in τ decays: the well-measured **CP** asymmetry in $K_L \rightarrow \pi^\mp l^\pm \nu$ produces a difference in $\Gamma(\tau^+ \rightarrow \bar{\nu} K_L \pi^+)$ vs. $\Gamma(\tau^- \rightarrow \nu K_L \pi^-)$, where K_L is defined as the neutral kaon decaying on a time scale $\sim \mathcal{O}(\Gamma_L^{-1})$, and – assuming **CPT** symmetry – the same asymmetry also in $\Gamma(\tau^+ \rightarrow \bar{\nu} K_S \pi^+)$ vs. $\Gamma(\tau^- \rightarrow \nu K_S \pi^-)$ with the K_S defined as the neutral kaon decaying on the much shorter time scale $\sim \mathcal{O}(\Gamma_S^{-1})$. We explain how the apparent conflict with **CPT** invariance enforcing equal τ^+ and τ^- lifetimes is resolved. Such **CP** asymmetries, which of course are absent in $\Gamma(\tau^+ \rightarrow \bar{\nu} K^+ \pi^0)$ vs. $\Gamma(\tau^- \rightarrow \nu K^- \pi^0)$, have to be taken into account; it also provides a powerful calibration, when searching for manifestations of New Physics through **CP** studies.

2 SM CP violation in τ decays

The SM predicts for the transition amplitudes

$$T(\tau^- \rightarrow \bar{K}^0 \pi^- \nu) = T(\tau^+ \rightarrow K^0 \pi^+ \bar{\nu}), \quad (1)$$

since there is no weak phase and the strong phase has to be the same. Yet the observed kaons are the mass and not the flavour eigenstates, i.e. K_S and K_L , rather than K^0 and \bar{K}^0 . Ignoring **CP** violation in $\Delta S \neq 0$ dynamics, one has $\langle K_L | K_S \rangle = 0$, and the K_L and K_S are unambiguously distinguished by their decay modes in addition to their vastly different lifetimes: $K_S \rightarrow 2\pi$, $K_L \not\rightarrow 2\pi$, $K_L \rightarrow 3\pi$. Then one has

$$\Gamma(\tau^- \rightarrow \nu K_S \pi^-) = \Gamma(\tau^- \rightarrow \nu K_L \pi^-) = \frac{1}{2} \Gamma(\tau^- \rightarrow \nu \bar{K}^0 \pi^-) \quad (2)$$

$$\Gamma(\tau^+ \rightarrow \bar{\nu} K_S \pi^+) = \Gamma(\tau^+ \rightarrow \bar{\nu} K_L \pi^+) = \frac{1}{2} \Gamma(\tau^+ \rightarrow \bar{\nu} K^0 \pi^+) \quad (3)$$

and thus no **CP** asymmetry due to Eq.(1).

The situation becomes considerably more complex and intriguing, once **CP** violation in $\Delta S = 2$ transitions is included. (We can safely ignore *direct* **CP** violation for our

purposes here.) Imposing **CPT** invariance we can write

$$\begin{aligned} |K_S\rangle &= p|K^0\rangle + q|\bar{K}^0\rangle \\ |K_L\rangle &= p|K^0\rangle - q|\bar{K}^0\rangle \end{aligned} \quad (4)$$

with $|p|^2 + |q|^2 = 1$. We then have

$$\langle K_L|K_S\rangle = |p|^2 - |q|^2 \simeq 2\text{Re}\epsilon_K \simeq (3.27 \pm 0.12) \times 10^{-3} \neq 0 \quad (5)$$

as deduced from

$$\frac{\Gamma(K_L \rightarrow \pi^- l^+ \nu) - \Gamma(K_L \rightarrow \pi^+ l^- \bar{\nu})}{\Gamma(K_L \rightarrow \pi^- l^+ \nu) + \Gamma(K_L \rightarrow \pi^+ l^- \bar{\nu})} = |p|^2 - |q|^2 \quad (6)$$

The mass eigenstates thus are no longer orthogonal to each other and both $K_S \rightarrow 2\pi$ and $K_L \rightarrow 2\pi$ can occur. I.e., the 2π final state by itself no longer distinguishes strictly between K_S and K_L . Yet the difference in lifetimes still provides a discriminator: Considering the decay rate evolution for $\tau \rightarrow \nu\pi[\pi^+\pi^-]_K$ as a function of t_K , the (proper) time of the kaon decay, one has for *short* decay times – $t_K \sim \mathcal{O}(\Gamma_S^{-1})$ – we have for all practical purposes only $K_S \rightarrow 2\pi$ decays and find

$$\frac{\Gamma(\tau^+ \rightarrow [\pi^+\pi^-]_{K_S} \pi^+ \bar{\nu}) - \Gamma(\tau^- \rightarrow [\pi^+\pi^-]_{K_S} \pi^- \nu)}{\Gamma(\tau^+ \rightarrow [\pi^+\pi^-]_{K_S} \pi^+ \bar{\nu}) + \Gamma(\tau^- \rightarrow [\pi^+\pi^-]_{K_S} \pi^- \nu)} = |p|^2 - |q|^2 \simeq (3.27 \pm 0.12) \times 10^{-3}. \quad (7)$$

For *long* decay times – $t_K \sim \mathcal{O}(\Gamma_L^{-1})$ – we have again for all practical purposes only $K_L \rightarrow 2\pi$ and find

$$\frac{\Gamma(\tau^+ \rightarrow [\pi^+\pi^-]_{K_L} \pi^+ \bar{\nu}) - \Gamma(\tau^- \rightarrow [\pi^+\pi^-]_{K_L} \pi^- \nu)}{\Gamma(\tau^+ \rightarrow [\pi^+\pi^-]_{K_L} \pi^+ \bar{\nu}) + \Gamma(\tau^- \rightarrow [\pi^+\pi^-]_{K_L} \pi^- \nu)} = |p|^2 - |q|^2 \quad (8)$$

Strictly speaking it does not even matter which forces generate $|q| \neq |p|$, whether it is due to SM dynamics or not, as long as τ decays *themselves* are described by the SM.

Measuring the asymmetry of Eq.(8) seems hardly feasible, since the K_L acts basically like a second neutrino, yet it raises an intriguing question: With the asymmetries in Eqs.(7,8) having the same sign, how is the equality of the τ^+ and τ^- lifetimes restored as required by **CPT** invariance, which we have explicitly invoked?

To answer this question, let us recall how we might actually measure the asymmetries of Eqs.(7,8). These asymmetries are obtained by studying the elapsed time between the τ decay and the time at which " $\pi\pi$ " is formed. The first asymmetry is obtained by looking at events with short time difference, whereas the second asymmetry is obtained by looking decays with large time difference. **CPT** constraint applies only to the total decay rate where we include events at all times of decay.

Because $\langle K_L|K_S\rangle \neq 0$, the decay rate evolution for $\tau \rightarrow \nu\pi[f]_K$, where f is an arbitrary final state, now contains three terms: in addition to the two contributions listed above with a time dependence $\propto e^{-\Gamma_S t_K}$ and $\propto e^{-\Gamma_L t_K}$, respectively, we have an interference term

$e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t_K}$ most relevant for intermediate times $\Gamma_S^{-1} \ll t_K \ll \Gamma_L^{-1}$.¹ Note that because of the interference term, observing only $\pi\pi$ final state does not allow us to understand the **CPT** constraint. Measuring all three terms for all f and integrating over all t_K – possible in principle, though maybe not in practice – recovers the full information on the production of \overline{K}^0 and K^0 with the relation of Eq.(1).

To be more explicit: one has to track the full decay rate evolution into a general state f when the initial state was a $K^0 - \Gamma(K^0(t_K) \rightarrow f)$ – versus a $\overline{K}^0 - \Gamma(\overline{K}^0(t_K) \rightarrow \overline{f})$. The most general expression reads

$$\begin{aligned} \Gamma(K^0(t_K) \rightarrow f) = & \frac{1}{2|p|^2} \left[|T(K_S \rightarrow f)|^2 e^{-\Gamma_S t_K} + |T(K_L \rightarrow f)|^2 e^{-\Gamma_L t_K} \right. \\ & \left. + 2e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t_K} \operatorname{Re}(e^{i\Delta M_K t_K} T(K_S \rightarrow f) T(K_L \rightarrow f)^*) \right] \quad (9) \end{aligned}$$

$$\begin{aligned} \Gamma(\overline{K}^0(t_K) \rightarrow \overline{f}) = & \frac{1}{2|q|^2} \left[|T(K_S \rightarrow \overline{f})|^2 e^{-\Gamma_S t_K} + |T(K_L \rightarrow \overline{f})|^2 e^{-\Gamma_L t_K} \right. \\ & \left. - 2e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t_K} \operatorname{Re}(e^{i\Delta M_K t_K} T(K_S \rightarrow \overline{f}) T(K_L \rightarrow \overline{f})^*) \right] \quad (10) \end{aligned}$$

For short times of decay the first term dominates, which describes K_S decays, and Eq.(7) applies; for very long times the second term does producing the same **CP** asymmetry as stated in Eq.(8). Yet for the intermediate range in times of decay the third term reflecting $K_S - K_L$ interference becomes important.

By rewriting the *interference* term in terms of K^0 and \overline{K}^0 , integrating over t_K and finally summing over all possible final state f and \overline{f} , we have

$$\begin{aligned} & \frac{1}{|p|^2} \sum_f \int dt_K e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t_K} [e^{i\Delta M_K t_K} T(K_S \rightarrow f) T(K_L \rightarrow f)^*] \\ & = \frac{1}{\frac{\Gamma_L+\Gamma_S}{2} - i\Delta M} [2(|p|^2 - |q|^2)\Gamma_{11} + i4\operatorname{Im}(qp^*\Gamma_{12})] \\ & = 2(|p|^2 - |q|^2) + \frac{2i}{\frac{\Gamma_L+\Gamma_S}{2} - i\Delta M} [2\Delta M \operatorname{Re}\epsilon - \Delta\Gamma \operatorname{Im}\epsilon], \quad (11) \end{aligned}$$

where we have used relations valid for this problem to first order in the **CP** violating parameters: $\Gamma_{11} = \frac{\Gamma_L+\Gamma_S}{2}$, $p = \frac{1}{\sqrt{2}}(1 + \epsilon)$, $q = \frac{1}{\sqrt{2}}(1 - \epsilon)$, $\Delta\Gamma = 2\Gamma_{12}$. Finally using $\arg \epsilon = \arctan\left(\frac{2\Delta M}{\Delta\Gamma}\right)$ we find that the square bracket in the last line of Eq.(11) vanishes; i.e.

$$\frac{1}{|p|^2} \sum_f \int dt_K e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t_K} [e^{i\Delta M_K t_K} T(K_S \rightarrow f) T(K_L \rightarrow f)^*] = 2(|p|^2 - |q|^2) \quad (12)$$

¹It was this interference term in $K^0(t) \rightarrow \pi^+\pi^-$ and $\overline{K}^0(t) \rightarrow \pi^+\pi^-$, which established originally that the Fitch-Cronin observation of $K_L \rightarrow \pi^+\pi^-$ could not be reconciled with **CP** symmetry by suggesting that they had actually observed $K_L \rightarrow \pi^+\pi^-U$ with U denoting a hitherto unknown neutral particle with odd **CP** parity that had escaped detection.

Using Eq.(12) it is simple to show that the interference term indeed restores the constraints from **CPT** symmetry:

$$\sum_f \int dt_K \Gamma(\tau^+ \rightarrow [f]_{K^0(t_K)} \pi^+ \bar{\nu}) = \sum_{\bar{f}} \int dt_K \Gamma(\tau^- \rightarrow [\bar{f}]_{\bar{K}^0(t_K)} \pi^- \nu). \quad (13)$$

While this is as it must be, it is still instructive to see how it comes about.

In talking about the time of decay t_K we were referring to the proper time of the neutral kaon decay. In a real experiment one has two times of decay, namely that of the τ lepton and of the kaon. The explicit formulae have been given for the even more involved case of $D^0 \rightarrow K_S X$ allowing even for $D^0 - \bar{D}^0$ oscillations to take place [6]. Yet for the experimentally most accessible case involving K_S decays at short values of t_K there is no practical need for the full machinery.

3 CPT violation

While we are not suggesting that **CPT** violation is likely to surface in τ decays, it is not inappropriate to address this issue. It has been searched for extensively in semileptonic K decays; thus it is convenient to employ the notation used there. Without imposing **CPT** invariance Eq.(4) is generalized to

$$\begin{aligned} |K_S\rangle &= p_S |K^0\rangle + q_S |\bar{K}^0\rangle \\ |K_L\rangle &= p_L |K^0\rangle - q_L |\bar{K}^0\rangle \end{aligned} \quad (14)$$

with [5]

$$\begin{aligned} p_S &= N_S \cos \theta/2, & q_S &= N_S e^{i\phi} \sin \theta/2 \\ p_L &= N_L \sin \theta/2, & q_L &= N_L e^{i\phi} \cos \theta/2 \end{aligned} \quad (15)$$

where ϕ and θ are both *complex* numbers constrained by the discrete symmetries as follows:

$$\text{CPT or CP invariance} \implies \cos \theta = 0 \quad (16)$$

$$\text{CP or T invariance} \implies \phi = 0 \quad (17)$$

The normalization constants N_S and N_L are given by:

$$N_S = \frac{1}{\sqrt{|\cos \frac{\theta}{2}|^2 + |e^{i\phi} \sin \frac{\theta}{2}|^2}}, \quad N_L = \frac{1}{\sqrt{|\sin \frac{\theta}{2}|^2 + |e^{i\phi} \cos \frac{\theta}{2}|^2}} \quad (18)$$

If **CPT** symmetry is violated $\cos \theta \neq 0$ and $\text{Im } \phi \neq 0$.

$$\begin{aligned} \frac{\Gamma(\tau^+ \rightarrow "K_S" \pi^+ \bar{\nu}) - \Gamma(\tau^- \rightarrow "K_S" \pi^- \nu)}{\Gamma(\tau^+ \rightarrow "K_S" \pi^+ \bar{\nu}) + \Gamma(\tau^- \rightarrow "K_S" \pi^- \nu)} &= \text{Im } \phi + \text{Re } \cos \theta \\ \frac{\Gamma(K_L \rightarrow \pi^- l^+ \nu) - \Gamma(K_L \rightarrow \pi^+ l^- \bar{\nu})}{\Gamma(K_L \rightarrow \pi^- l^+ \nu) + \Gamma(K_L \rightarrow \pi^+ l^- \bar{\nu})} &= \text{Im } \phi - \text{Re } \cos \theta \end{aligned} \quad (19)$$

where as before " K_S " is understood as the *short*-time component in $K \rightarrow \pi^+\pi^-$; we have also assumed the $\Delta S = \Delta Q$ selection rule for these decay amplitudes. We look forward to new information on $\text{Re} \cos \theta$ from τ decays.

To be consistently heretical, one could also entertain the idea of the $\Delta S = \Delta Q$ rule being violated and possibly by different amounts in K and τ decays. The relevant expressions are quite straightforward and can be derived using similar techniques described, for example, in Ref.[5]. We will not write them down here, since we feel there is even less space for heresy in $\Delta S = 1$ than in $\Delta S = 2$ dynamics.

4 Decays of other particles

The power of $K_{L,S}$ to discriminate matter against antimatter affects the decays of other particles as well. In $B_d/\bar{B}_d \rightarrow \psi K_S$ its effect is covered up by the huge **CP** violation in $\Delta B \neq 0$ dynamics. It is of relevance when SM forces generate no or only small **CP** violation like in $D^\pm \rightarrow K_S \pi^\pm$ [7] or in $D^\pm \rightarrow l^\pm \nu K_S$.

5 Conclusion

As stated before, the asymmetries expressed in Eqs.(8,7) have to be there, since they are caused by a well established effect, namely that the $K_{L,S}$ are ever so slightly, yet definitely sensitive to the matter-antimatter distinction. In principle it does not even matter, whether the SM can reproduce the observed size of ϵ_K . In that sense observing this asymmetry does not teach us anything we do not already know.

This, however, is an incomplete evaluation of the situation. For **CP** asymmetries in the channels $\tau \rightarrow \nu K + \text{pions}$ are natural portals for the emergence of New Physics. For the final state is sufficiently complex to allow for **CP** odd observables also in distributions beyond fully integrated widths; secondly they should be particularly sensitive to non-SM charged Higgs exchanges [8]. Obviously it is then essential to note that known physics produces a reliably predicted asymmetry in the full width, but not in final state distributions for some channels.

There are some more subtle points as well: it is actually most useful experimentally when not all modes are predicted to exhibit a null effect within the SM. For if one does not observe the effect predicted by Eqs.(8,7), then there has to be New Physics, which (partially) cancels the effect from known physics – or one does not understand one's experiment with the required accuracy. The small expected **CP** asymmetry discussed above thus provides a most helpful calibration.

Finally it behooves us to allow for the admittedly exotic possibility of **CPT** invariance being violated in general and in the dynamics of the $K^0 - \bar{K}^0$ complex in particular. Eq.(19) provides a novel test for it.

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