

Test of FSR in the process $e^+e^- \rightarrow \pi^+\pi^-\gamma$ at DAΦNE *

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Abstract

In this paper we consider the possibility to test the FSR model in the reaction $e^+e^- \rightarrow \pi^+\pi^-\gamma$ at DAΦNE. We propose to consider the low Q^2 region (Q^2 is the invariant mass squared of the di-pion system) to study the different models describing $\gamma^* \rightarrow \pi^+\pi^-\gamma$ interaction. As illustration we compare the scalar QED and Resonance Perturbation Theory prediction for the $e^+e^- \rightarrow \pi^+\pi^-\gamma$ cross section. We also consider the contribution coming from the ϕ direct decay ($\phi \rightarrow \pi^+\pi^-\gamma$). We find the low Q^2 region is sensitive to FSR models.

1. Final state radiation (FSR) is the main irreducible background in radiative return measurements of the hadronic cross section [1] which is important for the anomalous magnetic moment of the muon [2]. Besides being of interest as an important background source, this process could be of interest in itself, because a detailed experimental study of FSR allow us to get information about pion-photon interaction at low energies.

Usually the FSR tensor is evaluated in the scalar QED (sQED) model, or more exactly in the combined sQED*VMD model, i.e. the pions are treated as point-like particles and then the total FSR amplitude is multiplied by the pion form factor calculated in the VMD model [3]. Additional contributions to the FSR amplitude are possible. In [4] the FSR tensor was estimated in the framework of the Chiral Perturbation Theory with the explicit inclusion of the vector and axial-vector mesons, $\rho_0(770)$ and $a_1(1260)$, called Resonance Perturbation Theory (RPT) [5]. We apply the results obtained in [4] for the case of the ϕ -factory DAΦNE.

This paper is organized as follows. In Section 2 we briefly repeat the main results of [4]. In Section 3 we introduce the corresponding results into a Monte Carlo (MC) program, based on the EVA generator of the process $e^+e^- \rightarrow \pi^+\pi^-\gamma$ [6]. In Section 4 we present our conclusions.

2. Based on charge-conjugation symmetry, photon crossing symmetry and gauge invariance the general amplitude for $\gamma^*(Q) \rightarrow \pi^+(p_+)\pi^-(p_-)\gamma(k)$, when the final photon is real, can be

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expressed by three gauge invariant tensors (see Appendix A in [4] and Refs. [23-25] therein)

$$\begin{aligned}
M^{\mu\nu}(Q, k, l) &= -ie^2(\tau_1^{\mu\nu} f_1 + \tau_2^{\mu\nu} f_2 + \tau_3^{\mu\nu} f_3) \equiv -ie^2 M_F^{\mu\nu}(Q, k, l), \quad l = p_+ - p_-, \quad (1) \\
\tau_1^{\mu\nu} &= k^\mu Q^\nu - g^{\mu\nu} k \cdot Q, \\
\tau_2^{\mu\nu} &= k \cdot l (l^\mu Q^\nu - g^{\mu\nu} k \cdot l) + l^\nu (k^\mu k \cdot l - l^\mu k \cdot Q), \\
\tau_3^{\mu\nu} &= Q^2 (g^{\mu\nu} k \cdot l - k^\mu l^\nu) + Q^\mu (l^\nu k \cdot Q - Q^\nu k \cdot l).
\end{aligned}$$

We would like to point out that this decomposition is model independent, while the exact value of the scalar functions f_i are determined by the specific FSR model.

In sQED for the functions f_i we have [3]

$$f_1^{sQED} = \frac{2k \cdot Q}{(k \cdot Q)^2 - (k \cdot l)^2}, \quad f_2^{sQED} = \frac{-2}{(k \cdot Q)^2 - (k \cdot l)^2}, \quad f_3^{sQED} = 0, \quad (2)$$

Because of Low's theorem, these equations imply that for $k \rightarrow 0$ we have

$$\lim_{k \rightarrow 0} f_1 = \frac{2k \cdot Q F_\pi(Q^2)}{(k \cdot Q)^2 - (k \cdot l)^2}, \quad \lim_{k \rightarrow 0} f_2 = \frac{-2F_\pi(Q^2)}{(k \cdot Q)^2 - (k \cdot l)^2}, \quad \lim_{k \rightarrow 0} f_3 = 0, \quad (3)$$

where F_π is a VMD pion form factor describing $\gamma^* \rightarrow \pi^+ \pi^-$. Thus for soft photon radiation the FSR tensor is expressed in the term of one form factor F_π , but in general we have three independent form factors describing the FSR process.

It is convenient to rewrite f_i as

$$f_i = f_i^{(0)} + \Delta f_i, \quad (4)$$

where $f_i^{(0)} \equiv \lim_{k \rightarrow 0} f_i$. In [4] the functions Δf_i have been calculated in the framework of RPT, the result is

$$\begin{aligned}
\Delta f_1 &= \frac{F_V^2 - 2F_V G_V}{f_\pi^2} \left(\frac{1}{m_\rho^2} + \frac{1}{m_\rho^2 - Q^2} \right) \\
&\quad - \frac{F_A^2}{f_\pi^2 m_a^2} \left[2 + \frac{(k \cdot l)^2}{D(l)D(-l)} + \frac{(Q^2 + k \cdot Q)[4m_a^2 - (Q^2 + l^2 + 2k \cdot Q)]}{8D(l)D(-l)} \right], \quad (5)
\end{aligned}$$

$$\Delta f_2 = -\frac{F_A^2}{f_\pi^2 m_a^2} \frac{4m_a^2 - (Q^2 + l^2 + 2k \cdot Q)}{8D(l)D(-l)}, \quad (6)$$

$$\Delta f_3 = \frac{F_A^2}{f_\pi^2 m_a^2} \frac{k \cdot l}{2D(l)D(-l)}, \quad (7)$$

for all notations and the details of calculation see [4]. Taking the central value of the corresponding decay widths

$$\begin{aligned}
\Gamma(\rho^0 \rightarrow e^+ e^-) &= 6.85 \pm 0.11 \text{keV}, \quad (8) \\
\Gamma(\rho^0 \rightarrow \pi\pi) &= 150.7 \pm 2.9 \text{MeV}, \quad \Gamma(a_1 \rightarrow \pi\gamma) = 640 \pm 240 \text{keV}
\end{aligned}$$

and using the relations

$$\begin{aligned}
\Gamma(\rho^0 \rightarrow \pi\pi) &= \frac{G_V^2 m_\rho^3}{48\pi f_\pi^4} \left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{3/2}, \quad \Gamma(\rho^0 \rightarrow e^+ e^-) = \frac{4\pi\alpha^2 F_V^2}{3m_\rho}, \quad (9) \\
\Gamma(a_1 \rightarrow \pi\gamma) &= \frac{\alpha F_A^2 m_a}{24f_\pi^2} \left(1 - \frac{m_\pi^2}{m_a^2}\right)^3,
\end{aligned}$$

we have the following values for the parameters of the model

$$F_V = 0.156\text{GeV}, \quad G_V = 0.066\text{GeV}, \quad F_A = 0.122\text{GeV}, \quad (10)$$

and $f_\pi = 92.4\text{MeV}$. In [4] it was shown that the $\gamma^* \rightarrow \rho^\pm \pi^\mp \rightarrow \pi^+ \pi^- \gamma$ is negligible and we will discard it henceforward.

3. Our MC code for $e^+e^- \rightarrow \pi^+\pi^-\gamma$ is based on the MC EVA structure [6].

The matrix element we use for the cross section in the MC simulation is

$$\begin{aligned} d\sigma &\sim |M_{ISR} + M_{FSR} + M_\phi|^2 \\ &\simeq |M_{ISR} + M_{FSR}|^2 + |M_\phi|^2 + 2\text{Re}(M_{FSR}^{(sQED)} \cdot M_\phi^*), \end{aligned} \quad (11)$$

where we apply the EVA result for the initial state radiation (ISR) matrix element (M_{ISR}), while the FSR matrix element (M_{FSR}) is taken from the RPT prediction ((1) and (4, 5)) and M_ϕ is the amplitude for the ϕ direct decay. (We should mention here, that for initial state we will consider only the case of *one* photon radiation, that corresponds to the LO approximation for the EVA generator.) To estimate the $\phi \rightarrow \pi^+\pi^-\gamma$ decay we apply the Achasov four quark parametrization [7] with the parameters taken from the fit of the KLOE data for $\phi \rightarrow \pi^0\pi^0\gamma$ with only the f_0 intermediate state [8] (for different parametrizations of the ϕ direct decay and its contribution to the asymmetry and the cross section see [9], [10]).

To begin with, we estimate the relative magnitude of the different contributions to cross section (11) considering the inclusive kinematics:

$$\begin{aligned} 0^\circ &\leq \theta_\gamma \leq 180^\circ, \\ 0^\circ &\leq \theta_\pi \leq 180^\circ. \end{aligned} \quad (12)$$

In the left part of Fig.1 the value of the different contributions to Eq. (11) are shown for $s = m_\phi^2$. One can see that the ϕ resonant contribution (i.e. proportional to $|M_\phi|^2$) is quite large and the additional RPT contribution to FSR (i.e. the contribution to FSR not included in the sQED*VMD model) can be revealed only in the case of the destructive interference ($\text{Re}(M_{FSR}^{sQED} \cdot M_\phi^*) < 0$). In this case the interference term and the ϕ resonant contribution almost cancel each other at the low Q^2 region. The preliminary data from the KLOE experiment are in favour of this assumption [11].

Within this assumption we consider the cuts used in the KLOE large angle analysis [12]:

$$\begin{aligned} 50^\circ &\leq \theta_\gamma \leq 130^\circ, \\ 50^\circ &\leq \theta_\pi \leq 130^\circ. \end{aligned} \quad (13)$$

In Figs. 2 and 3 we show our numerical results for the cross section (11) for hard photon radiation with energies $\omega > 20$ MeV. In Fig.3 the term $d\sigma_{sQED}$ corresponds to the right side of Eq. (11) without the ϕ -meson decay and FSR in sQED*VMD, the term $\sigma_{sQED+\phi}$ includes the ϕ contribution and $d\sigma_{TOT}$ is for the cross section (11) with the ϕ term and FSR calculated by RPT theory. As we can see, in the low Q^2 region, the additional FSR term is up to 30% of the total contribution coming from sQED and $\phi \rightarrow \pi^+\pi^-\gamma$ decay. In the left part of Fig.3 the peak about 1GeV^2 corresponds to the f_0 intermediate state for the $\phi \rightarrow \pi\pi\gamma$ amplitude.

It has been proposed to take data outside the ϕ peak ($s < m_\phi^2$), in order to reduce the background from $\phi \rightarrow 3\pi$ decay [13]. In this case the ϕ resonant contribution (the term $\sim |M_\phi|^2$ in Eq.(11)) is suppressed (see Fig.1, right) and does not cancel anymore with the

interference term at low Q^2 . The interference term however still survives at $s = 1\text{GeV}^2$, so that even for $s \lesssim m_\phi^2$ we cannot neglect the ϕ direct decay to the cross section (11) (as it is shown in Fig. 4). At the same time the value of the interference can be comparable with the FSR contribution not covered by the sQED*VMD model. Therefore for a precise evaluation of the total contributions at low Q^2 the interference term and the contributions beyond sQED should be included.

4. According to our numerical results the low Q^2 region is sensitive to the inclusion of additional FSR contribution of RPT both for on-peak and off-peak energies.

Our results are limited by the main following reasons:

- we approximate the interference term by $M_{FSR}^{(sQED)} \cdot M_\phi^*$
- in the the pion form-factor in RPT we use only the ρ -meson contribution, whereas the actual VMD results include also ω and ρ'
- we do not include multiple photon emission (from initial and/or final state)
- we parametrize the ϕ direct decay amplitude only through f_0 intermediate state .

While the first three items can be improved in a refined version of the code, the precise description of the amplitude $\phi \rightarrow \pi\pi\gamma$ is a difficult task, especially at low Q^2 . (We would like to stress again that this energy region is of our interest because an essential FSR contribution beyond sQED*VMD can exist only for the low Q^2 region, when the radiated photon is energetic.) The new data on $\phi \rightarrow \pi^0\pi^0\gamma$ from KLOE will certainly help to refine the ϕ direct decay amplitude; some information can be extracted by a fit of the spectrum of $e^+e^- \rightarrow \pi^+\pi^-\gamma$ itself, using, for example, the charge asymmetry, as discussed in [9].

Clearly, a model independent analysis of FSR contribution will be very useful. We hope to disentangle FSR and the ϕ direct decay contributions in a model-independent way by using the information from cross section, asymmetry measurement for different beam energies.

Work is in progress.

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References

- [1] J. Kühn, presentation at this conference;
A. Aloisio et al., The KLOE Collaboration, Phys.Lett. B 606, 12 (2005) (hep-ex/0407048);
E.P. Solodov, The BaBar Collaboration (hep-ex/0107027).
- [2] S. Eidelman, F. Jegerlehner, Z. Phys. C 67, 585 (1995);
M. Davier, S. Eidelman, A. Hocker, Z. Zhang, Eur. Phys. J. C 27, 497 (2003) (hep-ph/0208177);
F. Jegerlehner, presentation at this conference.
- [3] J.S. Schwinger, Particles, Sources and Fields. Vol. 3, Redwood City, USA: Addison-Wesley (1989) p.99;
H. Czyż, A. Grzelinska, J. Kühn, G. Rodrigo, Eur.Phys.J. C 39, 411 (2005).

- [4] S. Dubinsky, A.Yu. Korchin, N.P. Merenkov et al., Eur. Phys. J C40 (2005) 41.
- [5] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B321 (1989) 311;
G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Phys. Lett. B223 (1989) 425.
- [6] S. Binner, J.H. Kühn, K. Melnikov, Phys. Lett. B459 (1999) 279.
- [7] N.N. Achasov and V.V. Gubin, Phys. Rev. D63 (2001) 094007 (hep-ph/0101024).
- [8] A. Aloisio et al., The KLOE Collaboration, Phys.Lett. B 537 (2002) 21 (hep-ex/0204013).
- [9] H. Czyż, A. Grzelinska, J.H. Kühn, Phys. Lett B611 (2005) 116 (hep-ph/0412239);
H. Czyż, presentation at this conference.
- [10] K. Melnikov, F. Nguyen, B. Valeriani, G. Venanzoni, Phys.Lett B477 (2000) 114.
- [11] P. Gauzzi, presentation at this conference.
- [12] S. Müller, presentation at this conference
- [13] F. Ambrosino *et al.* KLOE Memo 307;
S. Müller, *PhD thesis* (www-ekp.physik.uni-karlsruhe.de/pub/web/thesis/smueller_phd.pdf)

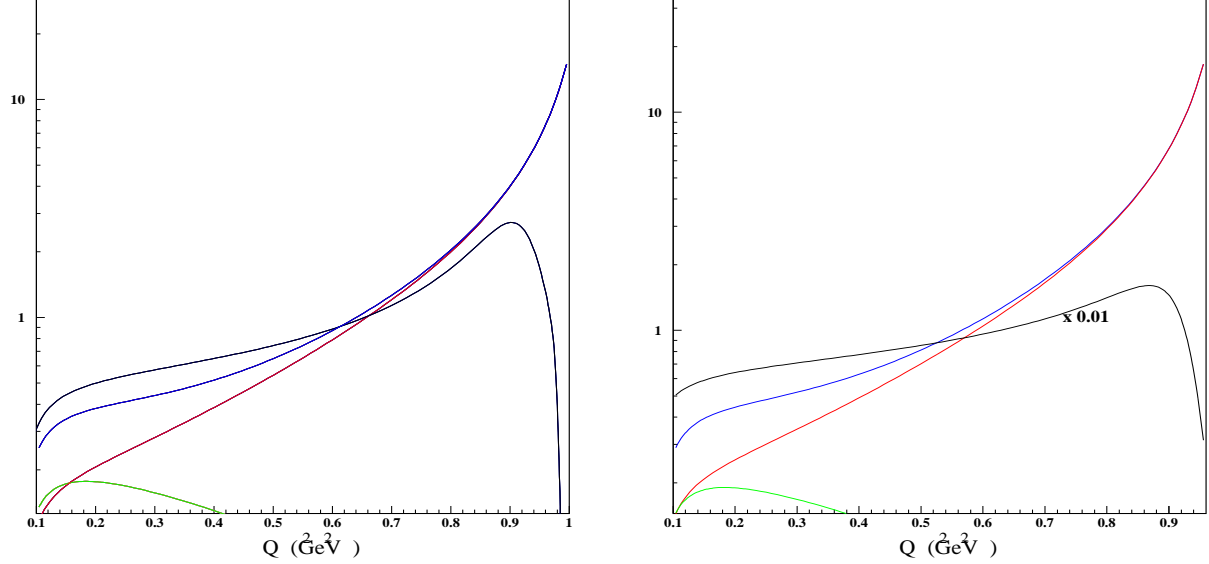


Figure 1: Contributions of FSR and ϕ direct decay to the cross section $e^+e^- \rightarrow \pi^+\pi^-\gamma$, see Eq. (11). The black line corresponds to the ϕ resonant contribution. The blue line is the FSR contribution in the framework of RPT, the red line is FSR in sQED, the green line is the additional RPT FSR contribution, beyond sQED. The left figure corresponds to $s = m_\phi^2$, the right one is for $s = 1 \text{ GeV}^2$ (i.e. below the ϕ resonance), where the ϕ resonant contribution is amplified by a factor 100.

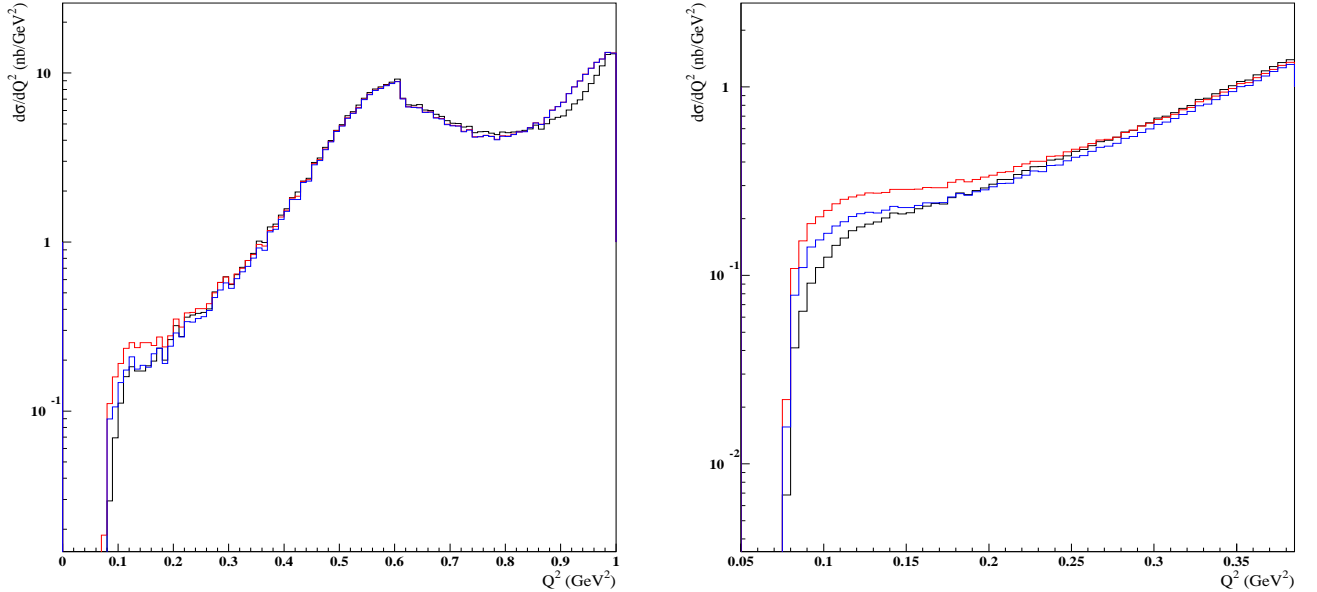


Figure 2: The cross section $e^+e^- \rightarrow \pi^+\pi^-\gamma$ with cuts (13). The black line corresponds to the result for Eq. (11) that includes FSR in the framework of the sQED*VMD model and does not include the ϕ direct decay. The blue line is the same including the ϕ decay. The red line corresponds to the cross section (11) where FSR is calculated in RPT. The left figure is for the entire energy region, the right one is for the low Q^2 region.

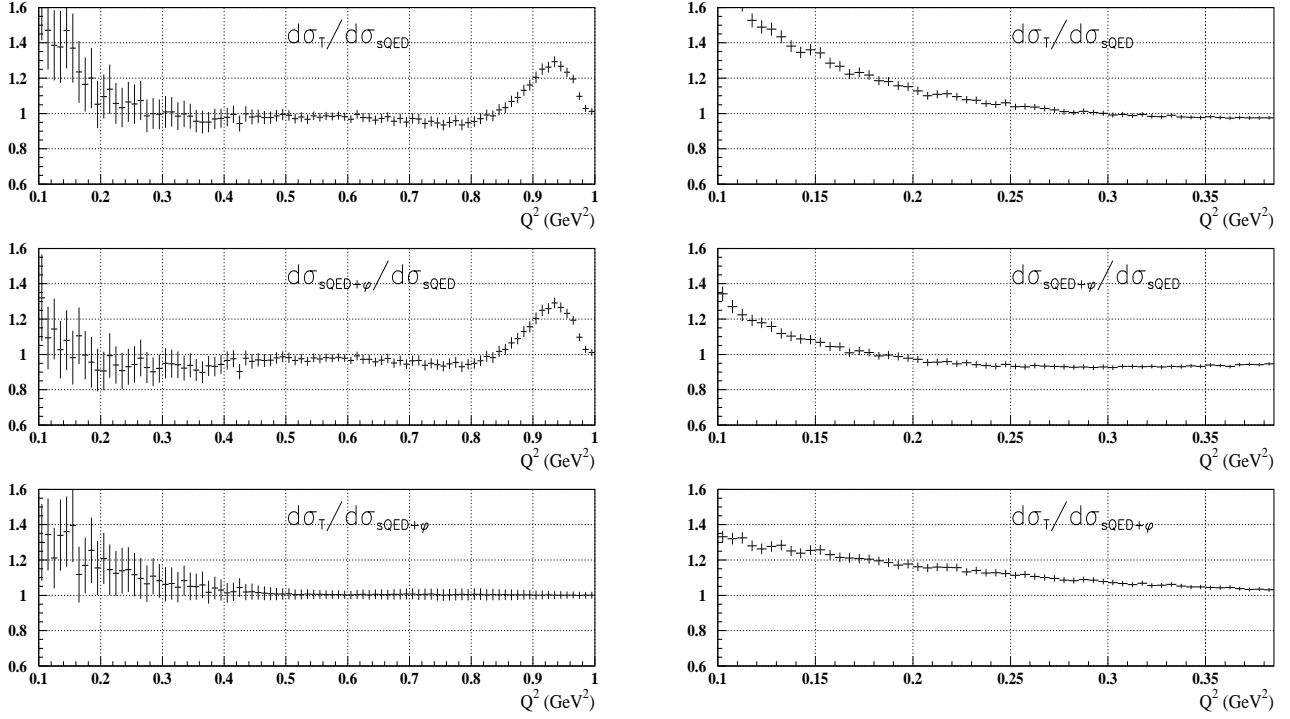


Figure 3: The relative value of the different contributions to the cross section $e^+e^- \rightarrow \pi^+\pi^-\gamma$ process, see Eq. (11), for $s = m_\phi^2$ and with cuts (13). The left figure corresponds to the entire Q^2 region, the right one is for the low Q^2 region.

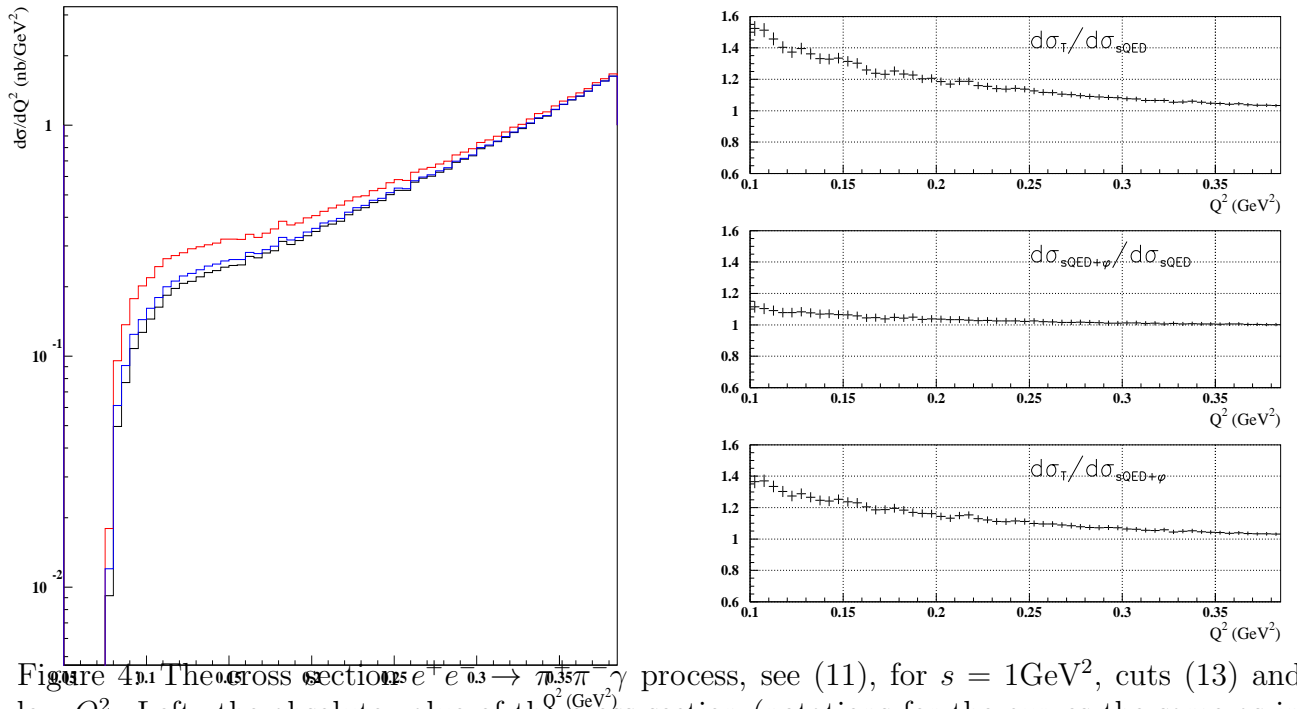


Figure 4: The cross section for the $e^+e^- \rightarrow \pi^+\pi^-\gamma$ process, see (11), for $s = 1\text{GeV}^2$, cuts (13) and low Q^2 . Left: the absolute value of the cross section (notations for the curves the same as in Fig.2). Right: the relative value of the different contributions to the cross section (11).