

Successes and problems of chiral soliton approach to exotic baryons

MICHAŁ PRASZAŁOWICZ^a AND KLAUS GOEKE^b

^a M. Smoluchowski Institute of Physics, Jagellonian University,
Reymonta 4, 30-059 Kraków, Poland

^b Institut für Theoretische Physik II, Ruhr-Universität Bochum,
D-44780 Bochum, Germany

We briefly review the formulation of chiral quark soliton model and explain the difference and similarities with the Skyrme model. Next, we apply the model to calculate non-exotic and exotic mass spectra. We concentrate on large N_c counting both for mass splittings and decay widths. It is shown that pure large N_c arguments do not explain the small width of exotic pentaquark states.

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1. Introduction

There is still a lack of consensus whether the lightest member of the exotic antidecuplet has been discovered [1]. Four months after this conference results from high statistics G11 experiment at CLAS have been presented at the APS meeting with negative result for the photoproduction of Θ^+ on proton [2]. Even more problematic is the sighting of the heaviest members of $\overline{10}$ that were seen only by NA49 experiment at CERN [3]. These states were predicted within the chiral soliton models [4, 5, 6, 7, 8]. Early estimates of the antidecuplet octet splitting, $\Delta M_{\overline{10}-8} \sim 600$ MeV, obtained in a specific modification of the Skyrme model can be found in Ref. [5]. The estimates of *both* Θ^+ and $\Xi_{\overline{10}}$ masses from the second order mass formulae obtained in the Skyrme model in 1987 are in a surprising agreement with present experimental findings [6]. Already at that time, however, the doubts whether these predictions were trustworthy had been raised [5, 6, 11]. Today they were scrutinized and rephrased by other authors [12, 13, 14].

In 1997 the masses, as well as the decay widths of the exotic states were estimated within the chiral quark soliton model [7]. One of the most striking predictions of this seminal paper by Diakonov, Petrov and Polyakov [7] was

the small width of antidecuplet states. Despite some misprints in this paper (see *e.g.* [9, 10]) and the model dependent corrections, the narrow width is one of the key features of the chiral model predictions which is in line with recent experimental findings.

In this paper we examine the successes and problems of chiral soliton models ability to predict properties of exotic baryons. In Section 2 we argue that soliton models *are* in fact quark models and explain the difference between quark–soliton models and Skyrme model. Then in Sect. 3 we list different predictions for masses of exotic baryons and discuss the N_c counting for the mass splittings. In Sect. 4 we repeat the same analysis for the decay widths. Summary is given in Sect. 5.

2. Soliton models

Soliton models are often regarded as orthogonal to the quark picture. Very often they are generally referred to as Skyrme type models where only mesonic degrees are present. In this Section we will demonstrate that they are deeply rooted in QCD, take into account quark degrees of freedom maybe even in a more complete way than the quark models themselves, and that they are fully operative providing predictions of static baryon properties, structure functions, skewed and off-forward amplitudes and light-cone distribution amplitudes for baryons (for review see *e.g.* Refs. [15, 16, 17]), not to mention properties of pseudoscalar mesons [18]. That of course does not mean that they capture all physics, since — for example — they do not possess confinement. They rely on large N_c limit and chiral symmetry breaking. We shall also make distinction between quark–soliton and Skyrme model.

Let us take as a starting point the chiral Lagrangian density of the form

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M U^{\gamma_5}[\varphi])\psi \quad (1)$$

which looks like a Dirac Lagrangian density for a massive fermion ψ if not for matrix U . In fact ψ is a 3-vector in flavor space and also in color. Matrix

$$U^{\gamma_5} = e^{(i/F_\varphi)\vec{\lambda}\cdot\vec{\varphi}\gamma_5}$$

parameterized by a set of pseudoscalar fields $\vec{\varphi}$ has been introduced to restore chiral symmetry given by a global multiplication of the fermion field by a phase factor

$$\psi \rightarrow e^{i\vec{\lambda}\cdot\vec{\alpha}\gamma_5}\psi. \quad (2)$$

Indeed, the term $M\bar{\psi}\psi$ is not invariant under (2), however $M\bar{\psi}U^{\gamma_5}[\varphi]\psi$ is, provided we also transform meson fields

$$U^{\gamma_5}[\varphi] \rightarrow e^{-i\vec{\lambda}\cdot\vec{\alpha}\gamma_5}U^{\gamma_5}[\varphi]e^{-i\vec{\lambda}\cdot\vec{\alpha}\gamma_5}. \quad (3)$$

Since matrix U “lives” in the flavor space, the color indices are here contracted producing simply an overall factor N_c in front of (1).

Lagrangian (1) does not contain kinetic term for meson fields, so φ 's are expressed in terms of fermion fields themselves. The kinetic term appears only when we integrate out the quark fields. Then the resulting effective action contains only meson fields and can be organized in terms of a derivative expansion

$$S_{\text{eff}}[\varphi] = \frac{F_\varphi^2}{4} \int \text{Tr} \left(\partial_\mu U \partial^\mu U^\dagger \right) + \frac{1}{32e^2} \int \text{Tr} \left(\left[\partial_\mu U U^\dagger, \partial_\nu U U^\dagger \right]^2 \right) + \Gamma_{\text{WZ}} + \dots, \quad (4)$$

where constants F_φ and e can be calculated from (1) with an appropriate cut-off. Γ_{WZ} is the Witten Wess–Zumino term which takes into account axial anomaly and does not require regularization. Perhaps the most important part are the ellipses which encode an infinite set of terms that are effectively summed up by the fermionic model of Eq. (1). The truncated series of Eq. (4) is the basis of the Skyrme model. Hence the Skyrme model is (a somewhat arbitrary, because it does not include another possible 4 derivative term) approximation to (1).

At this point both models, chiral quark model of Eq. (1) and Skyrme model of Eq. (4) (without the “dots”), look like mesonic theories devised to describe meson–meson scattering, for example [19]. Baryons are introduced in two steps, following large N_c strategy described by Witten in Refs. [20]. First, one constructs a soliton solution, *i.e.* solution to the classical equations of motion that corresponds to the extended configuration of the meson fields, *i.e.* to matrix U which cannot be expanded in a power series around unity. Second, since the classical soliton has no quantum numbers (except baryon number, see below), one has to quantize the system. Perhaps this quantization procedure, which reduces both models to the nonrelativistic quantum system analogous to the symmetric top [21, 4] with two moments of inertia $I_{1,2}$, makes chiral-soliton models look odd and counterintuitive.

It is not our purpose to give the full review of the soliton models which can be found elsewhere [15, 16, 17], especially their connection with QCD, to some extent obvious from Eq. (1), was extensively reviewed in Ref. [22]. Here we want to discuss the interconnection of the chiral soliton models and quark models. In fact Lagrangian (1) *is* an interacting quark model, despite the fact that there are no gluons there. The interaction proceeds through the chirally invariant coupling $\bar{\psi} U \gamma^5 \psi$ and information about gluons (which are integrated out) is encoded in the coupling strength M (constituent quark mass).

For the purpose of illustration it is convenient to use the variational approach for the soliton solution [23]. To this end one uses a hedgehog ansatz for the static U_0 field:

$$U_0 = \begin{bmatrix} e^{i\vec{n}\cdot\vec{\tau}P(r)} & 0 \\ 0 & 1 \end{bmatrix}, \quad (5)$$

where 2×2 matrix in the upper left corner depends on one variational function $P(r) = P(r/r_0)$ characterized by an effective size r_0 and being a subject to the boundary conditions $P(0) = \pi$ and $P(\infty) = 0$. For $r_0 = 0$ matrix $U_0 = 1$ and the spectrum of the Dirac operator corresponding to (1) looks like a spectrum of a free fermion of mass M (see the right panel of Fig. 1). Once we increase the size r_0 , the levels rearrange and one distinct level “sinks” rapidly into a mass gap (see the left panel of Fig. 1).

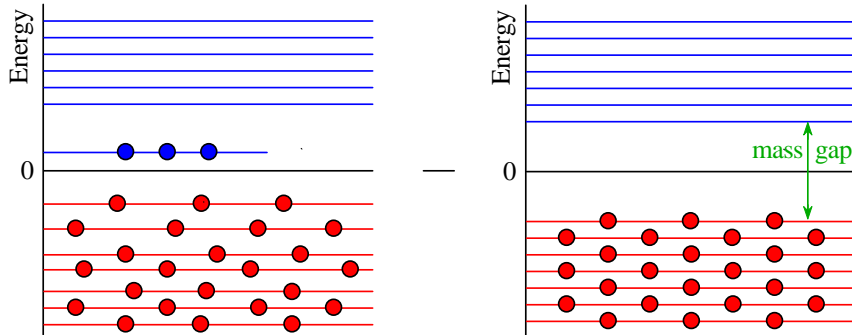


Fig. 1. Spectrum of the Dirac operator in the presence of the valence level and without. The soliton energy is calculated as the regularized difference of two contributions

The energy of this *interacting* fermionic system is given as a sum of the valence level and the sea levels (filled levels of the Dirac “sea”) calculated with respect to the vacuum ($r_0 = 0$) configuration as depicted in Fig. 1. Stable minimum is achieved for some intermediate soliton size $r_0 = r_{\text{sol}}$, usually of the order of a fermi, as an interplay between *decreasing* energy of the valence level and *increasing* energy of the Dirac sea. In this case the baryon number of the soliton is given simply as the baryon number of the valence level.

An interesting limit can be considered by *artificially* tuning the size of the soliton to $r_0 = 0$ [24]. In this limit the valence level goes back to the upper edge of the mass gap and the contribution of the sea levels

cancels out. Hence the soliton in this limit looks like the constituent quark model. Indeed, it has been shown that for $r_0 \rightarrow 0$ many static observables calculated in the soliton model are in agreement with the naive quark model predictions. These include $g_A = 5/3$, $\Delta\Sigma = 1$ and $\mu_n/\mu_p = -2/3$.

In the Skyrme model the soliton is constructed purely from the mesonic field (5). The baryon number is given as a charge of the conserved topological current. Stabilization is achieved by an interplay of the increasing energy of the kinetic term and the decreasing energy of the Skyrme term (4). This reflects the main difference between quark–soliton and Skyrme–soliton. Had we included all terms denoted by ellipses in Eq. (1) there would have been no minimum of energy as a function of r_0 .

The quantization on the other hand proceeds in both models almost identically [4]. The symmetric top Hamiltonian is supplemented by a constraint coming from the I_{WZ} term which selects SU(3) representations that contain states of $Y = 1$. Octet, decuplet, exotic antidecuplet and eikosiheptaplet (*i.e.* 27-plet) [25] are the lowest possible representations satisfying this constraint. The difference between the two models is buried in the analytical form of the expressions for the symmetric top parameters (overall mass and moments of inertia, *etc.*). Some of them are identically zero in the Skyrme model, whereas they are nonzero in the quark–soliton model due to the valence level contribution.

Is the tower of representations satisfying constraint $Y = 1$ infinite? Formally the answer is: yes, but physically: no, since we have to revise assumptions which led us to the quantization of the soliton. Namely, we have assumed *rigid rotation* which is (classically) very unlikely when the soliton angular velocities become large. Two phenomena are expected: deformation of the soliton and vibrations. Deformation will lead to instabilities resulting in radiation of pions (Goldstone bosons in general). Fast rotating solitons will have a cigar-like shape and will lie on linear Regge trajectories [26].

As discussed above there is only one representation of given dimension in the allowed series of representations selected by the Wess–Zumino constraint. So there is only one (nonexotic) octet, while the quark models inevitably require a cryptoexotic octet together with antidecuplet. Of course, the octet is not missing; it has to be of different origin. So far we have constructed only rotational states, however, there will be also vibrations.

Explicit construction of the vibrational states in the Skyrme model (with the dilaton field) has been carried on by Weigel in Refs. [8]. In this approach only one mode, namely the “breathing” mode of the soliton was quantized and a subsequent mixing with other states was investigated.

3. Mass estimates

In order to estimate the mass of Θ^+ we have to know two quantities, namely the strange moment of inertia, I_2 which contributes to antidecuplet–octet splitting

$$\Delta M_{\overline{10}-8} = \frac{3}{2I_2} \quad (6)$$

and the energy shift due to the nonzero strange quark mass. It has been observed by Guadagnini [27] that the mass splittings in the Skyrme model are well described in terms of 2 parameter effective Hamiltonian

$$H' = \alpha D_{88}^{(8)} + \beta Y,$$

where $\alpha \sim \Sigma_{\pi N}$. However $\beta \equiv 0$ in the minimal Skyrme model and the spectrum cannot be well described in the first order perturbation in H' . Second order correction, which can be schematically written as [6]

$$\Delta E_2 \sim -I_2 \times \alpha^2 \quad (7)$$

mimics the nonzero β . This second order correction cannot be smaller than the typical mismatch of the first order ΔM . Therefore, there is a lower bound on ΔE_2 , which translates into an upper bound on $\Delta M_{\overline{10}-8}$. The antidecuplet cannot be too heavy. On the other hand ΔE_2 cannot be too large for consistency reasons, hence too light antidecuplet is also excluded. This is how the original prediction $M_{\Theta^+} \sim 1535$ MeV was obtained [6]. The updated results of this analysis are given in Table I.

In the quark–soliton models one chooses different path. Instead of going to the second order in perturbative expansion in m_s one calculates non-leading terms in $1/N_c$ [28]. This generates $\beta \neq 0$ from the beginning and the lower bound on I_2 does not exist. One can try either to constrain I_2 by identifying some known nucleon resonance with $N_{\overline{10}}^*$, as it was done in Ref. [7], or resort to explicit model calculations which, however, cover rather broad range of allowed values [9].

In the original paper of Diakonov, Petrov and Polyakov [7] the value of I_2 was fixed by the identification of $N_{\overline{10}}^*$ with $N^*(1710)$ and the equal spacing in antidecuplet by adopting the value of 45 MeV for $\Sigma_{\pi N}$.

Today we would take a different approach. We would use Θ^+ rather than $N_{\overline{10}}^*$ to fix the average $\overline{10}$ mass. In a recent paper [9] it has been shown that the set of parameters of the symmetry breaking Hamiltonian

$$\hat{H}' = \alpha D_{88}^{(8)} + \beta Y + \frac{\gamma}{\sqrt{3}} D_{8i}^{(8)} \hat{S}_i \quad (8)$$

(where $D_{88}^{(8)}$ are SU(3) Wigner matrices, Y is hypercharge and \hat{S}_i is the collective spin operator [28]) which reproduces well the nonexotic spectra,

TABLE I

Masses of baryons obtained by minimizing the square deviation with respect to M_{sol} , I_1 I_2 for fixed α .

| | exp. | $\alpha = 720$ MeV | push |
|-----------------|------|--------------------|------|
| N | 939 | 915 | -23 |
| Λ | 1116 | 1090 | -26 |
| Σ | 1193 | 1214 | +21 |
| Ξ | 1318 | 1323 | +5 |
| Δ | 1232 | 1231 | -1 |
| Σ^* | 1385 | 1389 | +4 |
| Ξ^* | 1533 | 1535 | +2 |
| Ω^- | 1672 | 1662 | -10 |
| Θ^+ | 1540 | 1535 | -5 |
| N_{10}^* | | 1667 | |
| Σ_{10}^* | | 1751 | |
| Ξ_{10}^* | 1862 | 1784 | -78 |

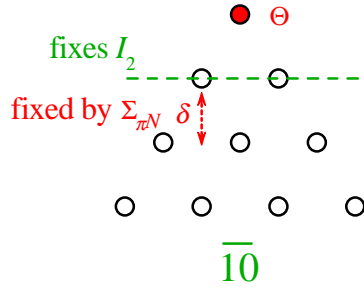


Fig. 2. Antidecuplet of SU(3) flavor including Θ^+ . In Ref. [7] I_2 was fixed by $N^*(1710)$ and the splitting δ by fixing $\Sigma_{\pi N}$.

as well as the measured mass of the $\Theta^+(1540)$, can be parametrized as follows¹:

$$\alpha = 336.4 - 12.9 \Sigma_{\pi N}, \quad \beta = -336.4 + 4.3 \Sigma_{\pi N}, \quad \gamma = -475.94 + 8.6 \Sigma_{\pi N}. \quad (9)$$

¹ We use here $m_s/(m_u + m_d) = 12.9$ [29].

Moreover, the inertia parameters which describe the representation splittings

$$\Delta M_{10-8} = \frac{3}{2I_1}, \quad \Delta M_{\overline{10}-8} = \frac{3}{2I_2} \quad (10)$$

take the following values (in MeV)

$$\frac{1}{I_2} = 152.4, \quad \frac{1}{I_1} = 608.7 - 2.9 \Sigma_{\pi N}. \quad (11)$$

If, furthermore, one imposes additional constraint that $M_{\Xi_{10}} = 1860$ MeV, then $\Sigma_{\pi N} = 73$ MeV [9] (see also [30]) in agreement with recent experimental estimates [31].

Hamiltonian (8) introduces mixing between different representations [9, 32]:

$$\begin{aligned} |B_8\rangle &= |8_{1/2}, B\rangle + c_{10}^B |\overline{10}_{1/2}, B\rangle + c_{27}^B |27_{1/2}, B\rangle, \\ |B_{10}\rangle &= |10_{3/2}, B\rangle + a_{27}^B |27_{3/2}, B\rangle + a_{35}^B |35_{3/2}, B\rangle, \\ |B_{\overline{10}}\rangle &= |\overline{10}_{1/2}, B\rangle + d_8^B |8_{1/2}, B\rangle + d_{27}^B |27_{1/2}, B\rangle + d_{35}^B |\overline{35}_{1/2}, B\rangle, \end{aligned} \quad (12)$$

where $|B_{\mathcal{R}}\rangle$ denotes the state which reduces to the SU(3) representation \mathcal{R} in the formal limit $m_s \rightarrow 0$. The m_s dependent (through the linear m_s dependence of α , β and γ) coefficients $c_{\mathcal{R}}^B$, $d_{\mathcal{R}}^B$ and $a_{\mathcal{R}}^B$ in Eq. (12) can be found *e.g.* in Ref. [9].

Although the model seems to describe the spectrum of antidecuplet rather well (assuming that Θ^+ and Ξ_{10} exist and have masses as discussed in the Introduction), we encounter here the first potential problem. Namely, the exotic–nonexotic mass splitting (6) reads in fact

$$\Delta M_{\overline{10}-8} = \frac{N_c}{2I_2} \sim \mathcal{O}(1) \quad (13)$$

whereas

$$\Delta M_{10-8} = \frac{3}{2I_1} \sim \mathcal{O}(N_c^{-1}). \quad (14)$$

This N_c counting is in fact born by experiment

$$\Delta M_{10-8} \simeq 230 \text{ MeV}, \quad \Delta M_{\overline{10}-8} \simeq 600 \text{ MeV}, \quad (15)$$

however, it poses a serious problem to the validity of the quantization procedure for exotic states in large N_c limit. Indeed, in the imaginary world of very large N_c all nonexotic states are degenerate, whereas exotic ones are split by a quantity of the order $\mathcal{O}(1)$, similarly to the vibrations with which they will mix. This fact although known already in the late 80's have been recently revised critically in the literature [12, 13]

4. Decay widths

In the decay of $\Theta^+ \rightarrow NK$ the kaon momentum in the rest frame of the decaying particle

$$p_K = 267 \text{ MeV} \quad (16)$$

is almost identical to the pion momentum in Δ decay

$$p_\pi = 225 \text{ MeV}. \quad (17)$$

One would, therefore, naively expect that the decay width of Θ^+ should be at least as large as the one of Δ or even larger, since no suppression coming from the overlap of the wave functions is expected. Indeed, pentaquark states have — naively — so called fall-apart modes. One of the chief predictions of the quark–soliton models is that the Θ^+ width, contrary to the naive expectations, is very small [7]. This prediction stimulated experimental searches.

Whilst the mass spectra discussed in the previous section are given as systematic expansions in both N_c and m_s in a theoretically controllable way, reliable predictions for the decay widths cannot be organized in a similar manner. In fact the decay width is calculated by means of the formula for the decay width for $B \rightarrow B' + \varphi$:

$$\Gamma_{B \rightarrow B' + \varphi} = \frac{1}{8\pi} \frac{p_\varphi}{M M'} \overline{\mathcal{M}^2} = \frac{1}{8\pi} \frac{p_\varphi^3}{M M'} \overline{\mathcal{A}^2} \quad (18)$$

up to linear order in m_s . The “bar” over the amplitude squared denotes averaging over initial and summing over final spin (and, if explicitly indicated, over isospin). Anticipating linear momentum dependence of the decay amplitude \mathcal{M} we have introduced reduced amplitude \mathcal{A} which does not depend on the kinematics, *i.e.* on the meson momentum p_φ . For the discussion of the validity of (18) see [9].

Soliton models can be used to calculate the matrix element \mathcal{M} . In order to match former normalization [7] we shall define the decay amplitude as

$$\begin{aligned} \mathcal{M}_{B \rightarrow B' + \varphi} &= \langle B' | \hat{O}_\varphi^{(8)} | B \rangle \\ &= 3 \langle B' | G_0 D_{\varphi i} - G_1 d_{ibc} D_{\varphi b}^{(8)} \hat{S}_c - \frac{G_2}{\sqrt{3}} D_{\varphi 8}^{(8)} \hat{S}_i | B \rangle \times p_\varphi^i, \end{aligned} \quad (19)$$

where the sum over the repeated indices is assumed: $i = 1, 2, 3$ and $b, c = 4, \dots, 7$. It is assumed that coupling constants $G_{0,1,2}$ can be related to the elements of the axial current by means of the generalized Goldberger–Treiman relations [7]. Explicitly

$$\Gamma_{B \rightarrow B' + \varphi} = \frac{3 G_{\mathcal{R}}^2}{8\pi M_B M_{B'}} C_{B \rightarrow B' + \varphi}^{\mathcal{R}} p_\varphi^3.$$

For antidecuplet decays ($\mathcal{R} = \overline{10}$):

$$G_{\overline{10}} = G_0 - G_1 - \frac{1}{2}G_2, \quad C_{\Theta^+ \rightarrow N+K}^{\overline{10}} = \frac{1}{5}, \quad (20)$$

whereas for decuplet ($\mathcal{R} = 10$):

$$G_{10} = G_0 + \frac{1}{2}G_2, \quad C_{\Delta \rightarrow N+\pi}^{10} = \frac{1}{5}. \quad (21)$$

Hence the suppression of antidecuplet decay width may come only from the cancellation between $G_{0,1,2}$ entering $G_{\overline{10}}$. Indeed, in the nonrelativistic small soliton limit discussed above one can show that $G_1/G_0 = 4/5$, $G_2/G_0 = 2/5$ and $G_{\overline{10}} \equiv 0$! Although nonintuitive this cancellation explains the small width of antidecuplet as compared to the one of 10 for example.

One problem concerning this cancellation is that formally

$$G_0 \sim \mathcal{O}(N_c^{3/2}) + \mathcal{O}(N_c^{1/2}), \quad G_{1,2} \sim \mathcal{O}(N_c^{1/2}) \quad (22)$$

and it looks as if the cancellation were accidental as it occurs between terms of different order in N_c . That this is not the case was shown in Ref. [33]. Indeed for arbitrary N_c antidecuplet $\overline{10} = (0, 3)$ generalizes to “ $\overline{10}$ ” = $(0, \frac{N_c+3}{2})$, decuplet “10” = $(3, \frac{N_c-3}{2})$ and octet “8” = $(1, \frac{N_c-1}{2})$ [34], and the pertinent Clebsch–Gordan coefficients in fact depend on N_c :

$$G_{\text{“}\overline{10}\text{”}} = G_0 - \frac{N_c+1}{4}G_1 - \frac{1}{2}G_2. \quad (23)$$

So the subleading G_1 -term is enhanced by additional factor of N_c and the cancellation is consistent with N_c counting. Moreover

$$C_{\Theta^+ \rightarrow N+K}^{\text{“}\overline{10}\text{”}} = \frac{3(N_c+1)}{(N_c+3)(N_c+7)} \sim \mathcal{O}\left(\frac{1}{N_c}\right),$$

$$C_{\Delta \rightarrow N+\pi}^{\text{“}10\text{”}} = \frac{(N_c-1)(N_c+5)}{2(N_c+1)(N_c+7)} \sim \mathcal{O}(1) \quad (24)$$

and it looks like the antidecuplet width were suppressed with respect to decuplet. Unfortunately the phase space factor p_φ^3 spoils this counting. Indeed, because of (13) and (14)

$$p_\pi \sim \mathcal{O}\left(\frac{1}{N_c}\right), \quad p_K \sim \mathcal{O}(1) \quad (25)$$

and consequently

$$\Gamma_{\Delta \rightarrow N+\pi} \sim \mathcal{O}\left(\frac{1}{N_c^2}\right), \quad \Gamma_{\Theta^+ \rightarrow N+K} \sim \mathcal{O}(1) \quad (26)$$

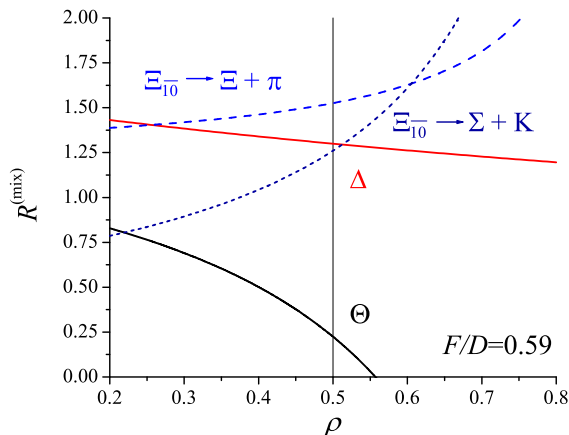


Fig. 3. The correction factors $R^{(\text{mix})}$ due to SU(3)-breaking representation mixing for the decays discussed in the text, as functions of the parameter $\rho \equiv G_1/G_0$.

in the chiral limit. This N_c counting contradicts experimental findings which suggest $\Gamma_{\Theta^+ \rightarrow N+K} \ll \Gamma_{\Delta \rightarrow N+\pi}$.

A few comments are in order. Firstly, let us note that in Nature neither π nor K mesons are massless and both p_π and p_K are of the same order of 230 MeV (16), (17) and the scaling (25) does not hold. So for $m_\pi \neq 0$ and $m_K \neq 0$ both meson momenta scale as N_c^0 , however Δ does not decay, because in the large N_c limit it is degenerate with nucleon and cannot emit a massive particle, whereas Θ^+ does decay. It is an instructive example how subtle is an interplay of theoretical limits $N_c \rightarrow \infty$ and $m_q \rightarrow 0$. Secondly, (25) holds only if the cancellation $G_{\overline{10}} = 0$ is not exact. Let us suppose that the leading N_c power cancels in such a way that $G_{\overline{10}} \sim \mathcal{O}(N_c^{1/2})$ rather than $\mathcal{O}(N_c^{3/2})$. That would make $\Gamma_{\Theta^+ \rightarrow N+K} \sim \mathcal{O}(1/N_c^2)$, *i.e.* of the same order as $\Gamma_{\Delta \rightarrow N+\pi}$. Finally let us remark that there is further suppression of $\Gamma_{\Theta^+ \rightarrow N+K}$ coming from the mixing (12). This is illustrated in Fig. 3 where we plot multiplicative correction factor $R^{(\text{mix})}$ as a function of the parameter $\rho \equiv G_1/G_0$. Phenomenological value of $\rho \sim 0.5$ [9].

5. Summary

The solitonic approach to baryons is very successful, as it describes not only spectra and other static properties, but also structure functions, skewed and nonforward parton distributions and also lighth-cone distribution ampli-

tudes. However, it relies on many approximations. Firstly, it is based on an ansatz and as such one must check the self-consistency of the approach. Evidently, following arguments by Witten [20], solitonic solutions are justified only for large N_c . Indeed, looking *e.g.* at Eq. (1) a calculation of baryonic properties requires functional integration over both bosonic (φ) and fermionic (ψ) degrees of freedom. In order to apply the stationary phase approximation, as it is commonly done in the soliton models, one has to omit the bosonic functional integral, using instead the *background* bosonic field that minimizes the effective action of the system. This is only justified for large N_c , where the bosonic fluctuations are suppressed. Secondly, soliton quantization proceeds by quantizing the rotations in space and flavor space. To this end one assumes the rotational motion to be adiabatic. This means that angular velocities go like $J/I \sim 1/N_c$ yielding frequencies (and hence excitation energies) of order $\mathcal{O}(N_c^{-1})$. This, in turn, implies a Born–Oppenheimer separation of the slow collective rotational motion from the faster modes associated with vibrations. Because of this scale separation the collective rotational modes can be quantized separately from the intrinsic vibrations. While this procedure has been applied with great success to many properties of the nonexotic baryons it has been criticized as far as exotic multiplets are concerned.

The question [12, 13] here is whether the rigid-rotor type semiclassical projection can be applied to exotic states. The fact that the standard semiclassical quantization gave excitation energies of the order $\mathcal{O}(N_c^0)$ for exotic states (13) means that the approach is not justified for such states unless further arguments can be invoked. In contrast, in view of Eq. (14), rigid-rotor quantization is certainly justified for the non-exotic states.

Diakonov and Petrov [35] have argued that while it is true that at large N_c rotational excitations are comparable to vibrational or radial excitations of baryons, both non-exotic and exotic, the corrections due to the coupling between rotations and vibrations die out as $1/N_c$. The collective rotational quantization description fails only when the exoticness, *i.e.* the number of $q\bar{q}$ valence pairs needed to construct the quantum numbers of a given state, becomes comparable to N_c . The newly discovered Θ^+ baryon belongs to the antidecuplet of exoticness = 1. The larger N_c , the more accurate would be its description as a rotational state of a chiral soliton. Diakonov and Petrov [35] support their estimates by considering a simple model consisting of a charged particle in the field of a monopole. However, if one generalizes the model by considering two charged particles interacting by a harmonic potential and moving in the field of a monopole, the coupling of rotational and vibrational degrees of freedom of this system is by no means vanishing but strong [13].

As we have discussed in Sect. 4 large- N_c arguments apply also to the

width of the baryons considered, because if the approach is justified, then at a formal level the width must approach zero at large N_c . Of course, for non-exotic states such as the decuplet, this is true. The reason is simply phase space (25). Unfortunately, as shown recently in Ref. [33] the width of the Θ^+ as calculated via the standard collective rotational approach is of the order N_c^0 in the chiral limit. This demonstrates that the procedure is not self consistent on the basis of pure large- N_c arguments. Thus, if the width of the Θ^+ is really small, as the experiments indicate, there must be particular dynamical reasons for the smallness, which exist on top of what is required for the validity of the large- N_c expansion alone. In this context the cancellation leading to $G_{\overline{10}} = 0$ in the small soliton limit is of particular importance.

For completeness one should also mention another approach to the quantization of chiral solitons based on the assumption that SU(3) symmetry is strongly broken [14]. This approach, known as a bound-state approach, was recently applied to Θ^+ by Klebanov *et al.* [11]. These authors reconsider the relationship between the SU(3) rigid-rotator and the bound-state approach to strangeness in the chiral soliton models. For non-exotic $S = -1$ baryons the bound-state approach matches for small kaon mass m_K onto the rigid-rotator approach, and the bound-state mode turns into the rotator zero-mode. However, for small m_K , there are no $S = +1$ kaon bound states or resonances in the spectrum (unless $m_K \equiv 0$). This shows that for large N_c and small (but non-zero) m_K the exotic state is an artifact of the rigid-rotator approach. An $S = +1$ near-threshold state with the quantum numbers of the Θ^+ pentaquark comes into existence only when SU(3) breaking is sufficiently strong or vector mesons are introduced [36]. Therefore, one argues that pentaquarks are not generic predictions of the chiral soliton models.

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