

Strong Phases in the Decays B to $\pi\pi$

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Abstract

Two sources of strong phases in the decays B to $\pi\pi$ are identified: (1) “quasi-elastic scattering” corresponding to intermediate states like $\pi\pi$ and $\rho\rho$, (2) “ $c\bar{c}$ ” corresponding to intermediate states like $D\bar{D}$ and $D^*\bar{D}^*$. Possibilities of using data to identify these two sources are discussed and illustrated. Present data suggests both sources may be significant.

The decay of $B \rightarrow \pi\pi$ may be considered as due to the effective interaction

$$\lambda_u d(u\bar{u} - t\bar{t})b + \lambda_c d(c\bar{c} - t\bar{t})b, \quad (1)$$

where

$$\begin{aligned} \lambda_u &= V_{ub}^* V_{ud} = e^{i\gamma} A \lambda^3 \frac{\sin\beta}{\sin\alpha} \\ \lambda_c &= V_{cb}^* V_{cd} = -A \lambda^3. \end{aligned}$$

The decay amplitudes may be written, neglecting the small electroweak penguins and assuming isospin invariance,

$$-A(\pi^+\pi^-) = T e^{i\delta_T} e^{i\gamma} + P e^{i\delta_P}, \quad (2)$$

$$-\sqrt{2}A(\pi^0\pi^0) = C e^{i\delta_C} e^{i\gamma} - P e^{i\delta_P}, \quad (3)$$

$$-\sqrt{2}A(\pi^+\pi^0) = (T e^{i\delta_T} + C e^{i\delta_C}) e^{i\gamma}, \quad (4)$$

where δ_T , δ_C , and δ_P are phases due to the strong final state interaction. In terms of the isospin analysis of the λ_u terms, T and C may be replaced by A_2 and A_0 and (δ_C, δ_T) by (δ_2, δ_0)

$$T e^{i\delta_T} = e^{i\delta_0} (A_2 e^{i(\delta_2 - \delta_0)} + A_0), \quad (5)$$

$$C e^{i\delta_C} = e^{i\delta_0} (2A_2 e^{i(\delta_2 - \delta_0)} - A_0). \quad (6)$$

The T term is often referred to as the tree amplitude corresponding to the b quark decay into $u\bar{u}d$, while P is called the penguin corresponding to a loop diagram dominated by the virtual t quark. However, as can be seen from Eq (1) there is also a $t\bar{t}$ loop contributing presumably in a small way to T . This is because we are using what is called the c convention in contrast to the t convention that separates off the t loop[1][2]. Note that P makes a contribution to the $I = 0$ final state so that the complete amplitude for $I = 0$ (sometimes given the notation A_0) is here a sum of the A_0 and P terms.

If the final state scattering were purely elastic the phases δ_2 and δ_0 would be $\pi\pi$ scattering phases in accordance with the Watson theorem. In fact the final state scattering is expected to be very inelastic and described by the $N \times N$ S matrix at the center-of-mass energy equal to the B mass and $J = 0$. The simple Watson theorem can be applied only to final states that are eigenstates of the S matrix. In general for any weak interaction operator O the phase due to the strong interaction for the amplitude A_f is determined from

$$\text{Im}A_f = \text{Im} \sum_i \langle f | S^{1/2} | i \rangle \langle i | O | B \rangle. \quad (7)$$

In the case of A_0 and A_2 or, equivalently C and T , determining the strong phases (δ_2, δ_0) or (δ_C, δ_T) the intermediate states i are $u\bar{u}d(\bar{q})$ state where \bar{q} is the spectator. These include $\pi\pi$, $\rho\rho$, and many others. We refer to these as “quasi-elastic” since the intermediate states include $\pi\pi$ and arise from the same quark set.

In the case of δ_P there are two classes of intermediate states: once again there is a phase due to “quasi-elastic” rescattering which would yield a phase δ_{0P} , but also intermediate states of the form $c\bar{c}d(\bar{q})$ such as DD or D^*D^* which would yield a phase δ_D [3]. Thus the imaginary part of P , $\text{Im}P$, can be written

$$\text{Im}P = \text{Im}(\text{quasi-elastic}) + \text{Im}(c\bar{c}). \quad (8)$$

In the limit that $\text{Im}(c\bar{c})$ vanishes we have

$$\sin\delta_P = \sin\delta_{0P} \equiv \frac{\text{Im}(\text{quasi-elastic})}{|P|}. \quad (9)$$

In general δ_{0P} and δ_0 need not be equal although they involve the same final S matrix because the contributions of different states i to the sum in Eq (7) may not be the same for penguin and tree operators. There is also a contribution of the form $s\bar{s}d(\bar{d})$ corresponding to states such as $K\bar{K}$; we expect this to be very small but include it in δ_{0P} .

We turn to the question of theoretical expectations for these two types of strong phases. It is often stated that the outgoing $\pi\pi$ pair do not scatter thus ruling out the quasi-elastic source[4]. This is clearly wrong since reasonable estimates give a significant value for the $\pi\pi$ cross-section at the energy 5.3 GeV. It can be argued that the multi-particle states which dominate the $u\bar{u}d(\bar{q})$ final states in B decay are not likely to rescatter into the $\pi\pi$ state. However since the final $\pi\pi$ states are less than one in a thousand of the final states even a small rescattering can yield a significant phase. It can be argued that different terms in the sum in Eq (7) cancel, but statistical analysis[5] allows for a significant final phase in spite of this.

At first one might expect that the $c\bar{c}$ states would be unimportant since scattering from these states to $\pi\pi$ is expected to be small by the Zweig rule. Furthermore the total branching ratio into these states is expected to be no more than a factor of 2 larger than the total rate for the $u\bar{u}$ states so the quasi-elastic would be much larger than the $c\bar{c}$ contribution. However two-body states are much more common among the $c\bar{c}$ states and these are expected to rescatter more readily into $\pi\pi$ than the multi-particle states that dominate $u\bar{u}$. For example, the branching ratio to D^*D^* is 30 to 40 times larger than that to $\rho\rho$. Thus a number of papers have suggested that this should dominate the strong phase [6].

We now turn to what we can learn from experimental results. We assume the standard model and that the phase γ has been determined from other experiments. The experimental

Table 1:

	$Br[10^{-6}]$	$C_{\pi\pi}$	$S_{\pi\pi}$
$B^0 \rightarrow \pi^+\pi^-$	4.8 ± 0.5	-0.37 ± 0.10	-0.50 ± 0.12
$B^+ \rightarrow \pi^+\pi^0$	$5.6^{+0.9}_{-1.1}$		
$B^0 \rightarrow \pi^0\pi^0$	1.51 ± 0.28		

results are the branching fraction ratios for the three $\pi\pi$ decays and the asymmetries C_{+-} and S_{+-} ; recent experimental results are summarized in Table 1[7]. The values of C_{+-} and S_{+-} can be used to determine P/T and $\delta_{PT} \equiv \delta_P - \delta_T$. In the approximation that P/T is small and $\beta + \gamma < 90^\circ$

$$\tan(\delta_{PT}) \simeq \cos(2(\beta + \gamma)) \frac{C_{+-}}{S'}$$

where $S' = -(S + \sin(2(\beta + \gamma)))$. Exact results are shown in Table 2 for the central values in Table 1 and three values of γ . As γ becomes smaller δ_{PT} passes through -90 degrees. Phases with magnitudes greater than 90° would be interpreted as a final state strong phase less than 90 degrees with a reversal of the sign of P/T .

The three $\pi\pi$ rates can now be used to determine C/T and $\delta_{CT} \equiv \delta_C - \delta_T$, or, equivalently, A_2/A_0 and $\delta_{20} \equiv \delta_2 - \delta_0$. This is illustrated in Table 2 for the central values in Table 1. Using these values and Eq (5) we determine $\delta_T - \delta_0$ and then from δ_{PT} we obtain $\delta_{P0} \equiv \delta_P - \delta_0$. Two solutions for δ_{CT} with opposite signs are shown. The positive sign leads to lower and more reasonable value for δ_{P0} ; also if the difference between δ_2 and δ_0 is due to isovector exchange in the rescattering one obtains the positive sign. The two solutions may be distinguished by the values of C_{00} ; limited data available so far favors the negative sign for the central value of γ .

The values for δ_{20} are entirely due to “quasi-elastic” rescattering. The value is seen to depend significantly on the $\pi^0\pi^0$ rate as shown in Fig 1 and is quite large for the present central value. If we assume that $\delta_{0P} \sim \delta_0$ then in the absence of a $c\bar{c}$ contribution $\delta_P \sim \delta_0$ (Eq (9)); in this case the large values for $(\delta_P - \delta_0)$ shown in Table 2 would be evidence for $c\bar{c}$ states contributing to δ_P . However it is important to note that the experiments cannot determine δ_0 or δ_{0P} or $\delta_0 - \delta_{0P}$; the isospin-independent quasi-elastic strong phase in T cannot be determined.

In conclusion we have tried to show what can be learned as to the origin of the strong phases simply using data on B to $\pi\pi$ decays. A more ambitious attempt including B to $K\pi$ and using $SU(3)$ has been made by Christopher Smith[8]. Similar results to those shown in Table 2 have been given in a number of papers[9] devoted to determining γ . Here we do not try to analyze

Table 2: Results for the central values in Table 1.

γ	47°	57°	67°
$\frac{p}{T}$	0.26	0.28	0.43
$\delta_P - \delta_T$	-117.37°	-68.97°	-42.23°
$\frac{C}{T}$	0.587 [0.883]	0.768 [0.970]	0.906 [1.027]
$\delta_C - \delta_T$	$+45.29^\circ$ [-77.16°]	$+34.46^\circ$ [-61.83°]	$+26.67^\circ$ [-47.58°]
$\frac{A_2}{A_0}$	0.898 [0.737]	1.178 [0.958]	1.474 [1.227]
$\delta_2 - \delta_0$	$+31.18^\circ$ [-61.26°]	$+32.54^\circ$ [-59.40°]	$+31.53^\circ$ [-54.24°]
$\delta_P - \delta_0$	-102.64° [-142.88°]	-51.33° [-97.98°]	-23.37° [-72.34°]
C_{00}	0.108 [0.353]	0.488 [0.079]	0.746 [-0.085]

the data in detail but simply try to illustrate what strong phase information can be obtained. Our conclusions are:

(1) Definite information on quasi-elastic strong phases in the tree amplitude can be obtained and present data points to a value δ_{20} of order 25° or larger but this is very sensitive to the $\pi^0\pi^0$ branching ratio.

(2) A second strong phase ($\delta_P - \delta_0$), which appears to be quite large, is associated with the penguin and has in general both quasi-elastic and $c\bar{c}$ contributions. If ($\delta_{0P} - \delta_0$), which represents the difference between the quasi-elastic $I = 0$ phase for the penguin and that for the tree, is small then there must be a significant $c\bar{c}$ term. This represents the contribution of rescattering from $c\bar{c}$ states like $D\bar{D}$ or $D^*\bar{D}^*$. Thus present data suggests that both sources of strong phases may be significant.

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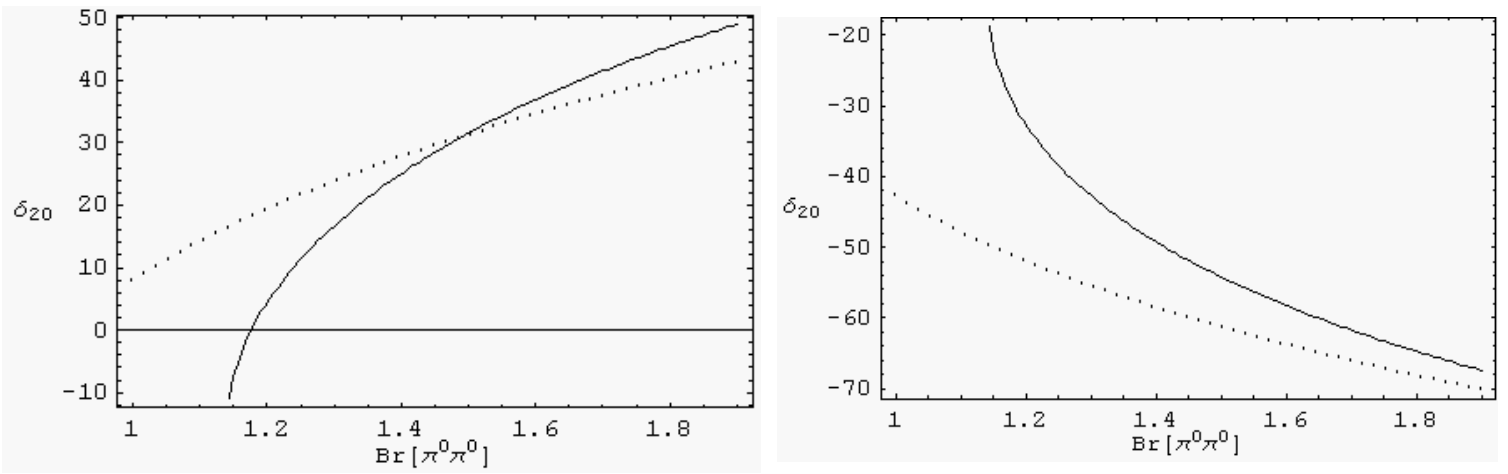


Figure 1: The two solutions for the strong phase difference δ_{20} as a function of the $\pi^0\pi^0$ branching ratio using central values for the other observables and two values of γ (Solid line $\gamma = 67^\circ$, Dashed line $\gamma = 47^\circ$).