

$B_d(\bar{B}_d) \rightarrow \rho^\pm \pi^\mp, \rho^+ \rho^-, \pi^+ \pi^-$: hunting for alpha

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Abstract

We determine the domains of the values of unitarity triangle angle α , allowed by the charmless strangeless $B_d(\bar{B}_d)$ decays.

1 Introduction

In paper [1] from the data on CP asymmetries in $B_d(\bar{B}_d) \rightarrow \rho^\pm \pi^\mp, \rho^+ \rho^-$ decays and BABAR data on CP asymmetries in $B_d(\bar{B}_d) \rightarrow \pi^+ \pi^-$ decays we determine the value of angle α of the unitarity triangle:

$$\alpha = 96^\circ \pm 3^\circ, \quad (1)$$

where only a tree quark decay amplitude $\bar{b} \rightarrow u\bar{u}\bar{d}$ ($b \rightarrow u\bar{u}d$) was taken into account. The numerical values of angle α obtained from the considered decays are consistent with each other and with the value which follows from the global CKM fit. This observation testifies to the validity of a proposed approach.

As the next step in the present paper we will study what changes in the values of α are induced by QCD penguins. Our aim is twofold. First, in this way we will get an estimate of the theoretical uncertainty of the value of α determined in paper [1]. Secondly, we will get the formulas for CP violating parameters describing these decays which vanish when penguins are neglected ($C_{\rho\pi}, A_{CP}^{\rho\pi}, C_{\rho\rho}, C_{\pi\pi}$).

The angle shifts $\Delta\alpha$ we are interested in were estimated in paper [2]; however, in that paper FSI phases were neglected, that is why $C_{\rho\pi} = A_{CP}^{\rho\pi} = C_{\rho\rho} = C_{\pi\pi} = 0$ follows from [2] (the nonzero penguin amplitudes are a

necessary, but not a sufficient condition for $C_{\rho\pi\dots}$ to be nonzero). We will take these phases into account. The asymmetries depend on the differences of FSI phases in the processes described by the tree and penguin diagrams. One source of these differences is an imaginary part of the quark penguin diagram, the so-called BSS mechanism of the strong phases generation [3] (see also paper [4]). The phase of the penguin diagram depends on the gluon q^2 which is transferred to $u\bar{u}$ pair, each quark of which goes to different π^\pm - or ρ^\pm -mesons. In this way the value of q^2 depends on the light meson wave functions and we can estimate it only roughly. Another source of FSI phases is hadron rescattering and even less is known about the values of the phase shifts between the penguin and tree diagrams generated in this way. In view of this we will determine FSI phases from the experimental data on CPV asymmetries, and investigate to what values of α it will lead.

In Appendix we present the weak interaction Hamiltonian which is responsible for $b \rightarrow u\bar{u}d$ transition and calculate the necessary matrix elements. Using these formulas in sections 2, 3, and 4 we study $B \rightarrow \rho\pi$, $\rho\rho$ and $\pi\pi$ decays correspondingly and extract the values of angle α from the experimental data on CP asymmetries in these decays. We conclude in section 5 with the averaged value of α and a general discussion.

2 α from $\bar{B}_d(B_d) \rightarrow \rho^\mp\pi^\pm$

The time dependence of the decay probabilities is given by [5]:

$$\begin{aligned} \frac{dN(B_d(\bar{B}_d) \rightarrow \rho^\pm\pi^\mp)}{dt} &= (1 \pm A_{CP}^{\rho\pi})e^{-t/\tau}[1 - q(C_{\rho\pi} \pm \Delta C_{\rho\pi}) \times \\ &\times \cos(\Delta mt) + q(S_{\rho\pi} \pm \Delta S_{\rho\pi}) \sin(\Delta mt)] \quad , \quad (2) \end{aligned}$$

where $q = -1$ describes the case when at $t = 0$ B_d was produced, while $q = 1$ corresponds to \bar{B}_d production at $t = 0$. In the case of $\Upsilon(4S) \rightarrow B_d\bar{B}_d$ decay the flavor of the beauty meson which will decay to $\rho\pi$ is tagged by the charge of a lepton in the other beauty meson semileptonic decay. A partner decay starts clocks as well. τ is $B_d(\bar{B}_d)$ life time, while Δm is the difference of masses of (B_d, \bar{B}_d) system eigenstates (it equals the frequency of $B_d - \bar{B}_d$ oscillations).

From Eqs. (A15) and (A17) we obtain:

$$\bar{M}^{-+} = AV_{ub}V_{ud}^*[1 - 0.07e^{i(\delta-\alpha)}] = AV_{ub}V_{ud}^*[1 - 0.07 \sin \delta + i0.07 \cos \delta] \quad , \quad (3)$$

$$M^{-+} = BV_{ub}^*V_{ud} \ , \quad (4)$$

$$\lambda^{-+} \equiv \frac{q}{p} \frac{\bar{M}^{-+}}{M^{-+}} = e^{2i\alpha} \frac{A}{B} [1 - 0.07 \sin \delta + i0.07 \cos \delta] \ , \quad (5)$$

where parameters q and p enter the expressions for (B_d, \bar{B}_d) eigenstates and we have substituted $\alpha = \pi/2$ in the (small) second term in square brackets in Eq. (A15). Analogously we get:

$$\bar{M}^{+-} = BV_{ub}V_{ud}^* \ , \quad (6)$$

$$M^{+-} = AV_{ub}^*V_{ud}[1 - 0.07e^{i(\alpha+\delta)}] = AV_{ub}^*V_{ud}[1 + 0.07 \sin \delta - 0.07i \cos \delta] \ , \quad (7)$$

$$\lambda^{+-} \equiv \frac{q}{p} \frac{\bar{M}^{+-}}{M^{+-}} = e^{2i\alpha} \frac{B}{A} [1 + 0.07 \sin \delta - i0.07 \cos \delta]^{-1} \ . \quad (8)$$

From the expressions for the quantities $C_{\rho\pi}$ and $\Delta C_{\rho\pi}$ [5]:

$$C_{\rho\pi} \pm \Delta C_{\rho\pi} = \frac{1 - |\lambda^{\pm\mp}|^2}{1 + |\lambda^{\pm\mp}|^2} \quad (9)$$

we obtain:

$$\begin{aligned} \Delta C_{\rho\pi} &= \frac{a^2 - b^2}{a^2 + b^2} \ , \quad \frac{a^2}{b^2} = \frac{1 + \Delta C_{\rho\pi}}{1 - \Delta C_{\rho\pi}} \ , \\ C_{\rho\pi} &= 0.28 \sin \delta \frac{(a/b)^2}{(1 + a^2/b^2)^2} \ , \end{aligned} \quad (10)$$

where $a \equiv |A|$, $b \equiv |B|$.

The Belle and BABAR averaged result for $\Delta C_{\rho\pi}$ is [6]:

$$\Delta C_{\rho\pi} = 0.22 \pm 0.10 \ , \quad (11)$$

which leads to:

$$\left(\frac{a}{b}\right)^2 = 1.56 \pm 0.33 \ . \quad (12)$$

From the averaged experimental result [6]

$$C_{\rho\pi} = 0.31 \pm 0.10 \quad (13)$$

and Eq. (10) we get:

$$\sin \delta = 4.6 \pm 1.5 \ . \quad (14)$$

We see that poor accuracy in the measurement of $C_{\rho\pi}$ does not allow to get any definite information on the value of phase δ .

The next observable we wish to discuss is CP asymmetry $A_{CP}^{\rho\pi}$:

$$\begin{aligned} A_{CP}^{\rho\pi} &= \frac{|M^{+-}|^2 - |\bar{M}^{-+}|^2 + |\bar{M}^{+-}|^2 - |M^{-+}|^2}{|M^{+-}|^2 + |\bar{M}^{-+}|^2 + |\bar{M}^{+-}|^2 + |M^{-+}|^2} = \\ &= 0.14 \sin \delta \frac{(a/b)^2}{1 + (a/b)^2} , \end{aligned} \quad (15)$$

which should be compared with the experimental result [6]:

$$A_{CP}^{\rho\pi} = -0.102 \pm 0.045 . \quad (16)$$

From (15) and (16) we get:

$$\sin \delta = -1.2 \pm 0.5 , \quad (17)$$

and it differs from given in Eq.(14) by 3.5 standard deviations. This is the largest discrepancy we encounter in this paper. Averaging these two numbers we obtain:

$$\sin \delta = -0.62 \pm 0.47 . \quad (18)$$

Finally we come to the discussion of the observables which are sensitive to the angle α :

$$S_{\rho\pi} \pm \Delta S_{\rho\pi} = \frac{2\text{Im}\lambda^{\pm\mp}}{1 + |\lambda^{\pm\mp}|^2} , \quad (19)$$

$$S_{\rho\pi} = \frac{2a/b}{1 + a^2/b^2} [\sin 2\alpha \cos \tilde{\delta} - 0.07 \cos \delta \cos \tilde{\delta} - 0.07 \frac{a^2/b^2 - 1}{a^2/b^2 + 1} \sin \delta \sin \tilde{\delta}] , \quad (20)$$

$$\begin{aligned} \Delta S_{\rho\pi} &= \frac{2a/b}{1 + a^2/b^2} [-\cos 2\alpha \sin \tilde{\delta} + 0.07 \cos \delta \sin \tilde{\delta} \sin 2\alpha - \\ &- 0.07 \frac{a^2/b^2 - 1}{a^2/b^2 + 1} \sin \delta \cos \tilde{\delta} \sin 2\alpha] , \end{aligned} \quad (21)$$

where the definition of the phase $\tilde{\delta}$ is $A/B \equiv (a/b)e^{i\tilde{\delta}}$ and in the small terms proportional to 0.07 in the expression for $S_{\rho\pi}$ we have substituted $\cos 2\alpha = -1$.

Let us start the analysis of the experimental data from $\Delta S_{\rho\pi}$. According to [6]:

$$\Delta S_{\rho\pi} = 0.09 \pm 0.13 \quad , \quad (22)$$

which is much less than one. According to Eq. (12) the factor which multiplies square brackets in Eq. (20) (and in Eq. (21)) is very close to one, that is why it is the expression in square brackets which should be much less than one. The second and the third terms of this expression are really very small and we can neglect them. What concerns the first term, it is small when $\tilde{\delta}$ is close to zero or π :

$$\sin \tilde{\delta} = \Delta S_{\rho\pi} = 0.09 \pm 0.13 \quad , \quad (23)$$

where a small deviation of $\cos 2\alpha$ from -1 is neglected.

Now everything is ready and from Eq. (20) we get:

$$\sin 2\alpha = S_{\rho\pi} / \cos \tilde{\delta} + 0.07 \cos \delta \quad , \quad (24)$$

where we omit the last term in square brackets since it is negligibly small. In order to find angle α from the experimental data [6]:

$$S_{\rho\pi} = -0.13 \pm 0.13 \quad , \quad (25)$$

we should determine the values of $\cos \delta$ and $\cos \tilde{\delta}$.

To propagate errors from $\sin \delta$ to $\cos \delta$, we consider gaussian distribution for $\sin \delta$ truncated to physical region $|\sin \delta| < 1$, transform it into (non-gaussian) distribution for $\cos \delta$ and take an interval containing 68% of probability.

In this way from Eq. (18) we obtain:

$$|\cos \delta| = 0.88 \pm 0.12 \quad . \quad (26)$$

The average value of $|\cos \delta|$ appears to be close to one due to the so-called Jacobian pick.

Concerning $\tilde{\delta}$ it follows from Eq. (23) that $|\cos \tilde{\delta}| = 1$ with very good accuracy. Depending on the values of phases $\tilde{\delta}$ and δ we get the following four domains for the angle α :

$$\begin{aligned} \tilde{\delta} \approx 0 \quad , \quad \delta \approx 0 : \quad & \sin 2\alpha = -0.13 \pm 0.13 + 0.06 \\ & \alpha = 92^\circ \pm 4^\circ \end{aligned} \quad (27)$$

$$\begin{aligned} \tilde{\delta} \approx 0, \quad \delta \approx \pi : \quad & \sin 2\alpha = -0.13 \pm 0.13 - 0.06 \\ & \alpha = 96^\circ \pm 4^\circ \end{aligned} \quad (28)$$

$$\begin{aligned} \tilde{\delta} \approx \pi, \quad \delta \approx 0 : \quad & \sin 2\alpha = 0.13 \pm 0.13 + 0.06 \\ & \alpha = 84^\circ \pm 4^\circ \end{aligned} \quad (29)$$

$$\begin{aligned} \tilde{\delta} \approx \pi, \quad \delta \approx \pi : \quad & \sin 2\alpha = 0.13 \pm 0.13 - 0.06 \\ & \alpha = 88^\circ \pm 4^\circ . \end{aligned} \quad (30)$$

Thus QCD penguins split values of α obtained without taking them into account: $94^\circ \rightarrow 92^\circ, 96^\circ$; $86^\circ \rightarrow 84^\circ, 88^\circ$. If BSS mechanism is valid, then only the domains given by Eqs.(27) and (29) remain (see also [7]).

3 α from $\bar{B}_d(B_d) \rightarrow \rho^+ \rho^-$

The time dependence of CP violating asymmetry is described by the following formula:

$$a_{CP}(t) \equiv \frac{\frac{dN(\bar{B}_d \rightarrow \rho_L^+ \rho_L^-)}{dt} - \frac{dN(B_d \rightarrow \rho_L^+ \rho_L^-)}{dt}}{\frac{dN(\bar{B}_d \rightarrow \rho_L^+ \rho_L^-)}{dt} + \frac{dN(B_d \rightarrow \rho_L^+ \rho_L^-)}{dt}} = -C_{\rho\rho} \cos(\Delta mt) + S_{\rho\rho} \sin(\Delta mt) . \quad (31)$$

Let us remind that the longitudinal polarization fraction $f_L = 0.98 \pm 0.01 \pm 0.02$ [8]; $f_L = 0.95 \pm 0.03 \pm 0.03$ [9] and its closeness to one greatly simplify the extraction of CPV parameters from $B_d(\bar{B}_d) \rightarrow \rho^+ \rho^-$ decay data. These parameters are given by the following expressions:

$$C_{\rho\rho} = \frac{1 - |\lambda_{\rho\rho}|^2}{1 + |\lambda_{\rho\rho}|^2}, \quad S_{\rho\rho} = \frac{2\text{Im}\lambda_{\rho\rho}}{1 + |\lambda_{\rho\rho}|^2} . \quad (32)$$

From Eq. (A14) we obtain:

$$\lambda_{\rho\rho} \equiv \frac{q \bar{M}_{\rho\rho}}{p M_{\rho\rho}} = e^{2i\alpha} \frac{1 - 0.07e^{i(\delta-\alpha)}}{1 - 0.07e^{i(\delta+\alpha)}} = e^{2i\alpha} (1 + 0.14i \sin \alpha e^{i\delta}) , \quad (33)$$

where we use the same letter δ for FSI phase difference of the amplitudes generated by penguin and tree diagrams as in the case of $B \rightarrow \rho\pi$ decays. These differences would be really the same if BSS mechanism dominates.

Comparing the averaged experimental result for $C_{\rho\rho}$

$$C_{\rho\rho} = -0.03 \pm 0.18 \pm 0.09 \quad [8], \quad (34)$$

$$C_{\rho\rho} = 0.0 \pm 0.30 \pm 0.10 \quad [9]; \quad (35)$$

$$C_{\rho\rho}^{exp} = -0.02 \pm 0.17 \quad (36)$$

with the theoretical expression which follows from Eqs. (32), (33)

$$C_{\rho\rho} = 0.14 \sin \alpha \sin \delta \approx 0.14 \sin \delta \quad (37)$$

we get:

$$\sin \delta = -0.15 \pm 1.2 ; |\cos \delta| = 0.88 \pm 0.12 \quad , \quad (38)$$

where the same procedure of error propagation as in the case of $B \rightarrow \rho\pi$ was used and we sum statistical and systematic errors of $\sin \delta$ as independent.

From Eq. (32) we obtain:

$$S_{\rho\rho} = \sin 2\alpha + 0.14 \sin \alpha \cos 2\alpha \cos \delta = \sin 2\alpha - 0.14 \cos \delta \quad . \quad (39)$$

According to the recent measurements:

$$S_{\rho\rho} = -0.33 \pm 0.26 \quad [8], \quad (40)$$

$$S_{\rho\rho} = 0.09 \pm 0.43 \quad [9], \quad (41)$$

$$S_{\rho\rho}^{exp} = -0.21 \pm 0.22 \quad , \quad (42)$$

and we get two domains for α :

$$\delta \approx 0 : \alpha = 92^\circ \pm 7^\circ \quad (43)$$

$$\delta \approx \pi : \alpha = 100^\circ \pm 7^\circ \quad . \quad (44)$$

Just as in the case of $B \rightarrow \rho\pi$ decays only the first domain remains if $|\delta| < \pi/2$ [7].

Let us note that using the isospin analysis (which allows to prove the smallness of the penguin contribution) it was obtained:

$$\alpha = 100^\circ \pm 13^\circ \quad [8], \quad (45)$$

$$\alpha = 87^\circ \pm 17^\circ \quad [9], \quad (46)$$

where the error is mainly due to the uncertainty of the penguin contribution. Extracting this uncertainty and averaging last two numbers we get:

$$\alpha = 96^\circ \pm 7^\circ(exp) \pm 11^\circ(penguin) \quad . \quad (47)$$

4 α from $\bar{B}_d(B_d) \rightarrow \pi^+\pi^-$

The time dependence of CP violating asymmetry is given by the formula analogous to Eq. (31) with the evident substitution of π instead of ρ . Eq. (32) with the same substitution is valid as well, while for the quantity $\lambda_{\pi\pi}$ from Eq. (A13) we obtain:

$$\lambda_{\pi\pi} \equiv \frac{q}{p} \frac{\bar{M}_{\pi^+\pi^-}}{M_{\pi^+\pi^-}} = e^{2i\alpha}(1 + 0.28i \sin \alpha e^{i\delta}) , \quad (48)$$

and what concerns letter δ we should repeat the comment made after Eq. (33).

The experimental data of BABAR and Belle for CPV parameters $S_{\pi\pi}$ and $C_{\pi\pi}$ were controversial though at present (with the latest Belle results) the divergence diminishes. In view of this we will perform a two step analysis, taking at the beginning only BABAR results and then the averaged results of two collaborations.

Comparing the theoretical expression

$$C_{\pi\pi} = 0.28 \sin \alpha \sin \delta \approx 0.28 \sin \delta \quad (49)$$

with BABAR result [6]

$$C_{\pi\pi}^{\text{BABAR}} = -0.09 \pm 0.15 \quad (50)$$

we get:

$$\sin \delta = -0.32 \pm 0.54 , \quad (51)$$

while for $S_{\pi\pi}$ we have:

$$S_{\pi\pi} = \sin 2\alpha - 0.28 \cos \delta , \quad S_{\pi\pi}^{\text{BABAR}} = -0.30 \pm 0.17 . \quad (52)$$

From Eq. (51) we get:

$$|\cos \delta| = 0.9 \pm 0.1 \quad (53)$$

and two domains of α corresponding to two signs of $\cos \delta^1$:

$$\delta \approx 0 : \alpha = 91^\circ \pm 5^\circ(\text{exp}) \pm 1^\circ(\text{theor}) \quad (54)$$

¹Assuming $|\delta| < \pi/2$ [7] we would get only the first domain.

$$\delta \approx \pi : \alpha = 107^\circ \pm 5^\circ(\text{exp}) \pm 1^\circ(\text{theor}) . \quad (55)$$

Belle result:

$$C_{\pi\pi}^{\text{Belle}} = -0.56 \pm 0.13 \quad (56)$$

deviates by 2.5σ from BABAR and (if correct) would require considerably larger P/T ratio than we use in our paper.

Finally, averaging (50) and (56) one gets [6]:

$$C_{\pi\pi}^{\text{ex}} = -0.37 \pm 0.10 , \quad (57)$$

and comparing with the theoretical expression (49) we obtain:

$$\sin \delta = -1.32 \pm 0.35 , \quad (58)$$

which leads to:

$$|\cos \delta| = 0.55 \pm 0.25 . \quad (59)$$

From the averaged experimental result [6]:

$$S_{\pi\pi}^{\text{ex}} = -0.50 \pm 0.12(\text{exp}) \quad (60)$$

we get two domains²:

$$\delta \approx 0 : \alpha = 100^\circ \pm 4^\circ(\text{exp}) \pm 2^\circ(\text{theor}) \quad (61)$$

$$\delta \approx \pi : \alpha = 110^\circ \pm 4^\circ(\text{exp}) \pm 2^\circ(\text{theor}) . \quad (62)$$

5 Conclusions

We have analyzed CPV asymmetries in $B_d(\bar{B}_d) \rightarrow \rho^\pm \pi^\mp, \rho_L^\pm \rho_L^-$ and $\pi^+ \pi^-$ decays induced by the charmless strangeless b -quark decay $b \rightarrow u\bar{u}d$. This decay can proceed through a tree or penguin diagram. As it was noted in [1] when the penguin diagram is neglected, one obtains the values of the unitarity triangle angle α from CPV asymmetries in these decays which are consistent with each other as well as with the value of α which follows from the global CKM fit. However, in order to determine the theoretical accuracy

²Assuming $|\delta| < \pi/2$ [7] which follows from BSS mechanism of δ generation, we would get the first domain only.

of α extracted from the decays under study one should take the penguin amplitude into account. This was done in the present paper, where the moduli of penguin over tree ratios were calculated with the help of the factorization:

$$\langle M_1 M_2 | j_1 j_2 | B \rangle = \langle M_1 | j_1 | B \rangle \langle M_2 | j_2 | 0 \rangle , \quad (63)$$

while FSI phase shifts between the tree and penguin amplitudes were extracted from experimental data.

In order to determine numerical value of α one should average the values which follow from the considered decays. Since the phase shifts δ can be different in $B \rightarrow \rho\rho, \pi\pi$ and $\rho\pi$ decays, we get too many possibilities. That is why let us limit ourselves to the theoretically motivated case $|\delta| < \pi/2$.

Averaging Eqs. (43) and (61) we obtain:

$$\delta \approx 0 : \alpha_{\rho\rho, \pi\pi} = 98^\circ \pm 4^\circ . \quad (64)$$

Averaging it with (27) and (29) we get two possibilities:

$$\begin{aligned} \alpha_{b \rightarrow u\bar{u}d} &= 95^\circ \pm 3^\circ , \text{ or} \\ \alpha_{b \rightarrow u\bar{u}d} &= 91^\circ \pm 3^\circ , \end{aligned} \quad (65)$$

and the last one corresponds to the smallest possible value of α . In the case $|\delta_i| > \pi/2$, $\tilde{\delta} = 0$ averaging Eqs.(28), (44), (62) we get the largest possible value: $\alpha_{b \rightarrow u\bar{u}d} = 102^\circ \pm 3^\circ$.

The global fit results for α are:

$$\alpha_{\text{Ufit}}^{[10]} = 94^\circ \pm 8^\circ , \quad \alpha_{\text{CKMfitter}}^{[11]} = 94 \pm 10^\circ . \quad (66)$$

Thus the accuracy of the present day knowledge of α can be close to that of β :

$$\beta = 23^\circ \pm 2^\circ . \quad (67)$$

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Appendix

The strong interaction renormalization of the tree Hamiltonian which describes the beauty hadrons weak decays is much smaller than for the case of the strange particle decays since the masses of beauty hadrons are much closer to M_W in the logarithmic scale. In the leading logarithmic approximation for operators O_1 and O_2 we have:

$$\begin{aligned}
\hat{H}_{1,2} &= \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \left\{ \left[\frac{\alpha_S(m_b)}{\alpha_S(M_W)} \right]^{4/b} \left[\bar{u}\gamma_\alpha(1 + \gamma_5)b\bar{d}\gamma_\alpha/1 + \gamma_5)u - \right. \right. \\
&\quad - \left. \bar{d}\gamma_\alpha(1 + \gamma_5)b\bar{u}\gamma_\alpha(1 + \gamma_5)u \right] + \left[\frac{\alpha_S(m_b)}{\alpha_S(M_W)} \right]^{-2/b} \left[\bar{u}\gamma_\alpha(1 + \gamma_5)b \times \right. \\
&\quad \left. \left. \times \bar{d}\gamma_\alpha(1 + \gamma_5)u + \bar{d}\gamma_\alpha(1 + \gamma_5)b\bar{u}\gamma_\alpha(1 + \gamma_5)u \right] \right\} \quad (A1)
\end{aligned}$$

and substituting $\alpha_S(M_W) = 0.12$, $\alpha_S(m_b) = 0.2$, $b = 23/3$ we get:

$$\begin{aligned}
\hat{H}_{1,2} &= \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \{ 1.1 \bar{u}\gamma_\alpha(1 + \gamma_5)b\bar{d}\gamma_\alpha(1 + \gamma_5)u - \\
&\quad - 0.2 \bar{d}\gamma_\alpha(1 + \gamma_5)b\bar{u}\gamma_\alpha(1 + \gamma_5)u \} . \quad (A2)
\end{aligned}$$

NLO calculations confirm and refine this result. From Table 1 of [12] for the value $\Lambda_4 = 280$ MeV (which corresponds to $\alpha_S(M_Z) = 0.118$) we get 1.14 instead of our 1.1 and -0.31 instead of our -0.2.

At one loop the following QCD penguin operator is generated:

$$\begin{aligned}
\hat{H}_{3-6} &= -\frac{G_F}{\sqrt{2}} (V_{cb}V_{cd}^* + V_{ub}V_{ud}^*) \frac{\alpha_S(m_b)}{12\pi} \ln \left(\frac{M_W}{m_b} \right)^2 (\bar{d}\gamma_\mu(1 + \gamma_5)\vec{\lambda}b) \times \\
&\quad \times (\bar{u}\gamma_\mu\vec{\lambda}u + \bar{d}\gamma_\mu\vec{\lambda}d) = +\frac{G_F}{\sqrt{2}} V_{tb}V_{td}^* 0.03 \left\{ -\frac{2}{3} \bar{d}\gamma_\alpha(1 + \gamma_5)b \times \right. \\
&\quad \left. \times (\bar{u}\gamma_\alpha u + \bar{d}\gamma_\alpha d) + 2(\bar{d}_a\gamma_\alpha(1 + \gamma_5)b^c)(\bar{u}_c\gamma_\alpha u^a + \bar{d}_c\gamma_\alpha d^a) \right\} , \quad (A3)
\end{aligned}$$

where $\vec{\lambda}$ are eight colour SU(3) Gell-Mann matrices, and Fierz identity $\vec{\lambda}_{ab}\vec{\lambda}_{cd} = -2/3\delta_{ab}\delta_{cd} + 2\delta_{ad}\delta_{bc}$ as well as unitarity relation $V_{cb}V_{cd}^* + V_{ub}V_{ud}^* = -V_{tb}V_{td}^*$ were used.

Substituting $\bar{q}\gamma_\alpha q = \frac{1}{2}\bar{q}\gamma_\alpha(1 + \gamma_5)q + \frac{1}{2}\bar{q}\gamma_\alpha(1 - \gamma_5)q$ we find the renormalization factors +0.01 and -0.03 for operators O_3 , O_5 and O_4 , O_6 respectively.

At NLO the renormalization factors for the operators in which only the left-handed quarks are involved (O_3, O_4) are different from those for the operators in which both left- and right-handed quarks participate (O_5, O_6). From the same Table 1 of [12] we get 0.016 and 0.010 instead of 0.01 and -0.036 and -0.045 instead of -0.03.

Finally, the effective Hamiltonian which describes the charmless strangeless \bar{B}_d decays looks like:

$$\hat{H} = \frac{G_F}{\sqrt{2}} [V_{ub}V_{ud}^*(c_1O_1 + c_2O_2) - V_{tb}V_{td}^*(c_3O_3 + c_4O_4 + c_5O_5 + c_6O_6)] \quad , \quad (\text{A4})$$

$$\begin{aligned} O_1 &= \bar{u}\gamma_\alpha(1 + \gamma_5)b\bar{d}\gamma_\alpha(1 + \gamma_5)u & c_1 &= 1.14 \quad , \\ O_2 &= \bar{d}\gamma_\alpha(1 + \gamma_5)b\bar{u}\gamma_\alpha(1 + \gamma_5)u & c_2 &= -0.31 \quad , \\ O_3 &= \bar{d}\gamma_\alpha(1 + \gamma_5)b[\bar{u}\gamma_\alpha(1 + \gamma_5)u + \bar{d}\gamma_\alpha(1 + \gamma_5)d] & c_3 &= 0.016 \quad , \\ O_4 &= \bar{d}_a\gamma_\alpha(1 + \gamma_5)b^c[\bar{u}_c\gamma_\alpha(1 + \gamma_5)u^a + \bar{d}_c\gamma_\alpha(1 + \gamma_5)d^a] & c_4 &= -0.036 \quad , \\ O_5 &= \bar{d}\gamma_\alpha(1 + \gamma_5)b[\bar{u}\gamma_\alpha(1 - \gamma_5)u + \bar{d}\gamma_\alpha(1 - \gamma_5)d] & c_5 &= 0.010 \quad , \\ O_6 &= \bar{d}_a\gamma_\alpha(1 + \gamma_5)b^c[\bar{u}_c\gamma_\alpha(1 - \gamma_5)u^a + \bar{d}_c\gamma_\alpha(1 - \gamma_5)d^a] & c_6 &= -0.045 \quad , \end{aligned} \quad (\text{A5})$$

and the complex conjugate Hamiltonian describes B_d decays.

Our next task is to calculate the matrix elements of \hat{H} between \bar{B}_d and $\rho^\pm\pi^\mp$, $\rho^+\rho^-$ and $\pi^+\pi^-$ states, which is the most difficult part of the job. We will present the matrix elements of 4-fermion operators as the product of matrix elements of two 2-fermion operators between \bar{B}_d and a light meson and vacuum and another light meson. The validity of this factorization is questionable; in particular, in this approach the FSI phases due to the light meson rescattering vanish identically. However, we found the statement in the literature that the corrections to the factorization formulas are small, being proportional to Λ/m_b or powers of $\alpha_S(m_b)$ [13]³. In any case nowadays factorization is the only way to get expressions for the decay amplitudes from the fundamental Hamiltonian.

Since we are interested in \bar{B}_d decays to charged mesons and we will factorize 4-fermion operators, let us present Eqs. (A4), (A5) in the following form [2]:

$$\hat{H} = \frac{G_F}{\sqrt{2}} V_{ub}V_{ud}^* \{a_1 \bar{u}\gamma_\alpha(1 + \gamma_5)b\bar{d}\gamma_\alpha(1 + \gamma_5)u -$$

³Since $q^2 \equiv (P_{B_d} - P_{\pi,\rho})^2 = O(m_\pi^2, m_\rho^2)$ one can argue that the large distance contributions invalidate the factorization formula.

$$\begin{aligned}
& - \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} [a_4 \bar{u} \gamma_\alpha (1 + \gamma_5) b \bar{d} \gamma_\alpha (1 + \gamma_5) u - \\
& - 2a_6 \bar{u} (1 + \gamma_5) b \bar{d} (1 - \gamma_5) u] \} , \tag{A6}
\end{aligned}$$

where $a_1 = c_1 + \frac{1}{3}c_2 = 1.04$, $a_4 = c_4 + \frac{1}{3}c_3 = -0.031$, $a_6 = c_6 + \frac{1}{3}c_5 = -0.042$ and Fierz identities $\bar{\psi} \gamma_\alpha (1 + \gamma_5) \varphi \bar{\chi} \gamma_\alpha (1 + \gamma_5) \eta = \bar{\psi} \gamma_\alpha (1 + \gamma_5) \eta \bar{\chi} \gamma_\alpha (1 + \gamma_5) \varphi$, $\bar{\psi} \gamma_\alpha (1 + \gamma_5) \varphi \bar{\chi} \gamma_\alpha (1 - \gamma_5) \eta = -2\bar{\psi} (1 - \gamma_5) \eta \bar{\chi} (1 + \gamma_5) \varphi$ were used.

The matrix elements we are interested in were calculated in paper [2] assuming factorization. Up to a common factor which includes constant f_π and $B \rightarrow \pi$ transition formfactor $f_0(m_\pi^2)$ for the amplitude of $\bar{B}_d \rightarrow \pi^+ \pi^-$ decay it was obtained:

$$\frac{M(\bar{B}_d \rightarrow \pi^+ \pi^-)}{V_{ub}V_{ud}^*} \sim a_1 - \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \left[a_4 + \frac{2m_\pi^2}{(m_u + m_d)(m_b - m_u)} a_6 \right] e^{i\delta} , \tag{A7}$$

where we use the result of [2] and take into account the difference of the rescattering phases of the tree ($\sim a_1$) and the penguin ($\sim a_4$ and $\sim a_6$) amplitudes δ . One evident source of this phase is the imaginary part of the penguin diagrams with intermediate u - and c -quarks. Let us demonstrate that only the last one should be taken into account in (A7):

$$\begin{aligned}
M(\bar{B}_d \rightarrow \pi^+ \pi^-) & \sim V_{ub}V_{ud}^*(T + P(m_u)) + V_{cb}V_{cd}^*P(m_c) + V_{tb}V_{td}^*P(m_t) = \\
& = V_{ub}V_{ud}^*[T + P(m_u) - P(m_c)] - V_{tb}V_{td}^*[P(m_c) - P(m_t)] . \tag{A8}
\end{aligned}$$

As we are interested in CP asymmetries we should calculate $\lambda_{\pi^+ \pi^-} = e^{-2i\beta} M(\bar{B}_d \rightarrow \pi^+ \pi^-) / M(B_d \rightarrow \pi^+ \pi^-)$:

$$\begin{aligned}
\lambda & = e^{-2i\beta - 2i\gamma} \frac{1 + \frac{P(m_u) - P(m_c)}{T} - \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \frac{P(m_c) - P(m_t)}{T}}{1 + \frac{P(m_u) - P(m_c)}{T} - \frac{V_{tb}^*V_{td}}{V_{ub}^*V_{ud}} \frac{P(m_c) - P(m_t)}{T}} = \\
& = e^{2i\alpha} \left[1 + (e^{i(\alpha - \pi)} - e^{i(\pi - \alpha)}) \frac{\sin \gamma}{\sin \beta} \frac{P(m_c) - P(m_t)}{T} \right] , \tag{A9}
\end{aligned}$$

where α , β and γ are the angles of the unitarity triangle.

Thus the absorptive part of $P(m_c)$ contributes to δ .

Since the penguin operator $P(m_c)$ equals the correlator of two vector currents, one immediately picks up its imaginary part from the textbooks on QED. It depends on the gluon momentum transfer and when the square of this momentum transfer is much larger than $4m_c^2$, we have:

$$\frac{P(m_c)}{T} \sim -\ln \frac{M_W^2}{m_b^2} - i\pi \equiv -\left| \frac{P}{T} \right| e^{i\delta} , \quad \delta \approx 30^\circ . \tag{A10}$$

Since u - and \bar{u} -quarks to which gluon decays go to different light mesons, the value of the momentum transfer squared is determined by these mesons wave functions. It varies between m_b^2 and zero. Thus we see that the mechanism suggested in [3] leads to the small positive value of δ :

$$\delta \lesssim 30^\circ . \quad (\text{A11})$$

If BSS mechanism determines the value of δ , it would confirm the validity of our approach. If, on the contrary, the large distance rescattering of light hadrons changes δ substantially, one should await large corrections to the dispersive part (the ratio (P/T)) as well. In the present paper we will allow δ to vary between zero and 2π , but we use the expressions analogous to (A7) for the decay amplitudes.

Let us return to Eq. (A7). With the help of the following equation:

$$\frac{V_{td}^* V_{tb}}{V_{ud}^* V_{ub}} = e^{i(\pi-\alpha)} \frac{\sin \gamma}{\sin \beta} \quad (\text{A12})$$

and using the numerical values $m_u + m_d = 11$ MeV, $m_b = 4.5$ GeV we obtain:

$$\begin{aligned} M(\bar{B}_d \rightarrow \pi^+ \pi^-) &\sim V_{ub} V_{ud}^* \left[1 - e^{i(\pi-\alpha)} \frac{\sin \gamma}{\sin \beta} (-0.06) e^{i\delta} \right] = \\ &= V_{ub} V_{ud}^* [1 + 0.14 e^{i(\pi-\alpha+\delta)}] , \end{aligned} \quad (\text{A13})$$

where $\beta = 23^\circ$ and $\gamma = 63^\circ$ were substituted (we are using the value of γ from the global CKM fit in order to estimate a small correction to the amplitude) and we put a_1 equal to one. In Section 4 we analyze the experimental data on CP asymmetries in $\bar{B}_d(B_d) \rightarrow \pi^+ \pi^-$ decays.

Coming to $\bar{B}_d(B_d) \rightarrow \rho^+ \rho^-$ decays we should calculate the corresponding matrix element of the Hamiltonian presented in (A6). Factorizing 4-quark operators we observe that the term proportional to a_6 vanishes, since $\langle \rho | \bar{d}(1 - \gamma_5)u | 0 \rangle = 0$: the (pseudo) scalar current cannot produce a vector meson from vacuum. That is why instead of Eq. (A7) we get (see also [2]):

$$\begin{aligned} \frac{M(\bar{B}_d \rightarrow \rho_L^+ \rho_L^-)}{V_{ub} V_{ud}^*} &\sim a_1 - \frac{V_{tb} V_{td}^*}{V_{ub} V_{ud}^*} a_4 e^{i\delta} , \\ M(\bar{B}_d \rightarrow \rho_L^+ \rho_L^-) &\sim V_{ub} V_{ud}^* [1 + 0.07 e^{i(\pi-\alpha+\delta)}] . \end{aligned} \quad (\text{A14})$$

The production of transversely polarized ρ -mesons by the vector current is suppressed as $(m_\rho/m_B)^2$, and the experimental data confirm the dominance of ρ_L .

If BSS mechanism is responsible for the phase δ , then it should be the same as in Eq. (A13). Eq. (A14) is used in Section 3 to extract the value of angle α .

Our last problem is the calculation of the amplitudes $M(\bar{B}_d \rightarrow \rho^\mp \pi^\pm) \equiv \bar{M}^{\mp\pm}$. The amplitude \bar{M}^{-+} corresponds to ρ^- production from vacuum by $(\bar{d}u)$ current, so the term proportional to a_6 does not contribute to it:

$$\begin{aligned} \frac{\bar{M}^{-+}}{V_{ub}V_{ud}^*} &= A \left[a_1 - \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} a_4 e^{i\delta_-} \right] ; \\ \bar{M}^{-+} &= AV_{ub}V_{ud}^* \left[1 + 0.07 e^{i(\pi-\alpha+\delta_-)} \right] , \end{aligned} \quad (\text{A15})$$

where A is the complex number.

In the case of the amplitude \bar{M}^{+-} it is π^- which is produced from vacuum by $(\bar{d}u)$ current, so the term proportional to a_6 contributes as well:

$$\frac{\bar{M}^{+-}}{V_{ub}V_{ud}^*} = B \left\{ a_1 - \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \left[a_4 - \frac{2m_\pi^2}{(m_b + m_u)(m_u + m_d)} a_6 \right] e^{i\delta_+} \right\} , \quad (\text{A16})$$

see [2]. Here B is the complex number. Unlike the case of $\bar{B}_d \rightarrow \pi^+ \pi^-$ decay the terms proportional to a_4 and a_6 have opposite signs and as a result the expression in square brackets with good accuracy equals zero, leading to:

$$\bar{M}^{+-} = BV_{ub}V_{ud}^* , \quad (\text{A17})$$

and the penguin pollution is absent (a_1 is omitted since it is very close to one).

The amplitudes of B_d meson decays, M^{-+} and M^{+-} , equal to \bar{M}^{+-} and \bar{M}^{-+} correspondingly with the complex conjugate CKM matrix elements. We will use formulas (A15) and (A17) in order to determine angle α in Section 2 and will omit index “-” from δ_- , since δ_+ did not enter Eq. (A17).

Thus the penguin pollution is minimal in $\rho\pi$ mode, intermediate in $\rho\rho$ mode and maximal in $\pi\pi$ mode (as it was noted in [2]).

References

- [1] G. G. Ovanesyan and M. I. Vysotsky, *Pis'ma v ZhETF*, **81**, 449 (2005).
- [2] R. Aleksan, F. Buccella, A. Le. Yaouanc, L. Oliver, O. Pene and J. - C. Raynal, *Phys. Lett. B* **356**, 95 (1995).
- [3] M. Bander, D. Silverman and A. Soni, *Phys. Rev. Lett.* **43**, 242 (1979).
- [4] G. M. Gérard, W. - S. Hou, *Phys. Rev. D* **43**, 2909 (1991).
- [5] M. Gronau, *Phys. Lett. B* **233**, 479 (1989).
- [6] Heavy Flavor Averaging Group (HFAG), hep-ex/0505100 (2005).
- [7] M. Gronau, E. Lunghi and D. Wyler, *Phys. Lett. B* **606**, 95 (2005).
- [8] BABAR Collaboration, B. Aubert et al., *Phys. Rev. Lett.* **93**, 231801 (2005).
- [9] Belle Collaboration, K. Abe et al., hep-ex/0507039 (2005).
- [10] M. Bona et al. (UT fit Collaboration), hep-ph/0501199 (2005).
- [11] J. Charles et al. (The CKMfitter Group), *Eur. Phys. J. C* **41**, 1 (2005).
- [12] A. J. Buras et al., *Nucl. Phys. B* **370**, 69 (1992).
- [13] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, *Phys. Rev. Lett.* **83**, 1914 (1999).