

# Quark model description of quasi-elastic pion knockout from the proton at JLAB

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## Abstract

The interference term between  $s$ - and  $t$ -pole contributions to the  $p(e, e'\pi^+)n$  cross section is evaluated on the basis of the constituent quark model. It is shown that the contribution of baryon  $s$ -poles can be modeled by a nonlocal extension of the Kroll-Rudermann contact term. This contribution is in a destructive interference with the pion  $t$ -pole that is essential to improve the description of recent JLab data at the invariant mass  $W = 1.95$  GeV. Some predictions are made for a new JLab measurement at higher values  $W = 2.1 - 2.3$  GeV and  $Q^2$  centered at 1.6 and 2.45 GeV<sup>2</sup>/c<sup>2</sup>.

*PACS:* 12.39.Fe, 12.39.Mk, 13.25.Jx, 13.40.Hg

*Keywords:* quark model, hadron form factors, meson electroproduction.

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## 1 Introduction

In Ref. [1] the JLab F $\pi$ 1 Collaboration extracted from the data the charged pion form factor  $F_\pi(Q^2)$  using a Regge model for high energy meson electroproduction [2]. Along with this the recent JLab data on the Rosenbluth separation of longitudinal and transverse cross sections for the reaction  $p(e, e'\pi^+)n$  were presented for  $Q^2 = 0.6, 0.75, 1$  and  $1.6$  GeV<sup>2</sup>/c<sup>2</sup> and for the invariant mass  $W = 1.95$  GeV.

The procedure for reggeization according to [2] is quite natural for the meson

( $\pi$ ,  $\rho$ ,  $b_1$ , ...)  $t$ -pole diagrams, however, the authors of Ref. [2] have been forced to add a nucleon  $s$ -pole term to the Regge amplitude to ensure gauge invariance. In general, the full sum of  $s$ -channel resonances taken with their proper form factors should be dual to the full sum of reggeon exchanges (see, e.g. the recent review [3]), but actually the form factors for the transitions  $\gamma^* + N \rightarrow N^*$  are only known with large uncertainties, and thus one can stay on a phenomenological level and use only some separate terms from both sums.

The aim of this letter is to show that an alternative (microscopic) description of quasi-elastic pion knockout from the proton can be obtained on the basis of a constituent quark model (CQM). Then with the fixed parameters of model (which are a few in number) one can predict the absolute value and the  $t$ -dependence of longitudinal  $d\sigma_L/dt$  and transverse  $d\sigma_T/dt$  cross sections for the reaction of pion quasi-elastic knockout from nucleon by electrons in a wide interval of intermediate energies. We start from two assumptions, which are natural in the framework of the CQM:

(a) At energies  $W \gtrsim 2.1 - 2.3$  GeV and  $Q^2 \gtrsim 1 - 2$  GeV<sup>2</sup>/c<sup>2</sup> characteristic of the JLab new measurements [1,5] there are no resonance peaks in the cross section, and thus one need not to account for the details of the intermediate nucleon excitations. Only a combined effect of all the  $s$ - and  $u$ -channel contributions should be evaluated. In our opinion, a quark approach for such evaluation would be more convenient than the traditional baryon-resonance approach which characterized by many coupling constants and form factors.

The  $t$ -pole contributions to the quasi-elastic pion knockout correspond to trivial quark diagrams shown in Fig. 1a. Our hypothesis is that in the kinematical region discussed the full sum of nucleon and excited-nucleon  $s$ - and  $u$ -pole terms can be represented by the quark diagrams in Figs. 1b,c. Moreover, in the region of the pion  $t$ -pole,  $|t| \sim m_\pi^2$ , the sum of all the  $s$ - and  $u$ -channel contributions should be a small correction to the  $t$ -pole contribution  $\sim (t - m_\pi^2)^{-1}$ . Therefore, one can neglect the highest order corrections  $\sim (m_q^2/Q^2)^n$ , e.g. generated by diagrams with one- or two-gluon exchange, in comparison to the dominant  $s$ - and  $u$ -pole terms in Figs. 1b,c.

(b) The  $s$ - and  $u$ -pole quark diagrams in Figs. 1b,c at large momentum transfer  $Q^2$  imply using the strong and e.-m. quark form factors. In the CQM one can take the Fourier transform of the pion wave function for the strong  $\pi qq$  vertex form factor (e.g. this can be shown [7] in terms of the  $^3P_0$  model [8]). Then, the quark amplitude for the  $s$ - or  $u$ -channel transition  $\gamma^* + q \rightarrow q + \pi$  becomes proportional to the product of two form factors, the strong one  $F_{\pi qq}$  and the electromagnetic form factor of the constituent quark  $F_q(Q^2)$ <sup>1</sup>. In the

<sup>1</sup> It implies that the constituent quark is an extended object and has its own electromagnetic form factor, e.g.  $F_q(Q^2) = 1/(1 + Q^2/\Lambda_q^2)$  [6]. The parameter  $\Lambda_q$  is

considered region of large  $Q^2$  and  $W$  this product is equivalent to the pion electromagnetic form factor  $F_\pi(Q^2) \simeq F_q(Q^2)F_{\pi qq}(Q^2)$ .

As this relationship has not been considered earlier on, here in Appendix we obtain it with a simple calculation in terms of two diagrams pictured in Fig. 2. The quark diagram for pion form factor  $F_\pi$  is shown in Fig. 2b with the same notations of momentum variables as for the discussed  $s$ -pole diagram represented in Fig. 2a (with the scalar  $\bar{q}q$  vertex  $v$  of  ${}^3P_0$  model). The solid line in both pictures shows the propagation of a large momentum  $\mathbf{q} \gg \mathbf{k}$  transferred from photon to pion by a highly excited quark. It is intuitively clear that at fixed  $q^2$  the contribution of this deeply off-shell quark<sup>2</sup> into the both processes should be similar in value independently on details in diagrams in Fig.2a and Fig.2b. This assumption is formally confirmed by a simple calculation in terms of Gaussian wave functions (see Appendix). As a result, in the considered kinematical region of quasi-elastic pion knockout both the  $t$ - and  $s(u)$ -contributions to the amplitude become proportional to a common form factor  $F_\pi(Q^2)$  which can be factorized from the sum of  $t$ - and  $s(u)$ -pole diagram contributions. Note that in a phenomenological approach [2] such common form factor was introduced *by hand* to preserve gauge invariance of the calculated amplitude.

In this work we obtain an evaluation of a common effect of  $s$ - and  $u$ -pole contributions to the cross section. The overall contribution of such terms is  $Q^2$ - and  $W$ -dependent and vanishes with increasing  $W$  and  $Q^2$  in direct proportion to the factor  $(QM_N/W^2) F_\pi(Q^2)$  (the final expression looks like a generalization of the Kroll-Ruderman contact term by introduction of a  $Q^2$ - and  $W$ -dependent form factor into it). Our evaluation shows that for JLab data [1] at lower  $Q^2$  (0.6 and 0.75 GeV<sup>2</sup>/c<sup>2</sup>) this contribution is important and cannot be neglected<sup>3</sup>, while at higher values of  $Q^2$  and  $W$  it becomes considerably smaller and practically invisible in the cross section at  $W \gtrsim 3$  GeV. Here we predict the  $t$ -dependence of the longitudinal and transverse cross section for a new JLab measurement at  $W = 2.1 - 2.3$  GeV.

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believed to be set by the chiral symmetry scale  $\Lambda_\chi \simeq 4\pi f_\pi \sim 1$  GeV.

<sup>2</sup> In the CQM a quark has a fixed mass  $m_q \approx M_N/3$  and all the non-trivial  $q^2$ -dependence of the quark propagator takes effectively into account through the  $q^2$ -dependent vertex form factors  $F_q$  and  $F_{\pi qq}$ .

<sup>3</sup> Unfortunately at small  $Q^2$ , comparable with the cutoff parameter  $\Lambda \simeq 0.7$  GeV/c in  $\gamma^*NN^*$  vertices, one cannot exclude the direct contribution of intermediate baryon resonances to the cross section. Formally such small  $Q^2$  are out of the region of the quasi-elastic kinematics.

## 2 The $t$ - and $s(u)$ -channel contributions in terms of the CQM

The possibility of describing the high-energy pion electroproduction in terms of the pion  $t$ -channel mechanism has been discussed in the literature over many years [9,10]. An important argument [10] is that the contribution of the  $s$ -channel diagrams in Fig.1b is suppressed by a factor  $1/Q^4$  in comparison to the contribution of the  $t$ -channel diagrams. In Refs. [10,11,12], the pion-exchange ( $t$ -channel diagram) was calculated using the light-front dynamics. In principle, this made it possible to extract the strong  $\pi NN$  form factor  $F_{\pi NN}(t)$  from comparison of a calculated longitudinal cross section  $d\sigma_L/dt$  with the data at high momentum transfer  $Q^2 \gtrsim 2-3 \text{ GeV}^2/c^2$ . But in practice, the accessible data on high-quality Rosenbluth separation are limited by too small  $Q^2$  and  $W$ , where the  $s(u)$ -contributions cannot be neglected.

In contrast to those studies, in Ref. [7] the process  $p(e, e'\pi^+)n$  was considered in the laboratory frame. In this case one is able to single out, in a natural way, the kinematical region where the recoil momentum of the final nucleon  $P'^\mu = \{P'_0, -\mathbf{k}\}$  with  $P'_0 \simeq M_N + \mathbf{k}^2/(2M_N)$  is small and where only the momentum  $\mathbf{k}' = \mathbf{q} + \mathbf{k}$  of the knocked out meson is large. In that (quasi-elastic) region at  $|t| \lesssim 0.1 - 0.2 \text{ GeV}^2$  4-momentum  $k = P - P'$  transferred to the nucleon can be related to the 3-momentum  $\mathbf{k}$ ,  $k^\mu \simeq \{-\mathbf{k}^2/(2M_N), \mathbf{k}\}$ . Therefore, both the initial  $|N(P)\rangle$  and the final-state  $|N(P')\rangle$  nucleons are nonrelativistic ones. In the CQM each of them can be described by a nonrelativistic wave function

$$|N_{3q}(P)\rangle = \Phi_N(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) |[2^3]_C [3]_{SI} S_z, I_z\rangle. \quad (1)$$

Here,  $|[2^3]_C [3]_{SI} S_z, I_z\rangle$  is the color( $C$ )-spin( $S$ )-isospin( $I$ ) part and

$$\Phi_N(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \mathcal{N}_N^{-1} e^{-\boldsymbol{\rho}_1^2/4b^2 - \boldsymbol{\rho}_2^2/3b^2} \quad (2)$$

is the internal wave function normalized as  $\int d\boldsymbol{\rho}_1 d\boldsymbol{\rho}_2 |\Phi_N(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)|^2 = 1$ , where  $\boldsymbol{\rho}_1$  and  $\boldsymbol{\rho}_2$  are the Jacobi coordinates,  $\mathcal{N}_N$  is a normalization constant and  $b$  is a ‘‘quark radius’’ of the nucleon.

The  $t$ -pole amplitude for the  $\gamma^*q \rightarrow q'\pi^+$  process on the  $i$ -th quark (Fig.1a) is

$$\mathcal{M}_{q(t)}^{(i)\mu} = ie \hat{G}_{\pi qq}^{(i)} \frac{F_\pi(Q^2)}{t - m_\pi^2} (k + k')^\mu, \quad \hat{G}_{\pi qq}^{(i)} = g_{\pi qq} \tau_-^{(i)} \boldsymbol{\sigma}^{(i)} \cdot \mathbf{k}, \quad (3)$$

where  $F_\pi(Q^2)$  is the pion electromagnetic form factor and  $\hat{G}_{\pi qq}^{(i)}$  is the  $\pi qq$  vertex operator for the  $i$ -th quark, which is related to the pion-nucleon form factor  $G_{\pi NN}(\mathbf{k}^2)$  as

$$G_{\pi NN}(\mathbf{k}^2)\tau_-\boldsymbol{\sigma}\cdot\mathbf{k}\equiv\langle N_{3q}(P')|\sum_{i=1}^3\hat{G}_{\pi qq}^{(i)}|N_{3q}(P)\rangle=\frac{5\tau_-}{3}g_{\pi qq}\boldsymbol{\sigma}\cdot\mathbf{k}e^{-\mathbf{k}^2b^2/6}. \quad (4)$$

In the following, for convenience, we proceed with the normalized pion-nucleon form factor  $F_{\pi NN}(\mathbf{k}^2)\doteq G_{\pi NN}(\mathbf{k}^2)/g_{\pi NN}$  with  $g_{\pi NN}\equiv G_{\pi NN}(0)$ . We use a simple form of the operator  $\hat{G}_{\pi qq}^{(i)}$  neglecting the momentum dependence since the exchanged pion and the constituent quarks are approximately on their mass shells.

The quark-level amplitude  $\mathcal{M}_{q(t)}^\mu$  gives rise to the  $t$ -pole matrix element for  $\pi^+$  electroproduction from the nucleon

$$\mathcal{M}_{N(t)}^\mu=\langle N_{3q}(P')|\sum_{i=1}^3\mathcal{M}_{q(t)}^{(i)\mu}|N_{3q}(P)\rangle, \quad (5)$$

which is proportional to the strong form factor

$$F_{\pi NN}(t)=\int d\boldsymbol{\rho}_1d\boldsymbol{\rho}_2e^{i\frac{2}{3}\mathbf{k}\cdot\boldsymbol{\rho}_2}|\Phi_N(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)|^2=e^{-\mathbf{k}^2b^2/6}, \quad t\simeq-\mathbf{k}^2, \quad (6)$$

defined above. At  $b=0.6$  fm, which is a typical scale in the CQM (at small  $|t|\lesssim 0.2$  GeV<sup>2</sup> this Gaussian is very close to the monopole form factor with  $\Lambda_{\pi NN}\simeq 0.7$  GeV), the matrix element (5) gives a reasonable description of the  $t$ -dependence of the differential cross sections in the JLab experiment (see, e.g., Ref. [7]). However, since the  $s$ - and  $u$ - channel quark contributions (Fig. 1b,c), which are required for gauge invariance, have not been taken into account, the results of such simple quark evaluation (dashed lines in the upper panels of Fig. 3) are not so good in comparison with the Regge parametrization [2] used in Refs. [1,13] (dash-dotted lines in Fig. 3).

Here we shall evaluate these contributions, starting from a nonlocal (nl) variant of the pseudoscalar  $\pi qq$  coupling  $g_{\pi qq}^{\text{nl}}$ , which differs from the local coupling  $g_{\pi qq}$  by the presence of the quark form factor:  $g_{\pi qq}^{\text{nl}}(p^2)\doteq g_{\pi qq}F_{\pi qq}(p^2)$ . Hence, the vertex operator  $\hat{G}_{\pi qq}^{(i)}$  is modified accordingly. In this case the kinematics is the following. Two particles, pion and ingoing (outgoing) quark, are on mass shell:  $k'^2=m_\pi^2$ ,  $p_i^2=m_q^2$  ( $p_i'^2=m_q^2$ ), where  $m_q=M_N/3$  is the mass of the constituent quark. But the intermediate quark with a large momentum  $p_{i(s)}''=p_i+q$  ( $p_{i(u)}''=p_i'-q$ ) obtained by absorption (emission) of a momentum transfer from the  $\gamma^*$  should be considerably off its mass shell:  $p_i''^2\simeq Q^2$ , (see solid lines in Figs.1b,c).

The contribution of the quark diagrams in Figs.1b,c to the matrix element for absorption of the longitudinal virtual photon ( $\epsilon^{(0)\mu}=\frac{1}{Q}\{|\mathbf{q}|, q_0\hat{\mathbf{q}}\}$ ,  $Q=\sqrt{-q^2}$ )

$$\begin{aligned} \mathcal{M}_{q(s+u)}^{(i)\mu} &= -i\sqrt{2}g_{\pi qq}F_{\pi qq}(Q^2)F_q(Q^2) \\ &\times \left\{ e_u\gamma^5\frac{1}{\not{p}'_i + \not{k}' - m_q}\gamma^\mu + e_d\gamma^\mu\frac{1}{\not{p}'_i - \not{k}' - m_q}\gamma^5 \right\}^{(i)} \end{aligned} \quad (7)$$

was evaluated in the approximation  $W^2 \gg (Q^2 - M_N^2), M_N^2$ . Here  $e_u(e_d)$  is the quark charge and  $F_q(Q^2)$  is the quark electromagnetic form factor. In the considered region of a large momentum  $k'^\mu = (k'^0, \mathbf{k}')$  transferred to the pion (with  $k'^0 \simeq |\mathbf{k}'| \simeq |\mathbf{q}|$ ) and a small momentum  $k^\mu = \{t/(2M_N), \mathbf{k}\} \simeq \{0, \mathbf{k}\}$  transferred to the nucleon we can omit all contributions to the amplitude proportional to small parameters

$$\frac{Q^2 - M_N^2}{W^2}, \frac{|\mathbf{k}|}{2M_N}, \frac{m_q^2}{W^2}, \frac{m_\pi^2}{M_N^2}, \dots \ll 1. \quad (8)$$

Then, for the 4-momenta of intermediate (off-shell) quarks in the diagrams in Fig. 1b,c one can approximately write

$$p''_{i(s)}{}^2 \simeq 2m_q k'_0 \left( 1 + \frac{|\boldsymbol{x}_2|}{m_q} \cos\theta \right) - Q^2, \quad p''_{i(u)}{}^2 \simeq -p''_{i(s)}{}^2 - Q^2, \quad (9)$$

where  $\boldsymbol{x}_2$  is a relative momentum conjugated to the Jacobi coordinate  $\boldsymbol{\rho}_2$ , and  $\cos\theta = \hat{\mathbf{k}}' \cdot \hat{\boldsymbol{x}}_2 \simeq \hat{\mathbf{q}} \cdot \hat{\boldsymbol{x}}_2$ . By making use of the approximate equality  $|\mathbf{q}|/q_0 \simeq 1 + Q^2/(2q_0^2)$ , which holds within the conditions (8), one obtains the approximation  $(\not{k}'\gamma^5\gamma^\mu + \gamma^5\gamma^\mu\not{k}')\varepsilon_\mu^{(0)} \simeq -k'_0 Q/q_0 \text{diag}\{\boldsymbol{\sigma} \cdot \hat{\mathbf{q}}, \boldsymbol{\sigma} \cdot \hat{\mathbf{q}}\}$  useful for reducing the r.h.s. of Eq. (7) (the approximation implies that  $\hat{\mathbf{k}}' \simeq \hat{\mathbf{q}}$ , which is true for the forward pion knockout).

Finally, in the lowest order of expansion in the small parameters  $M_N^2/W^2$  and  $Q^2/W^2$  the matrix element (7) reduces to an effective  $\gamma\pi qq$  interaction<sup>4</sup>

$$\mathcal{M}_{q(s+u)}^{(i)\mu}\varepsilon_\mu^{(\lambda=0)} = -i\frac{eg_{\pi qq}}{2m_q}\frac{M_N Q}{W^2}\frac{\tau_-^{(i)}(\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathbf{q}})}{(1 + \frac{|\boldsymbol{x}_2|}{m_q}\cos\theta)}F_q(Q^2)F_{\pi qq}(Q^2), \quad (10)$$

in which one can use the relation

$$F_q(Q^2)F_{\pi qq}(Q^2) \simeq F_\pi(Q^2), \quad (11)$$

<sup>4</sup> At the photon point  $Q^2 = 0$ ,  $\varepsilon_\mu^{(\lambda)} = \{0, \hat{\mathbf{n}}_\perp\}$  and in the low-energy limit  $k'_0 \rightarrow m_\pi$  the same quark calculation gives exactly rise to the Kroll-Ruderman term [14] for the threshold pion photoproduction.

discussed in the Section 1 (see also Appendix). Note that the  $\boldsymbol{x}_2$ -dependent denominator in Eq. (10), being integrated with Gaussian functions (1), introduces only a small correction to the integral, and thus we shall omit it in the final expression (see next section) for the nucleon matrix element (but it has not been omitted in the numerical calculation).

Eq. (11) is very useful to check gauge invariance of the sum of  $t$ - and  $s(u)$ -pole amplitudes in the final expression, where these amplitudes destructively interfere (see Sect. 3). It should be noted that gauge invariance can also be restored in the general case, if one uses different vertex form factors (see detailed discussion in Refs. [4,15]).

### 3 Longitudinal and transeverse cross section

We calculate the longitudinal part of differential cross section

$$\frac{d\sigma_L}{dt} = \frac{M_N^2 \overline{|J_\pi^\mu \varepsilon_\mu^{(\lambda=0)}|^2}}{4\pi(W^2 - M_N^2) \sqrt{(W^2 - Q^2 - M_N^2)^2 + 4W^2 Q^2}} \quad (12)$$

for quasi-elastic pion knockout taking into account both the  $t$ -pole diagram in Fig.1a and the  $s$ - and  $u$ -pole contribution (10),

$$\overline{|J_\pi^\mu \varepsilon_\mu^{(\lambda=0)}|^2} = \frac{1}{2} \sum_{spin} |\langle N_{3q}(P') | \sum_{j=1}^3 [\mathcal{M}_{q(t)}^{(j)\mu} + \mathcal{M}_{q(s+u)}^{(j)\mu}] \varepsilon_\mu^{(\lambda=0)} | N_{3q}(P) \rangle|^2, \quad (13)$$

and compare the results to the JLab data [1] and to the Regge model [2] predictions [1,13] (see Fig. 3).

The CQM calculation of the right-hand side of Eq. (13) leads to a simple matrix element for a nonrelativistic nucleon wave function  $\Phi_N(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ . Using Eqs. (5) and (10) we obtain in the laboratory frame the following matrix element of the longitudinal hadron current for the transition  $\gamma^* + p \rightarrow n + \pi^+$ :

$$J_\pi^\mu \varepsilon_\mu^{(0)} = i\tau_- \frac{eg_{\pi NN}}{2M_N} F_\pi(Q^2) F_{\pi NN}(t) \left[ \frac{2\varepsilon^{(0)} \cdot k' \boldsymbol{\sigma} \cdot \mathbf{k}}{t - m_\pi^2} - \frac{M_N Q}{W^2} \boldsymbol{\sigma} \cdot \hat{\mathbf{q}} \right]. \quad (14)$$

Here the contribution of the  $b_1$ -meson pole (the P-wave excitation of the pion in the CQM) may be also taken into account. Then, the third term,  $+(g_{b_1 NN}/g_{\pi NN}) (g_{b_1 \pi \gamma} k_0 Q / (2M_N m_{b_1})) (\mathbf{k} \cdot \hat{\mathbf{q}} \boldsymbol{\sigma} \cdot \mathbf{k} / (t - m_{b_1}^2))$ , should be inserted into the square brackets in the r.h.s. of Eq. (14). However, the  $b_1$ -pole contribution is too small when compared to the  $\pi$ -pole contribution. The  $\rho$ -meson

pole is also not included as in the CQM it does not contribute to the longitudinal part of the cross section (in the CQM the quark spin-flip amplitude  $\rho^+ + \gamma^*(M1) \rightarrow \pi^+$  only contributes to the transverse part of the cross section where it is of prime importance [10,16]).

The quark model calculation leading to the r.h.s. of Eq. (14) has the advantage that the parameters of the vertices  $g_{\pi NN}$ ,  $g_{b_1 NN}$ ,  $g_{b_1 \pi \gamma}$  and their form factors  $F_{\pi NN}(t)$ ,  $F_{b_1 NN}(t)$ ,  $F_{b_1}(Q^2)$  are not free, but related to each other by the following relationships: (i) The integral part of Eq. (13) defines a nonrelativistic vertex form factor (6)  $F_{mNN}(t) = F_{\pi NN}(t) = F_{b_1 NN}(t)$  common to all the terms in Eq. (14). (ii) The relative phases of all the amplitudes in the r.h.s of Eq. (14) are fixed by the results of quark model calculations. Therefore, the negative sign of the interference term between  $t$ - and  $s$ -pole contributions (the destructive interference) is unambiguously determined: the spin average of the product of the first term in the squared bracket of Eq. (14) with the second one has a negative value:  $1/2 \sum_{S_z S'_z} \langle S'_z | \boldsymbol{\sigma} \cdot \hat{\mathbf{k}} | S_z \rangle \langle S_z | \boldsymbol{\sigma} \cdot \hat{\mathbf{q}} | S'_z \rangle = \cos(\widehat{\mathbf{kq}})$ . Recall that  $\cos(\widehat{\mathbf{kq}}) \simeq -1$  in the quasielastic pion knockout kinematics, where  $-\mathbf{k}$  is the momentum of the nucleon-spectator in the lab frame.

The  $\rho$ -pole contribution to the transverse part of differential cross section  $d\sigma_T/dt$  is proportional to the value  $|J_\rho^\mu \varepsilon_\mu^{(\lambda=1)}|^2$ , where

$$J_\rho^\mu \varepsilon_\mu^{(\lambda)} = i\tau_- e g_{\rho\pi\gamma} \frac{M_\rho}{M_\pi} F_{\rho\pi\gamma}(Q^2) \frac{1 + \varkappa_\rho}{2M_N} g_{\rho NN} F_{\rho NN}(t) |\mathbf{q}| \frac{i[\boldsymbol{\sigma} \times \mathbf{k}] \cdot \boldsymbol{\epsilon}^{(\lambda)}}{t - M_\rho^2} \quad (15)$$

and  $\varepsilon^{(\lambda)\mu} = \{0, \boldsymbol{\epsilon}^{(\lambda)}\}$  for  $\lambda = \pm 1$ . For coupling constants and form factors in Eq. (15) we use the results of calculations [16] in terms of  ${}^3P_0$  model from Ref. [7]:  $g_{\rho NN} = \frac{1}{5} g_{\pi NN} \sqrt{\frac{M_\rho}{M_\pi}}$ ,  $1 + \varkappa_\rho = 5$ ,  $F_{\rho NN}(t) = F_{\pi NN}(t)$ . For  $g_{\rho\pi\gamma}$  coupling constant we take the value  $g_{\rho\pi\gamma} = 0.103$  which is fixed by the experimental value of the decay width  $\Gamma_{\rho \rightarrow \pi\gamma} = 67$  KeV. The momentum dependence of  $F_{\rho\pi\gamma}(Q^2)$  will be discussed in the next section.

## 4 Results and outlook

In our calculation we use a standard (monopole-like) representation for the transition form factors  $F_\pi(Q^2)$  and  $F_{\rho\pi\gamma}(Q^2)$ :

$$F_\pi(Q^2) = \frac{1}{1 + Q^2/\Lambda_\pi^2}, \quad F_{\rho\pi\gamma}(Q^2) = \frac{1}{1 + Q^2/\Lambda_{\rho\pi}^2}. \quad (16)$$



For the pion charge form factor we use  $\Lambda_\pi^2 = 0.54 \text{ GeV}^2/c^2$  which is close to the recent theoretical evaluation [17] and correlates well with the recent JLab data [1]. In the case of the  $F_{\rho\pi\gamma}(Q^2)$  form factor we vary  $\Lambda_{\rho\pi}^2$  from  $\Lambda_\pi^2 = 0.54 \text{ GeV}^2/c^2$  to  $0.7 \text{ GeV}^2/c^2$ . We think that an accurate analysis of the transverse part of the differential cross section can shed light on the value of  $\Lambda_{\rho\pi}$ . From Fig. 3 one can see, that the value  $\Lambda_{\rho\pi}^2 = 0.7 \text{ GeV}^2/c^2$  is more appropriate in description of the transverse cross section. Apart from this value we vary only one free parameter in the standard representation of the strong  $\pi NN$  form factor  $F_{\pi NN}(Q^2) = \Lambda_{\pi NN}^2 / (\Lambda_{\pi NN}^2 + Q^2)$  where the range parameter  $\Lambda_{\pi NN} \simeq 0.6 - 0.7 \text{ GeV}/c$  corresponds to the reasonable value  $b \simeq 0.5 - 0.6 \text{ fm}$  for the radius of the three-quark configuration  $s^3$  used in the CQM nucleon wave function (1). At realistic values of  $\Lambda_{\pi NN} = 0.7 \text{ GeV}/c$  and  $g_{\pi NN} = 13.5$  our results (the solid lines in Fig.3) are in a better agreement with the data [1] than the simplified model [7] (dashed lines in two upper panels) taking into account only the pion  $t$ -pole not satisfying gauge invariance.

In our calculation we model the contribution of baryon resonances in the  $s$ - and  $u$ -channel by a nonlocal extension of the Kroll-Rudermann contact term. This contribution is negative, which is essential to improve the description of the measured cross section. At intermediate values of  $Q^2 \gtrsim 1 \text{ GeV}^2/c^2$ , which correspond to a quasi-elastic mechanism of pion knockout, the calculated cross section is close to the experimental data. At smaller  $Q^2 \lesssim 0.7 \text{ GeV}^2/c^2$  our results are close to the cross section calculated in Ref [1,13] (dash-dotted lines) on the basis of a model [2] which takes into account both the Reggeon-pole exchange and the nucleon-pole contribution.

For a new JLab measurement of  $d\sigma_L/dt$  and  $d\sigma_T/dt$  at higher invariant mass (the data analysis is presently underway) in Fig. 4 we give our prediction for the  $t$ -dependence of the cross sections at  $W = 2.1 - 2.3 \text{ GeV}$  and  $Q^2$  centered at  $1.6$  and  $2.45 \text{ GeV}^2/c^2$ . Here we use fixed (above defined) parameters of the model. Because of the large value of  $W$  the contribution of the effective contact term (10), which is proportional to the factor  $QM_N/W^2$ , becomes too small and not shown in Fig. 4.

Our calculations show that the longitudinal cross section practically does not depend on the contribution of the  $b_1$ -meson pole, and thus the data [1] cannot constrain the  $b_1 NN$  and  $b_1 \pi \gamma$  vertices. In contrast, data on the transverse cross section should be critically dependent on the  $\rho$ -pole contribution and on the  $\rho\pi\gamma^*$  spin-flip amplitude. This was first shown in Ref. [10] and then supported in Ref. [16]. Our results (Figs. 3 and 4) confirm this statement and show that the data on the  $d\sigma_T/dt$  at high  $Q^2$  and  $W$  can be used for the direct measurement of the  $F_{\rho\pi\gamma}(Q^2)$  form factor. On the other hand, a Regge description [2] of the transverse cross section is not as good as for the longitudinal case.

## Acknowledgements

The authors thank the Fpi2 Collaboration [Experiment E01-004 at JLab] for the interest to our work and informative discussion. We thank Vladimir Neudatchin, Nikolai Yudin, Garth Huber and Tanja Horn for fruitful discussions and suggestions. This work was supported by the DFG under contracts FA67/25-3 and GRK683 and the DFG grant 436 RUS 113/790/. This research is also part of the EU Integrated Infrastructure Initiative Hadron-physics project under contract number RII3-CT-2004-506078, President grant of Russia "Scientific Schools" No. 1743.2003 and grants of Russia RFBR No. 05-02-04000, 05-02-17394 and 03-02-17394.

## A Appendix

The contribution of the diagram in Fig. 2a is proportional to the overlap integral [7]

$$\begin{aligned}
 F_{diag}(\mathbf{q}, \mathbf{k}) &= F_q(\mathbf{q}^2) \frac{1}{\mathcal{N}} \int \frac{d\boldsymbol{\chi}_2}{(2\pi)^3} \Phi_N(\boldsymbol{\chi}_2) \Phi_N\left(\boldsymbol{\chi}_2 + \frac{2}{3}\mathbf{k}\right) \Phi_\pi\left(-\boldsymbol{\chi}_2 + \frac{\mathbf{q}-\mathbf{k}}{2}\right) \\
 &= e^{-\mathbf{k}^2 b^2/6} F_q(\mathbf{q}^2) \exp\left[-\frac{(\mathbf{q}-\frac{\mathbf{k}}{3})^2 b_\pi^2}{4(1+2x_\pi^2/3)}\right], \quad (\text{A.1})
 \end{aligned}$$

where  $\Phi_N$  is a wave function of the 3-rd quark in the nucleon and  $\Phi_\pi$  is a pion wave function in the momentum representation (here it plays the role of a strong  $\pi qq$  form factor). In this simple calculation we use  $\Phi_N(\boldsymbol{\chi}_2) \sim e^{-3\boldsymbol{\chi}_2^2 b^2/4}$  and  $\Phi_\pi(\boldsymbol{\chi}) \sim e^{-\boldsymbol{\chi}^2 b_\pi^2}$ , where  $\boldsymbol{\chi}_2 = (\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{p}_3)/3$ . Here  $b$  and  $b_\pi$  are the quark radii in nucleon and pion correspondingly with  $x_\pi = b_\pi/b$ . In the CQM one can obtain a very similar expression for the diagram in Fig. 2b:

$$F_\pi(\mathbf{q}^2) = F_q(\mathbf{q}^2) \int \frac{d\mathbf{p}}{(2\pi)^3} \Phi_\pi\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) \Phi_\pi\left(\mathbf{p} + \frac{\mathbf{q}-\mathbf{k}}{2}\right) = F_q(\mathbf{q}^2) e^{-\mathbf{q}^2 b_\pi^2/8}. \quad (\text{A.2})$$

One can see that the strong  $\pi NN$  form factor (in the CQM it is  $F_{\pi NN}(\mathbf{k}^2) = e^{-\mathbf{k}^2 b^2/6}$ ) appears as a multiplier in the second line of Eq. (A.1) and the remainder of this expression coincides with the pion form factor (A.2) in the limit  $\mathbf{q} \gg \mathbf{k}$  at a specific value  $x_\pi = \sqrt{3}/2$ , which is not very different from the realistic value  $x_\pi \approx 1$  characteristic of the CQM. Really the factor  $\sqrt{3}/2$  appears because of a specific procedure of center-of-mass motion eliminating, which is model dependent. We shall consider a small difference between  $F_\pi$  and  $F_q F_{\pi qq}$  as a small model-dependent correction which can be neglected in line of the assumption (a) given in Section 1.

It could also happen that the  $\gamma^*$  is absorbed by one quark while the pion is emitted from a different quark in the nucleon. Then a large momentum  $\mathbf{q}$  should be firstly transferred to the  $N$ - $3q$  vertex. In the CQM this amplitude can be evaluated as well. One can insert an intermediate nucleon (baryon) state between two points in the diagram where a large momentum  $\mathbf{q}$  or  $|\mathbf{k}'| \approx |\mathbf{q}|$  is absorbed or emitted. It is essential that the nucleon propagator and form factors (both electromagnetic and strong) depend on the large momentum. This leads to the following contribution to the amplitude of pion quasi-elastic knockout, which scales as:

$$\begin{aligned}
F_{nondiag}(\mathbf{q}, \mathbf{k}') &\sim F_q(\mathbf{q}^2) \int \frac{d\boldsymbol{\kappa}_2}{(2\pi)^3} \Phi_N(\boldsymbol{\kappa}_2) \Phi_N\left(\boldsymbol{\kappa}_2 + \frac{2}{3}\mathbf{q}\right) \frac{M_N \sqrt{Q^2}}{Q^2 - M_N^2} \\
&\times \int \frac{d\boldsymbol{\kappa}'_2}{(2\pi)^3} \Phi_N(\boldsymbol{\kappa}'_2) \Phi_N\left(\boldsymbol{\kappa}'_2 - \frac{2}{3}\mathbf{k}'\right) \approx G_E(Q^2) \frac{M_N}{\sqrt{Q^2}} F_{\pi NN}(Q^2), \quad (\text{A.3})
\end{aligned}$$

where  $G_E$  is a nucleon electric form factor. At large  $Q^2 \gtrsim 1 - 2 \text{ GeV}^2/c^2$  characteristic of the new  $F\pi 2$  measurement the ratio  $|F_{nondiag}(\mathbf{q}, \mathbf{k}')/F_{diag}(\mathbf{q}, \mathbf{k})|^2$  is too small. Hence one can consider the amplitude (A.3) as a second order correction to the  $\pi$ -pole amplitude, while the amplitude (A.1) is the first order correction.

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## LIST OF FIGURES

Fig.1: Quark diagrams for the  $t$ -,  $s$ - and  $u$ -channel mechanism of pion knock-out. Thick lines indicate large  $Q^2$  transition.

Fig.2: Quark diagrams: (a) for the  $s$ -pole mechanism of pion emission (in terms of  ${}^3P_0$  model), (b) for pion form factor.

Fig.3: Longitudinal and transverse cross sections for the  $p(e, e'\pi^+)n$  process. The  $F\pi 1$  data [1] for  $W \simeq 0.95$  GeV. 1) The upper row of panels:  $Q^2 = 0.6$  GeV<sup>2</sup>/c<sup>2</sup> (left panel) and  $0.75$  GeV<sup>2</sup>/c<sup>2</sup> (right panel). Dashed lines: the pion  $t$ -pole only. Solid lines: the total sum of the pion  $t$ -pole and the contact ( $s + u$ )-term with common e.-m. and strong form factors. Dotted lines: the same sum, but without form factors in the contact term. Dash-dotted lines: the Regge model [2] prediction. 2) The lower two rows of panels:  $Q^2 = 1.0$  GeV<sup>2</sup>/c<sup>2</sup> (upper panels) and  $1.6$  GeV<sup>2</sup>/c<sup>2</sup> (lower panels). Left panels ( $d\sigma_L/dt$ ): the same notations as in the first two panels. Right panels ( $d\sigma_T/dt$ ): the  $\pi$ -pole contribution only (dotted); the sum of  $\pi$ - and  $\rho$ -pole + ( $s + u$ ) contributions with a monopole  $\rho\pi\gamma$  form factor,  $\Lambda_{\rho\pi}^2 = 0.54$  GeV<sup>2</sup>/c<sup>2</sup> (solid) and  $0.7$  GeV<sup>2</sup>/c<sup>2</sup> (dashed).

Fig.4: Predicted cross sections for new JLab measurements [5] at a higher value of  $W = 2.1 - 2.3$  GeV and  $Q^2$  centered at  $1.6$  GeV<sup>2</sup>/c<sup>2</sup> (upper curves) and  $2.45$  GeV<sup>2</sup>/c<sup>2</sup> (lower curves). Left panel ( $d\sigma_L/dt$ ): the total sum of the pion  $t$ -pole and the contact ( $s + u$ )-term (solid); the Regge model prediction (dash-dotted). Right panel ( $d\sigma_T/dt$ ): the  $\pi$ -pole contribution only (dotted); the sum of  $\pi$ - and  $\rho$ -pole + ( $s + u$ ) contributions with a monopole  $\rho\pi\gamma$  form factor,  $\Lambda_{\rho\pi}^2 = 0.54$  GeV<sup>2</sup>/c<sup>2</sup> (solid) and  $0.7$  GeV<sup>2</sup>/c<sup>2</sup> (dashed).

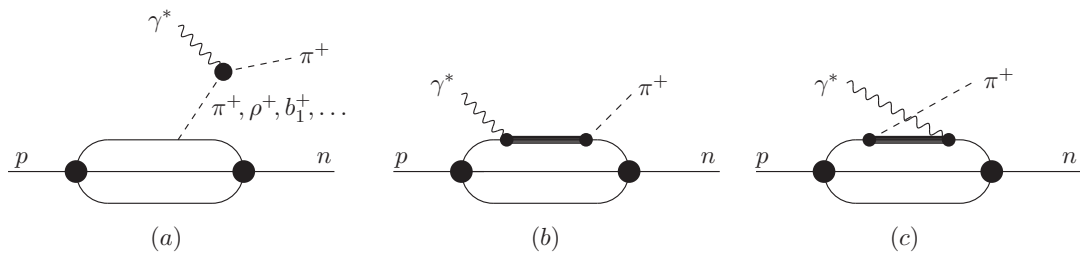


Fig.1

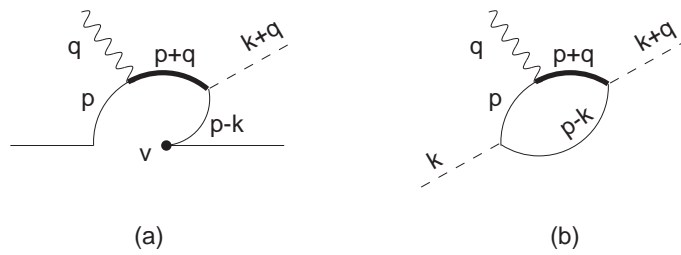


Fig.2

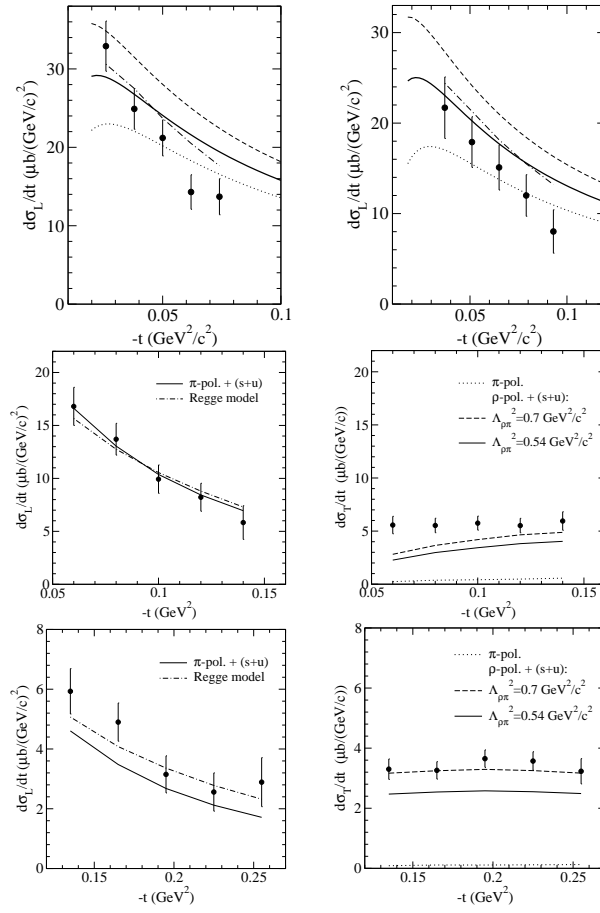


Fig.3

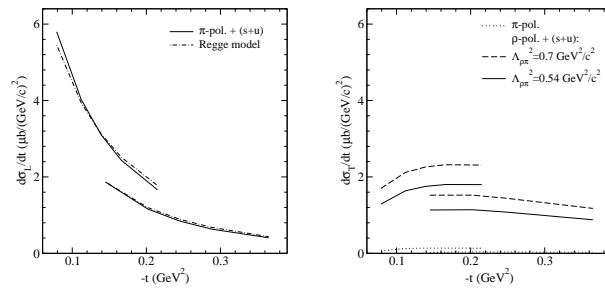


Fig.4